Observation of Audio-Frequency Edge Magnetoplasmons in the Classical Two-Dimensional Electron Gas

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The electrical admittance of a two-dimensional electron gas on liquid helium measured at audio frequencies ω is observed to oscillate as a function of magnetic field at strong magnetic fields. The oscillations can be attributed to the propagation of very-low-frequency ($\omega\tau \sim 10^{-6}$, τ scattering time; $\omega/\omega_c \sim 10^{-8}$, ω_c cyclotron frequency) edge magnetoplasmons. The directly determined dispersion relation agrees with theory and quantitatively with measurements in the collisionless regime ($\omega\tau \gg 1$). The attenuation, theoretically obtained by incorporating the screening in a simple local-capacitance model, agrees well with experiments.

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The two-dimensional electron gas (2DEG) on liquid helium has played an important role in clarifying the collective excitations (plasmons) of 2DEG's in general. The first experimental verification of the gapless plasmon dispersion in two dimensions (2D) was made in this system [1] and, more recently, an unexpected edge magnetoplasmon (EMP) excitation was discovered [2,3]. It is now realized that the first observation of an EMP was actually made for a small 2DEG (3 μm) in a GaAs-AlGaAs heterojunction [4]. The EMP is the 2D analog of a surface plasmon in three dimensions (3D) [5]. The peculiar properties are that it propagates around the edge of the sample at a frequency which is much lower than the cyclotron frequency (which is the minimum frequency for a normal plasmon) and which decreases with increasing magnetic field. In recent years, this new mode has attracted a great amount of interest, both theoretically and experimentally (for a review, see Ref. [6]).

The EMP has been observed in large size (~ 1 cm) 2DEG's in GaAs-AlGaAs heterojunctions at radio, microwave, and far-infrared (FIR) frequencies, under conditions of the integer quantum Hall effect (QHE) [7–10] and fractional quantum Hall effect [11]. These EMP's exist in the FIR down to sample sizes where quantum confinement of the electrons is important [12]. Very recently, the one-dimensional magnetoplasmon, which is closely related to the two-dimensional edge magnetoplasmon, was observed [13] in a GaAs-AlGaAs quantum wire.

In this Letter, it is shown that the edge magnetoplasmon exists in the 2DEG on liquid helium down to extremely low, audio, frequencies ω (~ 1 kHz), with $\omega\tau$ values (τ is the scattering time) down to 10^{-6}, about 4 orders of magnitude smaller than in any previous experiment. Theoretically [6,14], it is emphasized that EMP's should exist for frequencies $\omega\tau \ll 1$, and it is now clear [14] why, in early experiments [2] at $\omega\tau \lesssim 1$, the EMP damping was anomalously small. A key element to explain the low damping is that the EMP is carried by the

Hall currents, which in a strong magnetic field are almost perpendicular to the electric fields. This explains qualitatively why low-frequency ($\omega\tau \ll 1$) EMP's exist at (fractional) quantum Hall plateaus, where the Hall angle is very close to 90°, but rapidly damp out outside the plateaus [9,11,15]. The experimental situation with regard to damping is not so clear and the data are sometimes contradictory [8,15]. For the present data, taken on the nondegenerate 2DEG on helium, the QHE plays no role at all.

For the present work, data were taken on both high-temperature (1.8 K), low-mobility ($\mu \sim 2.5$ m^2/Vs) samples and low-temperature (0.87 K), high-mobility ($\mu \sim 3 \times 10^2$ m^2/Vs) samples, in the fluid phase in both cases, well above the electron crystallization temperature. For the low-mobility sample, the damping length is shorter than the sample circumference and the phase and amplitude of the wave are measured directly along the perimeter. This provides a novel way to directly measure the dispersion relation $\omega(q)$ (q wave vector) and its dependence on density n and magnetic field B. For the high-mobility sample, the fundamental EMP resonance is observed.

For the experiments at 1.8 K, an electrode array (see inset to Fig. 1) was placed a distance d (~ 0.5 mm) below the helium surface. The outer perimeter and a guard electrode have a circular geometry, so that the charge sheet is circular. The sheet diameter was calculated for given potentials on the confining electrodes [16]. One of the small electrodes around the perimeter was driven with an ac voltage V_0 and a current was detected at any one of the others. This was done with a capacitance bridge, so that the complex ac admittance $Y = I/V_0$ between two electrodes was measured, but the same results were also obtained with a lock-in amplifier.

A representative set of data for the components of $Y \equiv G + j\omega C$ between electrodes 1 and 5 is shown in Fig. 1. Similar data were taken using other electrode combinations. The phase angle ϕ of Y is defined as ϕ

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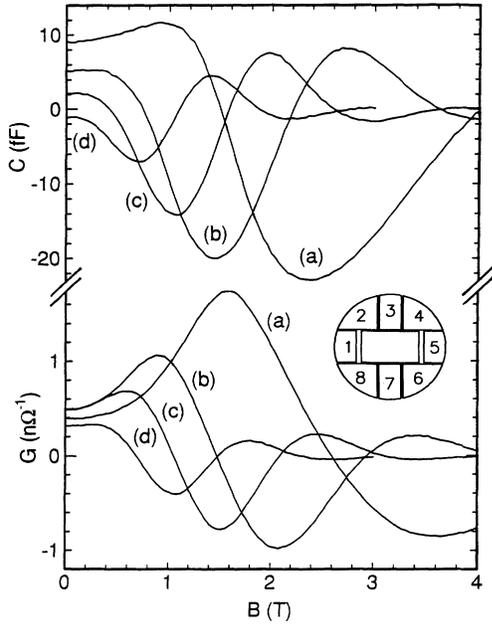
FIG. 1. The components C and G of the complex admittance $Y = G + j\omega C$ between electrodes 1 and 5 vs magnetic field, for the circular geometry with electrode configuration as shown in the inset. Data are shown for densities of (a) 3.5, (b) 1.8, (c) 1.2, and (d) 0.9 (in units of 10^{11} m^{-2}) as estimated from the dc charging potentials. The temperature was 1.8 K, the measuring frequency $\omega/2\pi$ was 10 kHz, the outer diameter of the electrode configuration was 19 mm, and the calculated diameter of the charge sheet was 15 mm.

$= \tan^{-1}(G/\omega C)$. For $\phi \ll 1$, the resistance of the electron sheet is proportional to ϕ [17]. In Fig. 2, the phase shifts at a fixed, large magnetic field and a fixed density and frequency are plotted, as measured at all electrodes around the perimeter as a function of their distance s from the driven electrode along the sheet perimeter. It follows that $\phi = q(\omega, B, n)s$ (ignoring the nonessential vertical shift). Note that ϕ increases linearly around the edge, over more than three-quarters of the perimeter. Also plotted in Fig. 2 is the (normalized) magnitude of the signal, which decreases with s, but slower than $\exp[-\phi(s)]$. With the direction of the magnetic field reversed, the same picture emerges if the sequence of edge electrodes is traversed in the opposite sense.

It thus follows that for sufficiently strong fields the current is confined to the edge of the sheet and is given by $I = I_0 \exp[-i(\omega t - qs) - q's]$. This is a well-defined traveling wave with a damping length $1/q'$ larger than $1/q$. The propagation direction is determined by the direction of the magnetic field. These are all the characteristics of an edge magnetoplasmon.

As $q = \phi/s$, the data to obtain $q(\omega, B, n)$ are shown in Fig. 3. At large fields, the phase varies linearly in B. The slopes of the linear portions are proportional to ω and inversely proportional to n. The experimentally determined
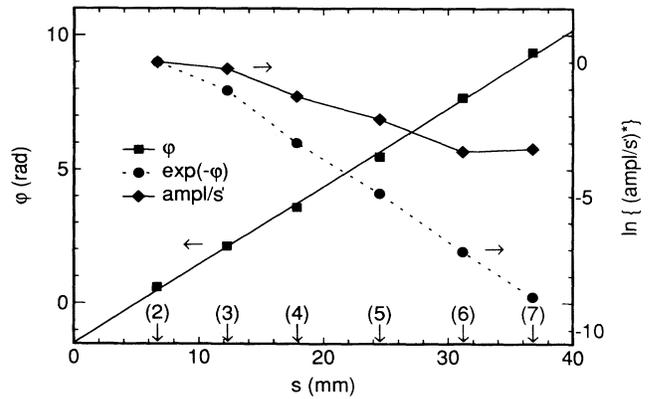
FIG. 2. Left scale: the phase ϕ of the current $[=\tan^{-1}(G/\omega C)]$ measured at successive electrodes (see inset to Fig. 1) as a function of electrode position s measured along the perimeter of the electron sheet, for fixed magnetic field (3 T), density $(2.6 \times 10^{11}$ m$^{-2})$, and frequency (10.1 kHz). The center positions of the electrodes are indicated. Right scale: natural logarithm of the amplitude of the signal, divided by the width s' of the electrodes along the sheet perimeter, vs electrode position. For comparison, the function $\ln\{\text{const} \times \exp[-\phi(s)]\}$ adjusted arbitrarily to the data at electrode 2 is also shown.

dispersion relation at large fields can hence be written as

$$\omega = \alpha(ne/\epsilon\epsilon_0 B)q = \alpha(\sigma_{xy}/\epsilon\epsilon_0)q , \qquad (1)$$

where σ_{xy} is the off-diagonal component of the conduc-
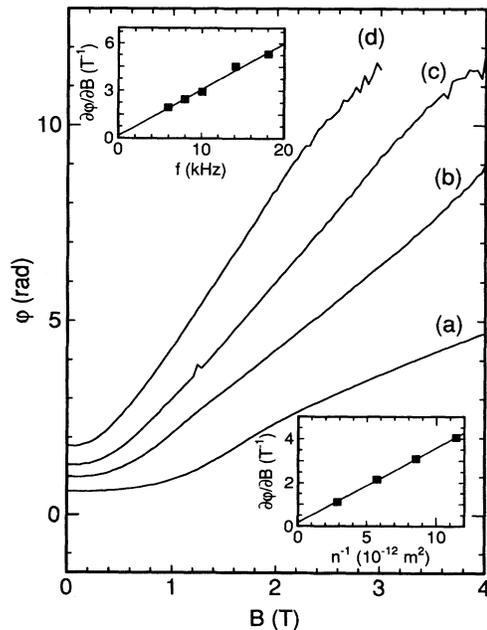
FIG. 3. The phase ϕ vs magnetic field, for the same data as Fig. 1; the curve labels are the same as for Fig. 1. Insets: The slopes of the high-field regions as a function of frequency f $(=\omega/2\pi)$ and of inverse density. The temperature is 1.8 K.

tivity tensor which is equal to ne/B in large fields, $\epsilon\epsilon_0$ the dielectric permittivity of liquid helium, and α a dimensionless constant. The experimental value for α is $\alpha = 0.25 \pm 0.05$.

Measurements for the high-mobility samples at 0.87 K were taken in a somewhat different rectangular geometry, shown in the inset to Fig. 4. Electrode A was excited at frequencies of 50.4 and 74.4 kHz and the differential Hall voltage between electrodes $H1$ and $H2$ measured with a lock-in amplifier. Figure 4 shows the squared magnitude of this voltage $|V|^2$ as a function of magnetic field up to 6.6 T. The initial rise at low fields is the normal Hall voltage. The voltage then decreases as the electron currents become confined to the edges, giving the first peak in $|V|^2$. As the field was increased further a resonance was observed at a field which is inversely proportional to ω, in agreement with (1). Because of the high mobility the damping length is now longer than the sample circumference L (48.8 mm) of the electron sheet and a traveling-wave resonance occurs. The peak is interpreted as the fundamental EMP resonance mode with $q = 2\pi/L$. Measurements of the relative phase of the Hall voltage and of the current to electrode B (which also showed resonance) indicate that this mode corresponds to a charge configuration with excess and deficient charge at opposite boundaries of the sample (dipolar mode), which rotates at the frequency ω. Inserting the measured resonance positions and $q = 2\pi/L$ in Eq. (1), we find $\alpha = 0.38$, in reasonable agreement with the value for the low-mobility samples, derived in another way.

Detailed theories for EMP's have been worked out by Fetter [5] and Volkov and Mikhailov [6,14]. The latter authors also consider the damping, which was ignored by Fetter. For most cases, Volkov and Mikhailov obtain a
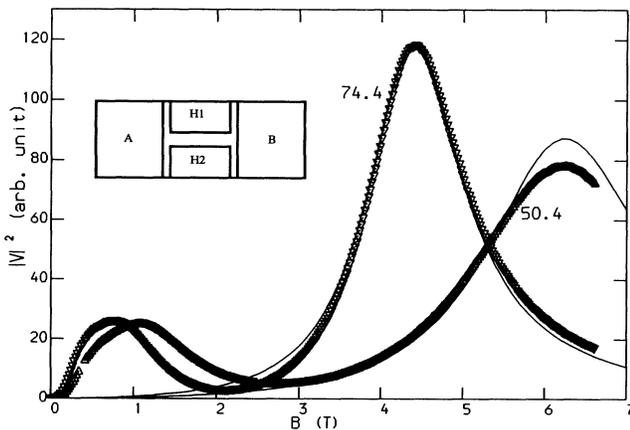
FIG. 4. The squared amplitude of the ac Hall voltage for a rectangular geometry (see inset; dimensions: 17.4×7.0 mm^2) as a function of magnetic field at frequencies of 50.4 and 74.4 kHz at a temperature of 0.87 K. The electron density was 2.8×10^{12} m^{-2}. The solid lines show calculations of the resonant response.

dispersion which has precisely the form (1). The relevant length of the problem, which appears in the expression for α, is given, for the case of electrons on helium, by the transition layer h over which the charge density falls to zero (of the order of d, ~ 0.5 mm). In the notation of Volkov and Mikhailov then $\alpha = F_3(h/d)/2\pi$ (Ref. [6], Eq. 49), where the function F_3 describes the screening by the metal electrodes and is of order unity. They also deduced the constants α from the earlier high-frequency experiments and found $\alpha \simeq 0.46$ for the experiments of Glattli et al. [3] and $\alpha \simeq 0.40$ for the experiments of Mast, Dahm, and Fetter [2]. These values are close to the values found in the present work ($\alpha \simeq 0.25$ and 0.38).

To analyze the present experiments, a more simple model for the EMP's has been developed using a local-capacitance approximation, which is appropriate for the present case of screening by two metallic electrodes. Recently, such a model [18] was used to find the response of the electron sheet assuming rigid boundaries [19]. It was found that at large fields a voltage wave is excited at the edge of the sheet that decays exponentially around the perimeter. The propagation constant for this case has equal real and imaginary parts and is given by

$$k_\| = (1-j)/\delta_\| \quad \text{with} \quad \delta_\| = (2/\omega C_s \rho_{xx})^{1/2}, \qquad (2)$$

with C_s the capacitance per unit area and ρ_{xx} the diagonal component of the magnetoresistivity tensor. Volkov and Mikhailov obtain, in the limit $h \ll d$ and for $\omega\tau \ll 1$, the expression $\omega(q) = -iq^2d/2\epsilon\epsilon_0\rho_{xx}$ (Ref. [6], Eq. 44b). With $C_s = 2\epsilon\epsilon_0/d$ and $q = k_\|$, this is exactly the same expression as Eq. (2). This model (Ref. [18]) has now been extended to include the finite transition layer h, of order d, by assuming an edge capacitance C_e (dimension F/m) at the boundaries of the sheet. The previous boundary condition that no current flows perpendicular to the boundary is now relaxed, because current may flow in the edge capacitance. The existence of an edge capacitance was previously shown by the calculations of Mehrotra [16]. The effect of the edge capacitance is to increase the real part of the propagation constant by $\Delta k_\| = C_e\omega B/ne$, which is equivalent to (1) if we take $C_e = \epsilon\epsilon_0/\alpha$. In the general case we obtain for the propagation constant along the boundary

$$k_\| = (1-j)/\delta_\| + C_e\omega B/ne. \qquad (3)$$

For large fields, $C_e\omega B/ne \gg 1/\delta_\|$ so that $\mathrm{Re}(k_\|) \gg \mathrm{Im}(k_\|)$ which indicates a propagating wave. The spatial width of this edge mode in a large sample is given by $\delta_\perp = (\rho_{xx}/\rho_{xy})\delta_\|$, with ρ_{xy} the off-diagonal component of the resistivity tensor. In high fields the width of the EMP will be the larger of $\delta_\perp$ and h, as discussed above. In the Drude model, $\delta_\|$ is independent of field whereas $\delta_\perp$ decreases as $1/B$. It should be noted that the expression for σ_{xy}, unlike ρ_{xx} and σ_{xx}, remains valid even for quantizing fields $\mu B \gg 1$, $\hbar\omega_c \gg kT$ [19,20]. In the present experiments with the low-mobility samples very low densities

($\sim 10^{11}$ m^{-2}) were used. For high densities the first term dominates in the field range used, as was verified from the frequency dependence of $\delta_{\parallel}$ ($\omega^{-1/2}$) [21]. In addition, we analyzed only data that exhibited large phase shifts ($> \pi$) with still appreciable amplitude, which assures the dominance of the second term. Because the data in Fig. 3 nearly extrapolate to the origin, the real part of the first term of (3) seems smaller than expected from (2), for reasons that are not yet understood.

Based upon the dispersion relation (3) a fit to the perimeter wave resonance has been made, shown as solid lines in Fig. 4. The experimentally determined magnetoresistance [19] $\rho_{xx} \propto B^2$ was used and the parameters were fitted to the peak at 4.3 T. The line shapes and peak positions are well reproduced by the calculations. The effective spatial width of the edge mode was estimated, from the magnitude of the current to electrode B at resonance, to be 0.6 ± 0.05 mm. As expected, this is comparable to the helium depth of 0.42 mm and much smaller than the sheet dimensions of 17.4×7.0 mm^2.

In conclusion, it is shown experimentally here that currents at audio frequencies in a 2DEG on helium in large magnetic fields ($\mu B \gg 1$) flow around the edges only, in the form of a relatively weakly damped propagating wave. The observation of edge magnetoplasmons at extremely low frequencies, $\omega\tau$ down to 10^{-6}, ω/ω_c down to 10^{-8}, convincingly shows that their existence does not depend on $\omega\tau$, but on $\omega_c\tau$ or σ_{xy}/σ_{xx}. An EMP will propagate on a screened 2DEG for $\omega_c\tau > 1$ as soon as the damping length $\delta_\perp$ (which decreases with field) becomes smaller than the width of the electron sheet. The mode will be heavily damped until $\delta_\perp < 2d/a$, when the last term in the dispersion relation (3) will dominate. It is emphasized that the phenomena observed here are entirely classical: The 2DEG is nondegenerate and, conceptually, the data are understood within the Drude transport model, although quantum effects enter at high fields through σ_{xx} and ρ_{xx}. There exist, however, some marked analogies with the quantum 2DEG in the quantum transport model [22]. There the currents, contributing to the external currents in the contacts, flow along the edges only, even at dc, in opposite directions at opposite faces of the sample. The mean free path for inelastic scattering is extremely large [23].

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