New Test of Nonsymmetric Theories of Gravity: Observational Limits on Gravity-Induced Depolarization of Solar Spectral Lines

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We show that theories which couple the antisymmetric part of a nonsymmetric-tensor gravitational field to the electromagnetic field predict a depolarization of the Zeeman components of spectral lines emitted by extended, magnetically active regions near the limb of the Sun. This new effect is a consequence of the violation of the Einstein equivalence principle by these nonsymmetric alternatives to general relativity. We show that existing solar-physics data limit the extent of such depolarization and imply that the Sun's antisymmetric charge $l_\odot^2$ must be less than $(535\ \mathrm{km})^2$ in Moffat's theory, the prototypical nonsymmetric theory of gravity.

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Metric theories of gravity like general relativity and Brans-Dicke theory [1] feature a symmetric, second-rank tensor gravitational field that couples universally [2] to matter. Any auxiliary gravitational fields, such as the Brans-Dicke scalar field, couple only to the metric-tensor field. This metric coupling scheme assures that the effects of gravity on matter are accounted for purely geometrically, purely in terms of the curvature of the metric-tensor field. In contrast, theories of gravity which couple auxiliary gravitational fields directly to matter violate the Einstein equivalence principle [2] and, so, do not attribute a unique operational geometry to spacetime.

Moffat's nonsymmetric gravitation theory [3] (NGT) is the prototype for a diverse class of Lagrangian-based nonmetric theories of gravity [4] that has been studied extensively during the past decade. These theories feature a nonsymmetric-tensor gravitational field whose antisymmetric part couples directly to the electromagnetic field. This coupling breaks the symmetries defined by the Einstein equivalence principle. It causes the gravitational acceleration of test bodies to depend upon their internal electrostatic structure [5,6]. It also causes the gravitational deflection and propagation delay of light to depend upon the light's polarization state [7]. In this paper we show that the propagation of light through a nonsymmetric gravitational field can alter the light's polarization and that propagation through the solar field can depolarize light emitted by an extended polarized source. We show that existing measurements of the polarization of the Zeeman components of spectral lines emitted from extended, magnetically active regions near the limb of the Sun [8,9] impose constraints on nonsymmetric theories that are far more severe than those implied by previous tests of these theories. These measurements provide a qualitatively new kind of evidence supporting general relativity and metric theories of gravity. We describe additional measurements that can strengthen this evidence even further.

The electromagnetic field equations which govern the propagation of light through a nonsymmetric gravitational field follow from an action principle whose most general form [7] is

$$I_{\mathrm{em}}=\int d^4x\sqrt{-g}\,\mathcal{F}g^{\mu\alpha}g^{\nu\beta}\times\{ZF_{\mu\nu}F_{\alpha\beta}+(1-Z)F_{\alpha\nu}F_{\mu\beta}+YF_{\mu\alpha}F_{\nu\beta}\}\,. \quad (1)$$

The electromagnetic field tensor $F_{\mu\nu}$ is related to a vector potential in the usual way, $F_{\mu\nu}\equiv A_{\nu,\mu}-A_{\mu,\nu}$. The matrix $g^{\mu\nu}$ is the inverse of the nonsymmetric gravitational field $g_{\mu\nu}$ defined by $g^{\mu\alpha}g_{\nu\alpha}=g^{\alpha\mu}g_{\alpha\nu}=\delta^\mu_\nu$. The constants Y and Z and the scalar function $\mathcal{F}=\mathcal{F}(\sqrt{-g}/\sqrt{-\gamma})$, where $g\equiv\det g_{\mu\nu}$ and $\gamma\equiv\det g_{(\mu\nu)}$, differ from one nonsymmetric theory to the next.

The Sun's essentially static, spherically symmetric gravitational field admits, in theories like NGT, a representation in spherical coordinates $g_{00}=-T(r)$, $g_{(0i)}=0$, $g_{(ij)}=H(r)\delta_{ij}$, $g_{[0i]}=L(r)n_i$, and $g_{[ij]}=0$, where $n_i\equiv x_i/r$ and $r\equiv|\mathbf{x}|$. We analyze the effect that the coupling between such a field and the electromagnetic field has on light propagation in nonsymmetric theories having $Y=1-Z$ and $\mathcal{F}=\sqrt{-g}/\sqrt{-\gamma}$. In these theories the action (1) takes the form

$$I_{\mathrm{em}}=\int d^4x\left(\epsilon E^2-\frac{1}{\mu}[B^2-\Omega(\hat{\mathbf{n}}\cdot\mathbf{B})^2]\right), \quad (2)$$

where $\epsilon=\mu=(H/T)^{1/2}$, $\Omega=L^2/TH$, and where we have introduced electric and magnetic fields via $F_{j0}\equiv E_j$ and $F_{jk}\equiv\epsilon_{jkl}B_l$. This action is metric in form [5] except for the term proportional to $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$. In general, a nonsymmetric theory's electromagnetic action also includes a term proportional to $(\hat{\mathbf{n}}\cdot\mathbf{E})^2$ which has an effect on the propagation of light analogous to that of the $(\hat{\mathbf{n}}\cdot\mathbf{B})^2$ coupling. We set the $(\hat{\mathbf{n}}\cdot\mathbf{E})^2$ coupling aside because atomic-physics experiments of the Hughes-Drever type can impose a sharper constraint on its strength than the observations we interpret in this paper [10].

The condition that the variational derivative of the action (2) with respect to the vector potential A_μ vanish implies the source-free electromagnetic field equations. From these equations we have derived [7] the eikonal equation which governs the propagation of locally plane electromagnetic waves,

$$\mathbf{E}=\mathbf{A}_E e^{i\Phi}, \quad \mathbf{B}=\mathbf{A}_B e^{i\Phi}, \tag{3}$$

in the geometric-optics limit. Letting k_μ denote the gradient of the phase function, $\partial_\mu \Phi \equiv (\partial\Phi/\partial t, \nabla\Phi) \equiv (-\omega, \mathbf{k})$, the field equations imply that the fields (3) are transverse, $\mathbf{k}\cdot\mathbf{A}_E = \mathbf{k}\cdot\mathbf{A}_B = 0$, orthogonal, $\mathbf{A}_B = \mathbf{k}\times\mathbf{A}_E/\omega$, and satisfy the eikonal equation

$$k^2\mathbf{A}_B - \epsilon\mu\omega^2\mathbf{A}_B - \Omega\hat{\mathbf{n}}\cdot\mathbf{A}_B[k^2\hat{\mathbf{n}} - \hat{\mathbf{k}}(\hat{\mathbf{n}}\cdot\mathbf{k})] = 0. \tag{4}$$

It is apparent from the structure of this equation that the coordinate speed of light propagating through the Sun's nonsymmetric field, the phase velocity ω/k, depends on the orientations of $\mathbf{k}$ and $\mathbf{A}_B$ relative to radial unit vector $\hat{\mathbf{n}}$, that is, on the direction in which the light is propagating and its polarization.

A linearly polarized electromagnetic wave whose magnetic field is perpendicular to $\hat{\mathbf{n}}$ propagates as specified by the dispersion relation $k^2 = \epsilon\mu\omega^2$. The coordinate speed of light having this polarization is simply $c_\perp \equiv \omega/k = (\epsilon\mu)^{-1/2}$. The transverse nature of electromagnetic waves implies that any wave propagating in the $\pm\hat{\mathbf{n}}$ direction travels at this speed. A wave propagating in any other direction travels at $c_\perp$ only when $\mathbf{A}_B$ is perpendicular to the plane spanned by $\mathbf{k}$ and $\hat{\mathbf{n}}$.

A linearly polarized wave whose magnetic field lies in the plane spanned by $\mathbf{k}$ and $\hat{\mathbf{n}}$ propagates as specified by the dispersion relation $k^2(1-\Omega\sin^2\theta) = \epsilon\mu\omega^2$, where θ is the angle between $\hat{\mathbf{n}}$ and $\mathbf{k}$. The coordinate speed of light having this polarization is $c_\theta = [(1-\Omega\sin^2\theta)/\epsilon\mu]^{1/2}$.

When light is linearly polarized with its magnetic field perpendicular to $\hat{\mathbf{n}}$ or with its magnetic field lying in the plane spanned by $\mathbf{k}$ and $\hat{\mathbf{n}}$, propagation through the solar field does not affect its polarization. However, light having any other polarization is a coherent superposition of these $\perp$ and $\parallel$ polarizations singled out by the structure of the Sun's nonsymmetric field and, since the phase velocities of the $\perp$ and $\parallel$ components differ, propagation alters the relative phase of these components and, thus, the light's polarization. In practice, the high frequency of visible light makes optical measurements of this effect far more sensitive to differences between $c_\perp$ and c_θ than direct time-of-flight measurements for $\perp$ and $\parallel$ polarized light.

We calculate the leading contribution to the phase difference that accumulates between $\parallel$ and $\perp$ polarized light by integrating the eikonal equation along straight light rays unperturbed by gravity [7]. The result for light of angular frequency ω is

$$\Delta\Phi = \tfrac{1}{2}\omega\int_{t_0}^{t_1}\Omega\sin^2\theta\, dt, \tag{5}$$

where Ω and θ are functions of t, an affine parameter along the ray of interest.

We are interested in rays which originate on the Sun's surface, but we still find it convenient to use the ray parametrization $\mathbf{x}(t) = \mathbf{b} + \hat{\mathbf{k}}_0 t$ employed in studies of light deflection. The unit vector $\hat{\mathbf{k}}_0$ specifies the direction of the unperturbed ray. By demanding that $\hat{\mathbf{k}}_0\cdot\mathbf{b} = 0$, we make $\mathbf{b}$ the impact vector that connects the Sun's center to the closest point on the ray. Of course, when b is smaller than the solar radius R, the portion of the ray inside the Sun is of no interest. We integrate Eq. (5) along only that portion of the ray that begins at the Sun's surface, $t_0 = (R^2 - b^2)^{1/2}$, and extends to a distant observer, $t_1 = \infty$.

According to Moffat's NGT, $\Omega = l_\odot^4/r^4$ to post-Newtonian order, where $l_\odot^2$ is the Sun's antisymmetric charge [3]. We evaluate the NGT phase shift by inserting this expression into Eq. (5), using the identities $\sin^2\theta = (b/r)^2 = (R/r)^2(1-\mu^2)$ and $r = (b^2+t^2)^{1/2} = [R^2(1-\mu^2)+t^2]^{1/2}$ and integrating. The result, in radians, is

$$\Delta\Phi(\mu) = \frac{\pi l_\odot^4}{\lambda R^3}\left\{\frac{3\pi}{16(1-\mu^2)^{3/2}} - \frac{\mu}{4} - \frac{3\mu}{8(1-\mu^2)} - \frac{3}{8(1-\mu^2)^{3/2}}\arcsin(\mu)\right\}, \tag{6}$$

where λ is the light's wavelength and where μ denotes the cosine of the heliocentric angle of the location on the solar surface from which a ray originates [11]. For a source at the center of the solar disk, $\mu = 1$, this phase shift vanishes. For a source at the solar limb, $\mu = 0$, it is $3\pi^2 l_\odot^4/16\lambda R^3$.

We are interested in the effect of the phase shift (6) on the polarization of light received by a distant observer from a source on the Sun's surface. In general, the light emitted by a source will be a mixture of polarized and unpolarized light. If we define the Stokes parameters I, Q, U, and V based on the $\perp$ and $\parallel$ polarizations singled out by the Sun's nonsymmetric field, the intensity I and the degrees of polarization Q/I, U/I, and V/I offer a particularly convenient representation of the light's state.

We have already seen that the polarization of light from a source of $\perp$ or $\parallel$ polarized light, $Q/I = \pm 1$, is unchanged by propagation through the Sun's field and that the phase shift (6) will alter the polarization of light from a source that emits a coherent superposition of the $\perp$ and $\parallel$ polarized light. Consider the effect of the phase shift on light emitted with a circular polarization, $V/I = \pm 1$, a superposition of equal amplitudes of $\perp$ and $\parallel$ polarized light. A distant observer who sees the light's source at the center of the solar disk, $\mu = 1$, receives light having the same circular polarization because the phase shift (6) vanishes. However, if we imagine moving the source away from the disk center, the phase shift increases as μ decreases. The polarization of the light received by the distant observer will be linear, $U/I = \pm 1$, when the phase shift reaches $\pi/2$. It will be circularly polarized again, $V/I = \mp 1$, when the phase shift reaches π and so on. In general, the polarization is elliptical with the axes of the polarization ellipse bisecting the angles between the $\perp$ and $\parallel$ polarization axes. Clearly, the same kind of oscillation between linear and circular polarization occurs for light emitted with any of these elliptical polarizations. Notice that the composite degree of "elliptical" polarization $P_e = (U^2 + V^2)^{1/2}/I$ remains constant as V/I and U/I vary.

The effect of propagation through the Sun's nonsymmetric gravitational field on light emitted with some arbitrary polarization is quite clear once we make use of the additive property of Stokes parameters. This property permits us to represent light characterized by an arbitrary set of Stokes parameters as an incoherent mixture of unpolarized light, linearly polarized light having only a Q/I degree of polarization, and elliptically polarized light having only U/I and V/I degrees of polarization. A distant observer receives light from a source on the Sun's surface whose unpolarized and linearly polarized components are identical to those of the light emitted by the source. The observer receives light whose elliptically polarized component differs from that of the light emitted by the source as described in the preceding paragraph.

To this point we have assumed implicitly that the source on the Sun's surface is essentially pointlike, that the phase shift (6) does not vary significantly across the source. If this is not the case, different phase shifts affect the elliptically polarized components of light emitted from different parts of the extended source. A distant observer who resolves regions over which the phase shift is essentially constant receives light having different elliptical polarizations from different parts of the source. We sum the different elliptically polarized components incoherently, using the additive property of Stokes parameters again, to determine the elliptically polarized component of the light received by an observer who does not resolve the source. Its intensity is less than that of the elliptically polarized component emitted by the source while the intensity of the unpolarized component received by the observer is correspondingly larger. It follows that the degree of elliptical polarization P_e of the light received is smaller than that of the light emitted by the source. It will be much smaller if the phase shift varies by more than a few π rad across the source.

Since the phase shift (6) varies most rapidly near the solar limb, light from a polarized source near the limb is most likely to be depolarized by propagation through a nonsymmetric solar gravitational field. The magnitude of the Sun's antisymmetric charge, the wavelength of the light observed, and the extent of the source determine whether or not the depolarization is appreciable.

Stenflo and co-workers [8,9] have measured the polarization of light from an extended, magnetically active region located at $\mu = 0.1$, roughly 5 arcsec from the Sun's limb. They measured $V/I = 0.043 \pm 0.002$ at the peak of the σ component of the 5250.2-Å Fe I spectral line split by the Zeeman effect. This value for the degree of circular polarization sets a lower limit on the degree of elliptical polarization P_e of light received from the extended source. Given determinations or reliable estimates of P_e for the light emitted by the source and of the source's extent, we can use this measurement to set an upper limit on the Sun's antisymmetric charge. For values of $l_\odot^2$ larger than this limit depolarization of the elliptically polarized component of the light emitted by the source drives the degree of circular polarization of the light received below the observed level of $(4.3 \pm 0.2)\%$.

The degree of elliptical polarization P_e of light received by a distant observer is proportional to the degree of elliptical polarization of the light emitted by the source. The lower this initial polarization is the less the effect of the Sun's nonsymmetric field can be permitted to depolarize the light if consistency with the measurement of Stenflo and co-workers is to be maintained. We can, therefore, infer a conservative limit on $l_\odot^2$ if we assume that the light was emitted with $P_e = \pm 1$. The limit we infer also depends upon the extent of the source. The more compact the source is the weaker the limit is. To be conservative again, we assume the source is circular to make it as compact as possible for a given area. Stenflo and co-workers use the ratio of the source's area to the area covered by their 5-arcsec observing aperture, the filling factor, as a measure of the source's size. From their extensive set of spectral and polarization measurements, Solanki, Keller, and Stenflo [9] infer that the source observed at $\mu = 0.1$ has a filling factor of 12.9%.

We have written a code based on Eq. (6) that evaluates the degree of elliptical polarization of 5250-Å light received from a source with the properties we have just described as a function of the source's location on the solar disk. Having run the code for a range of $l_\odot^2$ values, we conclude that the Sun's antisymmetric charge can be no larger than $(535\ \text{km})^2$ if the light received from the source at $\mu = 0.1$ is to have a degree of elliptical polarization P_e of at least 4.1%. Figure 1 shows P_e as a function of the source's location near $\mu = 0.1$ for $l_\odot^2 = (535\ \text{km})^2$. Since physical effects of the antisymmetric part of the Sun's nonsymmetric field are proportional to $l_\odot^4$ in NGT, our new limit on $l_\odot^2$ reduces the magnitude of all such effects by a factor of about 1325 from the levels consistent with the previous upper limit of $(3230\ \text{km})^2$.

In this paper we have imposed a sharp new constraint on nonsymmetric theories of gravity by means of observational data that limits the extent to which propagation of light through a nonsymmetric solar gravitational field alter's the light's polarization. We are presently studying several ways to use observations of this qualitatively new effect to sharpen the limit on $l_\odot^2$ further. For example, note that both the relative amplitudes of the σ and π

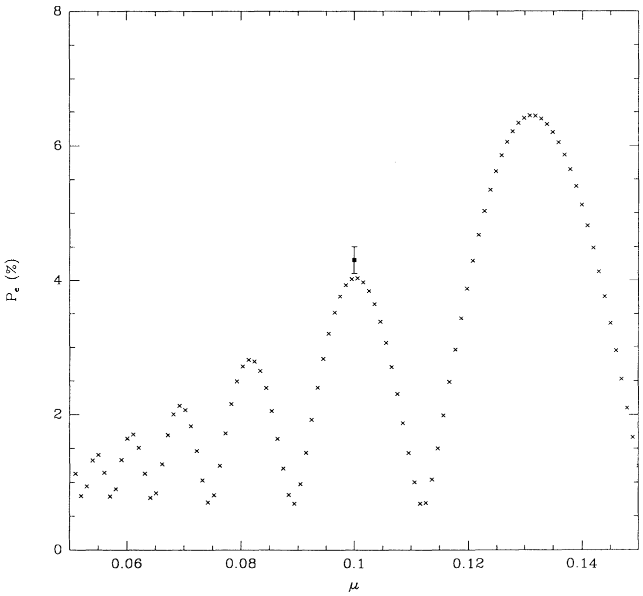

FIG. 1. The degree of circular polarization of 5250-Å light received from a source with the properties described in the text as a function of the source's location on the solar disk as computed for $l_\odot^2=(535\ \text{km})^2$. A single data point represents the observation of Stenflo and co-workers.

components of spectral lines split by the Zeeman effect and the polarization of these σ and π components carry information about the orientation of the magnetic field in an active region on the Sun's surface. If propagation through the solar gravitational field affects the polarization of light as predicted by nonsymmetric theories of gravity like NGT, then the information regarding magnetic field orientation garnered from spectral measurements may or may not be compatible with that garnered from polarization measurements depending on the magnitude of the phase shift (6) for a source's location. We are examining the practicality of using spectral and polarization data for a large number of sources scattered over the solar surface to search for evidence of a pattern of compatibility and incompatibility consistent with the phase shift (6). If observations at optical wavelengths fail to reveal such a pattern, it should be possible to conclude that any phase shift induced by a nonsymmetric solar gravitational field is no larger than $\pi/2$ rad even at the Sun's limb. This would imply that $l_\odot^2$ is less than $(100\ \text{km})^2$.

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