Hall Effect in the Two-Dimensional Luttinger Liquid

P. W. Anderson

Joseph Henry Laboratories of Physics, Jadwin Hall, Princeton University, Princeton, New Jersey 08544
(Received 17 July 1991)

The temperature dependence of the Hall effect in the normal state is a common theme of all the cuprate superconductors and has been one of the more puzzling observations on these puzzling materials. We describe a general scheme within the Luttinger liquid theory of these two-dimensional quantum fluids which correlates the anomalous Hall and resistivity observations on a wide variety of both pure and doped single crystals, especially the data in the accompanying Letter of Chien, Wang, and Ong.

PACS numbers: 74.70.Vy, 72.15.Gd, 72.15.Qm

One of the most striking and puzzling anomalies of the properties of the "normal" metallic state of the cuprate superconductors is the strong temperature dependence of the Hall resistance in the a-b plane. *Ad hoc* fits using complicated cancellations among hypothecated bands do not address the universality of this observation seriously nor explain the close correlation which is seen with T_c upon doping [1]. (Note that plotting $n_H \propto 1/R_H$ makes the T dependence appear to decrease with impurity additions just because the scale of n goes down. The temperature dependence is basically similar in all samples as we shall see.)

Recent plots of the data in a form suggested by a preliminary version of the present theory have demonstrated considerable regularities in this phenomenon, when plotted in terms of the *Hall angle* $\theta_H = R_H/\rho$ rather than R_H. (See, for instance, the accompanying Letter by Chien, Wang, and Ong [2].) We see that a reasonable fit to a sequence of doped single-crystal samples is given by

$$\theta_H^{-1} = (\omega_c \tau)^{-1} = A + BT^2 ,$$

where A vanishes in most pure samples. The quantity A is proportional to Ni or Zn doping in YBCO and B is constant for all dopings. We argue that this is a measurement of a "transverse" relaxation rate $(\tau_\perp)^{-1}$ which is the intrinsic relaxation rate $\tau_{\rm sp}$ of quasiparticlelike elementary excitations (spinons) of the Luttinger liquid.

In a sequence of papers [3–5] we have developed the idea of the "tomographic" Luttinger liquid. We showed that the anomalous forward scattering which accompanies the existence of an upper Hubbard band leads to an effective Landau interaction between up and down spins:

$$f_{kk'} = \eta |\epsilon_k - \epsilon_{k'}|/(k-k')^2 . \qquad (1)$$

This has the form of a "statistical interaction," a kind of fractional exclusion principle [6] in that the addition of an up-spin electron restricts the Hilbert space available for the down spins, and in particular does not allow the occupancy of the state $k' \equiv k$. Equation (1) follows from this and the incompressibility of Hilbert space.

We note that for states near the Fermi surface, (1) acts only for almost exactly parallel k and k'. To remove the singular interaction, a zeroth-order tomographic rediagonalization at each point on the Fermi surface is necessary. Luther [7] has shown that kinetic energy and the exclusion principle lead to a "bosonized" effective Hamiltonian which is summed over density waves $\rho_\sigma(Q)$ localized to the various directions "$\hat{\Omega}$" on the Fermi surface:

$$\mathcal{H}^0_{\rm eff} = 2\pi v_F \sum_{\hat{\Omega},Q,\sigma} \rho_\sigma(Q,\hat{\Omega})\rho_\sigma(-Q,\hat{\Omega}) , \qquad (2)$$

where ρ_σ are tomographic Tomonaga bosons made up from fermion operators with k's only along the specific direction $\hat{\Omega}$, and unrelated to the RPA density waves. They satisfy

$$[\rho_\sigma(Q,\hat{\Omega}),\rho_\sigma(Q',\hat{\Omega}')] = \frac{QL}{2\pi}\delta(\hat{\Omega},\hat{\Omega}')\delta(Q,-Q') . \qquad (3)$$

The interaction (1) simply adds a mixing term of the same form as (2) between up and down spins:

$$\text{const} \times \sum_{Q,\hat{\Omega}} \rho_\uparrow(Q,\hat{\Omega})\rho_\downarrow(-Q,\hat{\Omega}) . \qquad (4)$$

This extra term is not innocuous: After a Bogoliubov transformation it then leads to separate Fermi velocities for charge- and spin-density waves, $v_s(\hat{\Omega})$ and $v_c(\hat{\Omega}) > v_s(\hat{\Omega})$. For each radial direction, the zeroth-order "fixed-point" Hamiltonian is that of the one-dimensional Hubbard model as given by Haldane [8] (for instance):

$$\mathcal{H} = \sum_{\hat{\Omega}}\sum_{Q} \hbar k_F(\hat{\Omega})[v_s(\hat{\Omega})Qb^*_{s\hat{\Omega}}b_{s\hat{\Omega}} + v_c(\hat{\Omega})Qb^*_{c\hat{\Omega}}b_{c\hat{\Omega}}] .$$

Here, the b's are bosonized representations of the charge and spin fluctuations. It is not simple to reconstruct the original electron field variables from the b's and we refer to the references for that process. However, correlation and Green's functions calculated from the bosonized form of the one-dimensional Hubbard model are now known, and those of the 2D Luttinger liquid are just averages of these over the angles $\hat{\Omega}$. Most properties—such as, for instance, conductivity and magnetic susceptibility—can be expressed in terms of these correlation functions.

The physical meaning of these expressions can then be interpreted in terms of two kinds of solitons: spinons, which behave like electrons with spin and no charge and form a Fermi surface of the original size; and holons, which are like tomographic collective modes of the two Fermi surfaces at opposite ends of the vector $\hat{\Omega}$, and have momenta near $2k_F$. Holons carry the charge. Another way to think of them is an electron or hole of momentum near k_F, bound to an opposite-spin spinon. The two are connected by a gauge field which simply implies the "backflow" condition that what really flows macroscopically are real electrons, so that, for instance, when holons move one way the spinon gas moves oppositely. Neither can be necessarily thought of as true "topological solitons" of an order parameter. The essence of the physics is charge-spin separation.

When charge is accelerated by an electric field, we couple in to the tomographic degrees of freedom. One may discuss the resistivity in either of two equivalent ways: One may accelerate the "holons" which causes a backflow of spinons which scatter the holons, leading to a resistivity proportional roughly to the number of thermally excited spinons $\propto kT$ [9]. Or, one may accelerate the underlying electrons, which then decay into charge and spin degrees of freedom, because the two have different velocities, at a rate roughly proportional to their energy, again $\sim kT$ [3,4]. While these processes are momentum conserving, they lead to incoherent flows of holons and spinons which eventually scatter on the lattice: Lattice scattering determines the *outgoing boundary conditions* on holon and spinon wave functions. Thus $(\tau_{\rm tr})^{-1}$, the transport relaxation rate for resistivity, is not a true scattering rate and is independent of impurity or phonon scattering, but, in essence, is the decay rate of an accelerated electron into elementary excitations. If there is magnetic scattering of the spinon quasiparticle, τ_m^{-1}, this adds to $(\tau_{\rm tr})^{-1}$. But the holon charge fluctuations are not separately scattered; they are really to be thought of as collective excitations of the spinon Fermi surface. (A good way to understand this is the "marginal-Fermi-liquid" concept that spinons are the limit of electron quasiparticles as $Z \to 0$, and since Z and q are exactly zero the quasiparticles have only spin and cannot be scattered by potential fluctuations. Nonetheless, they form the Fermi surface.)

An entirely different situation arises when we accelerate electrons with a magnetic field. The electric-field interaction term is

$$\mathcal{H}' = \mathbf{A}\cdot e\mathbf{v} = \frac{e\mathbf{A}}{\hbar}\cdot\frac{\nabla\epsilon_k}{\nabla\mathbf{k}}$$

and acts precisely in such a way as to change the singular interaction (1) with other electrons in the same "tomograph," which is proportional to $\epsilon_k - \epsilon_{k'}$. The magnetic-field equation of motion is

$$\hbar\dot{k} = \frac{e}{\hbar c}\frac{\nabla\epsilon_k}{\nabla\mathbf{k}}\times\mathbf{B} - \frac{\hbar\delta\mathbf{k}}{\tau_\perp},$$

which specifically displaces all states in such a way as *not* to affect the energy ϵ_k and thus the singular interaction terms (1). States are accelerated only parallel to the Fermi surface, not affecting occupancies. Thus the effects which cause the conductivity to be anomalous do not affect the response to a magnetic field, and only genuine scattering processes enter into $1/\tau_\perp$. Another way to think is to note that we can turn on the magnetic field without, to lowest order, disturbing the Fermi surface, just rotating k space as a whole. Thus, in a sense, the singular interactions and the magnetic field *commute*: We can let the singular interaction turn on after the magnetic field, so the response to the magnetic field is like that of the original electrons. The opposite is true of ordinary potentials such as the electric field: The interactions change these responses in a singular fashion, and we have to turn on the field *after* we turn on the interactions to get correct answers.

As we have already pointed out, spinons—which are effectively the quasiparticles which form the Fermi surface—can only be scattered by scatterers which interact with the spin current. Of these, there are basically two varieties: the spinons themselves, and magnetic defects on the planes—bound spinons. Spinon-spinon scattering, like any other fermion-fermion interaction, leads to a T^2 process; magnetic impurities lead to a constant rate τ_m^{-1}. Thus we expect

$$\hbar(\tau^{-1})_\perp = \hbar\tau_m^{-1} + AT^2, \tag{5}$$

with $A \sim 1/J$ or 1/(spinon bandwidth) and $\hbar\tau_m^{-1} \propto n_{\rm scatt}$.

This provides a reasonably accurate fit to the data in its T and doping dependence. Both Ni and Zn are substitutional impurities in the Cu planar lattice, and both seem [10] to lead to a local, spin-$\frac{1}{2}$ impurity (a "bound spinon"). In the absence of such impurities or other spin scattering phenomena, one generally finds a quite accurate T^{-2} dependence for $\tau_\perp$. Numerically, inserting the maximum value of θ_H of $\sim\frac{1}{40}$ at 8 T observed in 1:2:3 materials, one finds, using

$$\frac{l_\perp}{r_{\rm apex}} = \theta_H = \frac{\epsilon B}{2\pi\hbar c}\frac{1}{n}k_F l_\perp,$$

that the Mott-Yoffe-Regel parameter $k_F l$ is ~ 100 at 100 K in the pure material. From the resistivity, using the universal formula

$$\sigma = (e^2/2\pi\hbar)k_F l,$$

we get $k_F l_{\rm tr} \simeq 40$.

The ratio of these two appears disturbingly low, as Chien, Wang, and Ong point out in the accompanying Letter [2]. This may be understood by recognizing that $l = v_F\tau$, but that in the Luttinger liquid—*unlike* any Fermi-liquid model—the Fermi velocity for spinons is *not* equal to that for charge fluctuations, but is probably of order at least 4 times slower. That is, $m^*_{\rm spinons} \sim 5$–10, $m_c^* \sim 2$–3. Using $\sim 4 = v_c/v_s$ (the photoemission data do not allow a smaller ratio), we get that $\tau_\perp \sim 10\tau_{\rm tr}$. This is

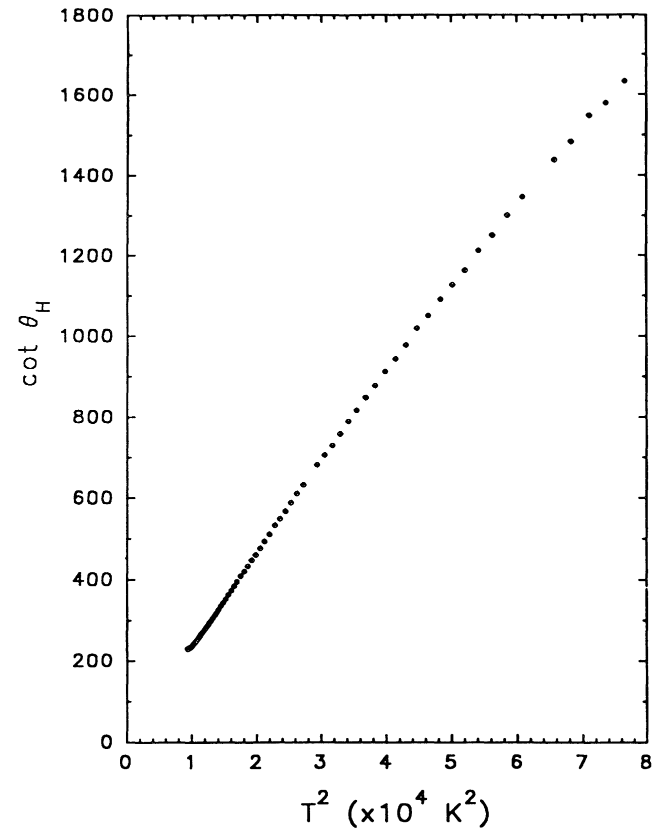

FIG. 1. Inverse Hall angle vs T^2 for a pure, untwinned crystal of $YBa_2Cu_3O_7$ from Ginsberg [12].

then compatible roughly with the observed ratio of $1/\tau_m$, and with a ratio of order J/T. The Hall angle will be reduced by any compensation effects as well as by the effect of anisotropy of m and τ around the Fermi surface, so that m_s may not have to be as large as quoted here and in Ref. [2].

The overall magnitude, as opposed to the ratio, is modified by another moderately large factor. This is that in order to satisfy the conductivity sum rule with $1/\tau_{tr} \propto \omega$—and, in fact, also appearing in the calculation of resistivity in our theory and in the marginal-Fermi-liquid theory [11]—a logarithmic correction is necessary, so that the formula for σ in terms of *dynamic* quantities is

$$\sigma = ne^2\tau_{tr}/m(\ln\omega_c\tau_{tr}) ,$$

so that there is an apparent logarithmic correction to the conductivity mass. This reduces the observed conductivity by a factor of 3–4 relative to the value of τ seen in the Drude peak. The same correction reduces E_H. This logarithm appears in the observed transport data in a number of other contexts [11], for instance, in the T dependence of high-frequency reflectivity data.

The same T^2 dependence is seen not only in other sources of pure YBCO data but in data on other materials (see Fig. 1) [12]. We emphasize how few parameters—basically, only overall scales—are required to fit a wide variety of experiments on several different quantities: θ_H, ρ, T_c, and $\sigma(\omega)$. I do not see a plausible alternative, though of course many implausible ones will continue to be brought forward.

We are indebted to Don Ginsberg for the unpublished data, and to N.-P. Ong, J. Clayhold, P. A. Lee, E. Abrahams, P. B. Weigmann, and others for many discussions. This work was supported by the NSF, Grant No. DMR-8518163 and AFOSR Grant No. 87-0392.

[1] J. A. Clayhold, thesis, Princeton University, 1989 (unpublished); J. Clayhold, N. P. Ong, Z. Z. Wang, J. M. Tarascon, and P. Barboux, Phys. Rev. B **39**, 7324 (1989).

[2] T. R. Chien, Z. Z. Wang, and N. P. Ong, preceding Letter, Phys. Rev. Lett. **67**, 2088 (1991); see also T. R. Chien, Ph.D. thesis, Princeton University, 1991 (unpublished).

[3] P. W. Anderson and Y. Ren, in *High Temperature Superconductivity*, edited by K. Bedell *et al.*, Proceedings of the International Conference on the Physics of Highly Correlated Electron Systems, Los Alamos, 1989 (Addision-Wesley, New York, 1989), pp. 3–33.

[4] P. W. Anderson, in Proceedings of the "Common Trends in Particle and Condensed Matter Physics" Workshop, Cargese, 1988 [Phys. Rep. **184**, 2–4 (1989)].

[5] P. W. Anderson, Phys. Rev. Lett. **65**, 2306 (1990); **66**, 3226 (1991).

[6] F. D. M. Haldane, Phys. Rev. Lett. **67**, 937 (1991).

[7] A. Luther, Phys. Rev. B **19**, 320 (1979).

[8] F. D. M. Haldane, Phys. Lett. **81A**, 153 (1981); Cargese Lectures, 1990 (Princeton Univ. Press, Princeton, NJ, to be published).

[9] L. B. Ioffe and P. B. Wiegmann, Phys. Rev. Lett. **65**, 653 (1990); N. Nagaosa and P. A. Lee, Phys. Rev. Lett. **64**, 2450 (1990).

[10] A. M. Finkelstein, V. E. Kataev, E. F. Kutovskii, and G. B. Tietelbaum, Physica (Amsterdam) **168C**, 370 (1990); see also M. Z. Cieplak, G. Xiao, A. Bakshai, and C. Z. Chien, Phys. Rev. B **39**, 4222 (1989); **39**, 315 (1989).

[11] Note that the longitudinal transport properties closely resemble those of the marginal-Fermi-liquid theory; it is the spinons and transverse properties which are characteristic of the Luttinger liquid.

[12] D. M. Ginsberg, in Proceedings of the International Conference on High-T_c Superconductors and Materials and Mechanisms of Superconductivity, Kyoto, edited by M. Tachiki [Physica (to be published)]; (to be published).