Kinetic Simulation of a Collisional Shock Wave in a Plasma

Michel Casanova and Olivier Larroche
Commissariat à l'Energie Atomique–Centre d'Etudes de Limeil-Valenton, 94195 Villeneuve St. Georges CEDEX, France

Jean-Pierre Matte
Institut National de la Recherche Scientifique-Energie, 1650 Montée Ste. Julie, CP 1020, Varennes, Québec, Canada J3X 1S2
(Received 14 March 1991)

The ion kinetic structure of a planar collisional shock front in a fully ionized plasma is investigated using a new Vlasov-Fokker-Planck code. The effects of ionic viscosity and ionic thermal conduction are found to be much larger than assumed in usual hydrodynamic plasma simulations with classical transport coefficients. This might have consequences on the numerical modeling of inertial-confinement fusion targets.

PACS numbers: 52.35.Tc, 52.25.Fi, 52.65.+z

There is a growing concern for ion kinetic effects beyond the Chapman-Cowling [1] approximation in laser-produced plasmas [2–5] where steep density and temperature gradients occur. The usual ionic thermal conductivity and ionic viscosity coefficients used in the fluid description of plasmas, which are strictly valid only when the gradient scale lengths are much larger than the mean free path for collision between plasma particles, might be in error in these regions. Similar effects in the case of electronic thermal conduction have been investigated recently [6,7]. Possible applications include inertial-confinement fusion [2,3] (ICF) and x-ray-laser studies [4].

In this Letter, we focus on a well-defined problem of plasma physics in which ion-ion collisions are important, namely, a plane collisional shock wave propagating in a fully ionized, homogeneous, unmagnetized plasma. We neglect radiation effects which, for a low Z, play a role on distances much longer than the width of the collisional shock structure [8]. The shock wave problem plays a crucial part in the optimization of temporal driver pulse shaping and target design in ICF [9].

The classical picture [10] of a collisional plasma shock wave is obtained in the frame of two-fluid plasma theory, using the transport coefficients of Spitzer and Härm [11] and Braginskii [12]. The main features of the shock structure are shown in Fig. 1. Regions 1 and 2 are supposed to be in full equilibrium and are connected by the well-known Rankine-Hugoniot relations. The ionic velocity and temperature change appreciably over a few ionic mean free paths, which violates the validity condition of the transport coefficients. It is thus necessary to investigate the shock structure in the frame of kinetic theory.

Former kinetic studies of the plasma shock structure were performed by Tidman [13] and Abe and Sakaguchi [14]. They obtained a solution of the Fokker-Planck equation governing the ionic distribution function in the form of a sum of two Maxwellians (the Mott-Smith ansatz [15]). However, this ansatz, which was originally used for neutral gases, might not be applicable to a plasma because collision cross sections strongly depend on particle velocity. Moreover, Ref. [13] neglects electron thermal conduction, which is certainly expected to be wrong: Because of the small electron-to-ion mass ratio, the electronic thermal conductivity is expected to be much larger than the ionic thermal conductivity (which is automatically taken into account in the kinetic treatment of the ions). Reference [14], on the other hand, uses the infinite electronic conductivity limit leading to a constant electronic temperature over the shock structure; this is valid in the vicinity of the ionic shock front (zone I in Fig. 1), but then the boundary conditions at the ends of this zone [16,17] are *not* the Rankine-Hugoniot relations. In particular, due to the electron-ion temperature relaxation in region E, the local Mach number in the vicinity of zone I always remains finite ($M < 2.66$ according to Ref.

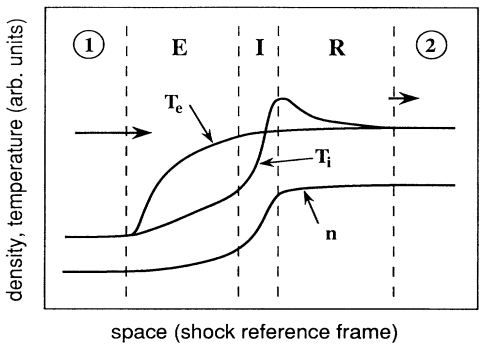

FIG. 1. Qualitative plot of the density (n) and the electronic (T_e) and ionic (T_i) temperatures across the shock structure in the frame of two-fluid plasma theory. I denotes the ionic shock front, whose width is of the order of the ion-ion mean free path λ_{ii}; R is the electron-ion temperature equilibration zone, of width $\approx (m_i/m_e)^{1/2}\lambda_{ii}$, connecting the ionic shock to the downstream equilibrium zone 2; E is the electronic preheating zone, of comparable width, extending up to the upstream equilibrium zone 1. The direction and amplitude of the flow velocity are illustrated by the horizontal arrows. Zone I has been strongly widened for clarity.

[17]), in contradiction to what is assumed in Ref. [14].

To go beyond these results, we computed the full shock structure using a kinetic treatment of the ions without any *a priori* assumption about the ionic distribution function, and taking electronic thermal conductivity into account in a more satisfactory way. We start from the full electronic and ionic Fokker-Planck equations, written in dimensionless form. The units of time and distance are the values, in region 2, of the ion-ion collision time,

$$\tau_{ii}=\frac{(k_BT_i)^{3/2}m_i^{1/2}}{4\pi e^4Z^4n_i\ln\Lambda_{ii}},$$

and the ionic mean free path $\lambda_{ii}=(k_BT_i/m_i)^{1/2}\tau_{ii}$. In the case (of interest to ICF) of the laser-driven implosion of a hollow solid-density D-T shell with typical parameters from Ref. [18], using the self-similar solution of Ref. [9] we obtain the following temperature and density values behind the reflected shock 160 ps after void closure: $k_BT_i\approx 5.5$ keV and $n_i\approx 4.0\times10^{23}$ cm^{-3}, which leads to $\lambda_{ii}\approx 0.35$ μm, $\tau_{ii}\approx 0.76$ ps (the Coulomb logarithm [19] is $\ln\Lambda_{ii}\approx 8.2$). From our results described below, the shock width is expected to be $\approx 200\lambda_{ii}$ and is thus comparable to the radius of the reflected shock (≈ 67 μm) at that time, which shows that kinetic effects must be taken into account at least up to that time. Let us point out that the results discussed in this Letter remain qualitatively valid (only with different time and length scales) on a broad range of values of T_i and n_i (several decades) since for a given Mach number they only depend on the value of the Coulomb logarithm which is a slow function of T_i and n_i. With the above figures, the Debye length is very small, $\lambda_D/\lambda_{ii}=6.1\times10^{-4}$, which allows us to assume quasineutrality, $n_e=n_i$.

The dimensionless distribution function $f_a(\mathbf{r},\mathbf{v})$ for particles of species a in six-dimensional phase space is expressed from the dimensional distribution $F_a(\mathbf{R},\mathbf{V})$ in terms of the corresponding thermal velocity $v_{Ta}=(k_BT_a/m_a)^{1/2}$ and density n_a (evaluated in region 2):

$$f_a\left(\mathbf{r}=\frac{\mathbf{R}}{\lambda_{ii}},\mathbf{v}=\frac{\mathbf{V}}{v_{Ta}}\right)=\frac{v_{Ta}^3}{n_a}F_a(\mathbf{R},\mathbf{V}).$$

We then expand the Fokker-Planck equations [20] in powers of the small parameter $\varepsilon=(m_e/m_i)^{1/2}$; m_e/m_i is the electron-to-ion mass ratio (see Ref. [21]). Keeping orders 0 and 1, the ionic equation becomes

$$\frac{\partial f_i}{\partial t}+v_x\frac{\partial f_i}{\partial x}-\frac{1}{n_i}\frac{\partial P_e}{\partial x}\frac{\partial f_i}{\partial v_x}=\left(\frac{\partial f_i}{\partial t}\right)_{ii}+\frac{4\pi\varepsilon n_i\ln\Lambda_{ei}}{3(2\pi T_e)^{3/2}}\frac{\partial}{\partial v_a}\left[(v_a-u_a)f_i+T_e\frac{\partial f_i}{\partial v_a}\right]. \quad (1)$$

The first term on the right-hand side is the full classical [20] ion-ion collision term, $u_a=(u_x,\ u_y=0,\ u_z=0)$ is the mean ion velocity. Summation on the repeated index α is assumed in the temperature relaxation term on the right-hand side of Eq. (1). The problem is cylindrically symmetric about the x axis. To this order in ε the sum of the electric field and the electron-ion collisional drag is equal to the electronic pressure gradient term $-(1/n_i)\,\partial P_e/\partial x$, where $P_e=n_eT_e$. The electronic equation reads

$$\frac{3}{2}\left(\frac{\partial T_e}{\partial t}+u_x\frac{\partial T_e}{\partial x}\right)+\frac{\partial u_x}{\partial x}T_e=\frac{1}{n_i}\frac{\partial}{\partial x}\left(K\frac{\partial T_e}{\partial x}\right)+\varepsilon\left(\frac{2}{\pi}\right)^{1/2}n_i\ln\Lambda_{ei}\frac{T_i-T_e}{T_e^{3/2}}. \quad (2)$$

The reduced temperatures T_e and T_i are the ratio of the dimensional temperatures to the temperature in region 2. See Ref. [19] for the definition of $\ln\Lambda_{ei}$. The electronic Fokker-Planck equation is reduced to the heat equation (2) due to the small electron-electron collision time $\tau_{ee}\approx\varepsilon\tau_{ii}$, so that a first-order Legendre polynomial expansion of f_e is sufficient [6] (f_e is nearly Maxwellian). It will be checked that the electronic heat flux is weak enough to use the Spitzer and Härm [11] conductivity, which reads for $Z=1$, in dimensionless units,

$$K\approx 12.04T_e^{5/2}/\varepsilon\ln\Lambda_{ei}.$$

Equations (1) and (2) are numerically solved with a new time-dependent code whose main features are the following: The ionic distribution function is discretized in the coordinate space $(x,v_x,v_\perp)$; the full Rosenbluth potentials [20] are computed directly (without expanding in Legendre polynomials) so that f_i is allowed arbitrary deviations from the local Maxwellian. Equation (2) is solved by an iterative Crank-Nicholson scheme to insure stability although the time step is much larger than the characteristic time of evolution of T_e through electronic heat conduction. An additional hydrodynamic code yields a stationary fluid solution of the problem which provides both initial and boundary conditions for the kinetic code. In the case of the stationary shock problem, the boundary conditions obey the classical Rankine-Hugoniot relations.

For definiteness we discuss here the case $M=5$; M is the usual Mach number relating fluid quantities in regions 1 and 2 (see Fig. 1). Starting from the fluid solution, the system first exhibits a transient behavior displaying a beamlike structure in the upstream region similar to that described in Ref. [21]. Then, after approximately 100 ion-ion collision times the solution reaches the stationary state displayed in Figs. 2 and 3.

Figure 2 shows a contour plot of the parallel ionic distribution function

$$f_{i\parallel}(x,v_x)=\int_0^\infty f_i(x,v_x,v_\perp)2\pi v_\perp\,dv_\perp.$$

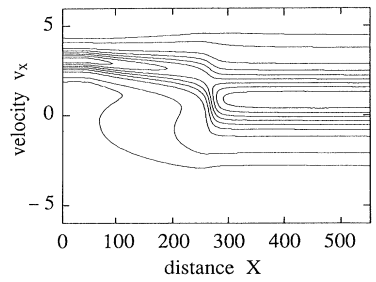

FIG. 2. Contour plot of the parallel ionic distribution function $f_{i\parallel}(x,v_x)$ across the shock structure in reduced units for a Mach number $M=5$. The values of $f_{i\parallel}$ on the three lowest contours, starting from the high velocity regions, are 5×10^{-4}, 5×10^{-3}, and 5×10^{-2}; on subsequent contours $f_{i\parallel}$ increases by the constant amount 5×10^{-2}.

The distribution is strongly distorted all over the preheating region (zone E in Fig. 1), which shows that the assumption of a short ionic shock front with constant T_e made in Refs. [14] and [21] is not satisfactory. However, beamlike features are much smoother than in the transient stage, and far less prominent than what was found in Ref. [14], which shows that the Mott-Smith ansatz used by Abe and Sakaguchi is strongly in error (see their Fig. 2). The acceleration of ions toward the upstream region arises from the balance between the collisional drag due to the bulk of the ionic distribution and the accelerating force $E=-(1/n_i)\partial P_e/\partial x$ due to the electric field plus the collisional drag due to the electrons. This effective field and the associated potential

$$\varphi(x)=\int_0^x E(x_1)dx_1$$

(normalized to k_BT_i in region 2) are plotted in Fig. 3(c). φ is strong enough in the vicinity of the ionic shock front to reflect a non-negligible number of incoming ions.

Figure 3 shows various macroscopic quantities obtained for $M=5$. The ionic heat flux and the ionic temperature anisotropy (related to ionic viscosity) are seen to be much larger than what would be inferred from the temperature and velocity profiles [Figs. 3(b) and 3(d)] using the formulas of Braginskii [12]. The deviation is stronger than in the case of electronic heat flow in steep temperature gradients [6,7]. This is due to the strong density gradient present in our case. Also, the strong temperature anisotropy enhances the effects of steep gradients, which does not happen in the case of electrons because electronic viscosity is negligible. As a result, the ionic heat flux is comparable to the electronic heat flux and cannot be neglected, contrary to what is often assumed in fluid codes. We check that for $M=5$ the electronic heat flux is a few percent of the free-streaming value $n_ev_{Te}k_BT_e$ almost everywhere, but reaches 10% near the foot of the electronic temperature profile [see Fig. 3(d)] where the gradient is the steepest. This allows us to use the Spitzer and Härm thermal conductivity, with possible errors restricted to a narrow zone far from the ionic shock front.

It is necessary to examine whether the strongly non-Maxwellian distributions found may lead to plasma instabilities [5,22,23]. According to Ref. [22], the fastest one should be the ionic two-stream instability driven by the

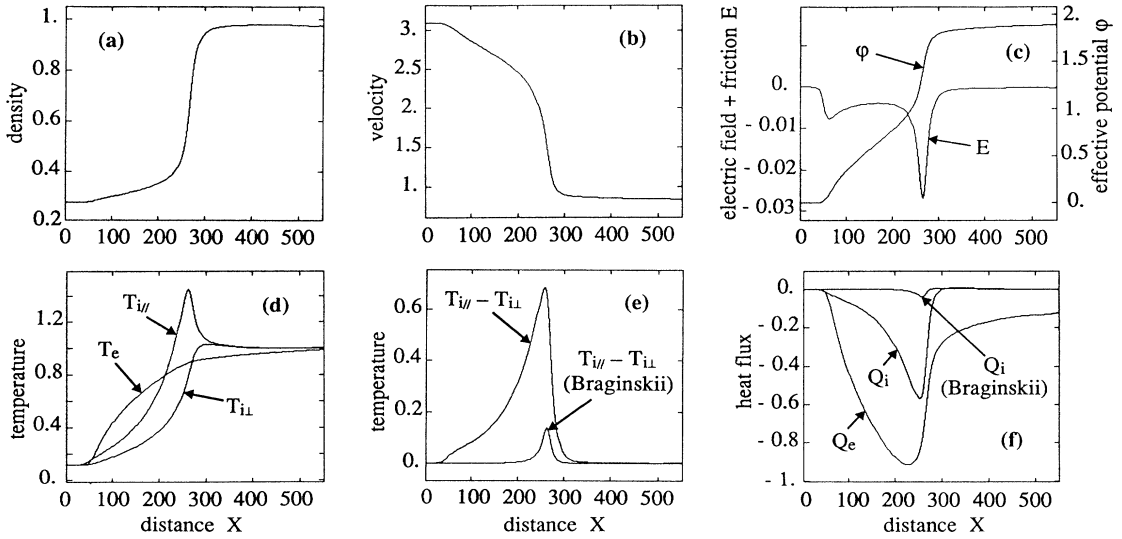

FIG. 3. The following fluid quantities are plotted in reduced units vs reduced distance across the shock structure for $M=5$: (a) density; (b) velocity; (c) electric force exerted on the ions plus collisional drag due to the electrons (E), together with the integral of E over space (φ); (d) electronic (T_e), parallel ionic ($T_{i\parallel}$), and perpendicular ionic ($T_{i\perp}$) temperatures; (e) ionic temperature anisotropy together with the corresponding value deduced from the velocity gradient in (b) using Braginskii's ionic viscosity; and (f) electronic (Q_e) and ionic (Q_i) heat fluxes together with the value of Q_i found using Braginskii's ionic thermal conductivity.

ion beam, which was examined in Ref. [14] in the case of a wave number parallel to the shock velocity. A preliminary analysis of our results, taking electronic Landau damping into account, indicates that this instability is not excited anywhere in the shock structure, contrary to what was found in Ref. [14]. This is due to the less prominent beam structure found and also to the fact that the growth rate is computed in Ref. [14] using much too high values of the ionic temperature jump across the ionic shock front. However, the question of stability in its full extent, with a possible transition to a collisionless magnetized shock, is beyond the scope of the present work.

In summary, we find a large enhancement of ion heat flow and viscosity effects in shock waves with respect to classical [10] fluid results, which is expected to occur unless the Mach number is close to 1. As a result, the effective shock width is comparable to the width of the electronic preheating layer rather than to the ionic mean free path, which means a several hundredfold increase over the classically admitted value. This should have important consequences for the numerical simulation of double-foil x-ray-laser experiments [4,5] and ICF target-implosion experiments [2] where shock waves play a crucial part [9]. Furthermore, nuclear reactivities [3] might be affected by the strongly altered ionic distributions found, but to reach a definite conclusion for D-T pellets it is necessary to shift to spherical geometry and to add a second ionic species. More refined simulations in specific cases are left over for future work.

We gratefully acknowledge stimulating discussions with Professor M. G. Haines and Dr. A. R. Bell, Dr. O. Daube, Dr. P. Mora, Dr. D. Pesme, Dr. W. Rozmus, and Dr. J. Virmont at the 1989 and 1990 Centre Européen de Calcul Atomique et Moléculaire workshops on laser-matter interaction and transport in Orsay (France). One of us (J.P.M.) also acknowledges support from NSERC (Canada) and the MEQ (Québec).

[1] S. Chapman and T. G. Cowling, *The Mathematical Theory of Nonuniform Gases* (Cambridge Univ. Press, London, 1970).

[2] T. Yabe and K. A. Tanaka, Laser Part. Beams **7**, 259 (1989)

[3] T. Nishikawa, H. Takabe, and K. Mima, Jpn. J. Appl. Phys. **28**, 2004 (1989); A. G. Petschek and D. B. Henderson, Nucl. Fusion **19**, 1678 (1979).

[4] R. Weber, P. F. Cunningham, and J. E. Balmer, Appl. Phys. Lett. **53**, 2596 (1988); T. Boehly *et al.*, Appl. Phys. B **50**, 165 (1990).

[5] R. L. Berger *et al.*, Phys. Fluids B **3**, 3 (1991).

[6] J.-P. Matte and J. Virmont, Phys. Rev. Lett. **49**, 1936 (1982); J. R. Albritton, Phys. Rev. Lett. **50**, 2078 (1983); A. R. Bell, Phys. Fluids **28**, 2007 (1985).

[7] J.-F. Luciani, P. Mora, and R. Pellat, Phys. Fluids **28**, 835 (1985).

[8] V. S. Imshennik, Fiz. Plazmy **1**, 202 (1975) [Sov. J. Plasma Phys. **1**, 108 (1975)].

[9] J. Meyer-ter-Vehn, in *Laser-Plasma Interactions 3,* edited by M. B. Hooper (SUSSP, Edinburgh, 1986), p. 215.

[10] M. Y. Jaffrin and R. F. Probstein, Phys. Fluids **7**, 1658 (1964).

[11] L. Spitzer and R. Härm, Phys. Rev. **89**, 977 (1953).

[12] S. I. Braginskii, in *Reviews of Plasma Physics,* edited by M. A. Leontovich (Consultants Bureau, New York, 1965), Vol. 1, p. 205.

[13] D. A. Tidman, Phys. Rev. **111**, 1439 (1958).

[14] K. Abe and G. Sakaguchi, Phys. Fluids **28**, 3581 (1985).

[15] H. M. Mott-Smith, Phys. Rev. **82**, 885 (1951).

[16] V. D. Shafranov, Zh. Eksp. Teor. Fiz. **32**, 1453 (1957) [Sov. Phys. JETP **5**, 1183 (1957)].

[17] M. S. Grewal, Phys. Fluids **16**, 561 (1973).

[18] J. D. Lindl, in *Inertial Confinement Fusion,* edited by A. Caruso and E. Sindoni (Editrice Compositore, Bologna, 1988), p. 595.

[19] D. V. Sivukhin, in *Reviews of Plasma Physics,* edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 4, p. 93.

[20] M. N. Rosenbluth, W. M. MacDonald, and D. L. Judd, Phys. Rev. **107**, 1 (1957).

[21] M. Casanova, O. Larroche, and J.-P. Matte, in "Rayleigh-Taylor Instabilities, Thermal Smoothing and Interactions in Laser Plasmas," 1989 CECAM Workshop Report, Orsay, 1990 (unpublished), p. 25.

[22] T. E. Stringer, J. Nucl. Energy C **6**, 267 (1964); N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973).

[23] D. A. Tidman and R. A. Shanny, Phys. Fluids **17**, 1207 (1974); E. M. Epperlein, T. H. Kho, and M. G. Haines, Plasma Phys. Controlled Fusion **28**, 393 (1986).