Microwave Propagation in Two-Dimensional Dielectric Lattices

S. L. McCall and P. M. Platzman
AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, New Jersey 07974

R. Dalichaouch, David Smith, and S. Schultz
Department of Physics, University of California, San Diego, 9500 Gilman Drive, La Jolla, California 92093-0319
(Received 15 February 1991)

We have calculated and measured the properties of X-band microwaves propagating in a 2D array of low-loss high-dielectric-constant cylinders. Transmission bands and photonic band gaps are conclusively identified in excellent agreement with the theoretical predictions. Detailed data on the properties of isolated defect states are also presented. We conclude that studies of this model scattering system allow the quantitative evaluation and testing of ideas regarding wave propagation and localization in strongly scattering media.

PACS numbers: 42.20.Ji, 41.90.+e, 84.90.+a

In recent years there has been renewed interest in the propagation characteristics of electromagnetic (EM) waves in media consisting of arrays of strong scatterers [1-4]. Experiments on such systems provide a nearly ideal analog for a single quantum-mechanical electron propagating in a given potential distribution [5-7]. Measurements of microwave transmission through samples comprised of periodic [2] and random [8] dielectric scatterers have been reported. Experiments at microwave frequencies are particularly interesting because high-dielectric-constant low-loss scatterers are readily available. In addition, we show here that point-by-point measurement of the EM field in such systems is also possible; i.e., we have, in practice, a way of measuring the analog of the wave function of the electron. In condensed-matter systems where strong scattering effects have been studied, electron wave functions are not easily measured. When the scatterers are arranged periodically the propagation is naturally described by a vector version of band theory [9-11]. The possibility of "photonic band gaps," [11] the suppression of spontaneous emission [1,2], and the existence of new types of localized states [12] are some of the phenomena which have been predicted. When the scatterers are arrayed randomly we can investigate the predictions of "strong localization" theory [13].

It is well known that dimensionality plays a crucial role in the behavior of these types of strongly scattering systems. The lower the dimension, the easier it is to have photonic band gaps since overlap of gaps in different directions is more probable (a certainty in 1D). When the experiments are performed in one and two dimensions it is possible to select a single polarization of the electric field so that a scalar calculation is correct, and a measurement of the field is relatively simple. The theory of waves in a random medium suggests that all states are localized in 1D and marginally localized in 2D, i.e., the localization length depends exponentially on the mean free path for scattering. For 3D it is generally believed that a mobility edge exists [13]. The mobility edge is the energy or frequency which separates extended from localized states. Direct observation of strong (EM wave) localization in random systems has not yet been reported although weak localization effects [14] and photon transport in optically thick diffusive media [15] have been studied.

In this Letter we present the results of calculations [16] and experiments for periodic arrays of an almost ideal 2D system consisting of low-loss high-dielectric-constant cylindrical scatterers. We begin our calculation by considering a wave propagating perpendicular to a periodic array of cylinders with a real dielectric constant ε. For $\mathbf{E}$ pointing along the z axis, the axis of the cylinders, Maxwell's equation reduces to a scalar 2D wave equation ($c=1$),

$$[\nabla^2+\varepsilon(r)\omega^2]\Psi(\mathbf{r})=0\,. \tag{1}$$

Here

$$\mathbf{E}=e^{-i\omega t}\Psi(\mathbf{r})\hat{\mathbf{z}}\,. \tag{2}$$

Since the medium is periodic, the function $\Psi(\mathbf{r})$ satisfies Bloch's theorem and can be expanded in terms of plane waves:

$$\Psi(\mathbf{r})=e^{i\mathbf{k}\cdot\mathbf{r}}\sum_{\mathbf{G}}\phi_{\mathbf{G}}e^{i\mathbf{G}\cdot\mathbf{r}}\,, \tag{3}$$

where the momentum $\mathbf{k}$ is in the first Brillouin zone and $\mathbf{G}$ is a reciprocal-lattice vector. The quantity $\phi_{\mathbf{G}}$ satisfies the usual matrix equation,

$$[\omega^2-(\mathbf{G}+\mathbf{k})^2]\phi_{\mathbf{G}}=-\sum_{\mathbf{G}'}\omega^2\chi_{\mathbf{G}-\mathbf{G}'}\phi_{\mathbf{G}'}\,, \tag{4}$$

where for cylinders of radius a,

$$\chi_{\mathbf{G}}=2\pi(\varepsilon-1)aJ_1(Ga)/Gd^2\,, \tag{5}$$

where J_1 is a Bessel function.

Equations (4) and (5) were solved numerically for the photon band structure $\omega(\mathbf{k})$ taking enough terms in reciprocal space to ensure convergence. In Fig. 1 we have plotted this band structure for a square lattice with lattice constant d whose dielectric constant for Fig. 1(a) was $\varepsilon=4$ (quartz) and for Fig. 1(b) was $\varepsilon=9$ (alumina composite). The solid lines are the numerical results. The

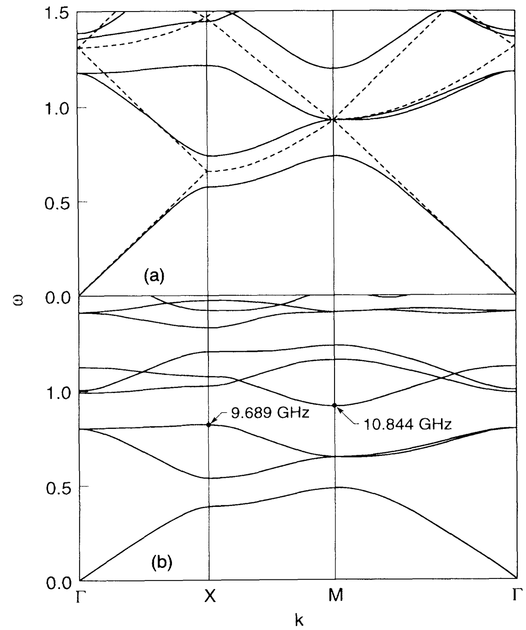

FIG. 1. Calculated band structure. Frequency ω is measured in units of $\omega_0=\pi c/d$. For the experimental case $d=1.27$ cm, $\omega_0/2\pi=11.811$ GHz. Points in the Brillouin zone Γ,X,M are $k=\pi/d$ times [0,0], [0,1], and [1,1], respectively. (a) $\varepsilon=4$. The dashed lines illustrate the band structure using a homogeneous average dielectric constant $\varepsilon_A=1+(\varepsilon-1)\pi a^2/d^2$. (b) $\varepsilon=9$. Gaps appear near $\omega/\omega_0=0.5$, 0.9, and 1.3. The numbers in GHz show the boundaries of the second gap for $a=0.48$ cm, $d=1.27$ cm.

dashed line is the free photon dispersion relation in a homogeneous medium with average dielectric constant $\varepsilon_A=[(\varepsilon-1)\pi a^2+d^2]/d^2$ folded back into the first zone. We note the following: (a) Photon propagation in the average medium along with simple ideas on degenerate perturbation theory give a correct qualitative picture of the lowest few bands [17]. (b) For $\varepsilon=4$ [Fig. 1(a)] there are gaps in some directions but either a very small "absolute gap" or no "absolute gap." For $\varepsilon=9$ there are unambiguously absolute gaps (for $d=1.27$ cm, $a=0.48$ cm) centered near 5, 10, and 15 GHz.

The microwave experiments were performed over the frequency range $\nu=7$–20 GHz in a 2D waveguide scattering chamber (1 cm high, 46 cm wide, and 51 cm long), machined into a single aluminum plate. The two 46-cm-wide sides taper down to the standard 8–12-GHz waveguide. The chamber is completed by a single aluminum plate. It contains a sheet of styrofoam ($\varepsilon=1.04$), capable of positioning up to 900 dielectric cylinders of radius $a\cong0.5$ cm. The square lattice spacing of the cylinders is established by placing them in holes drilled in the styrofoam sheets utilizing a numerically controlled mill. The walls of the tapers and the scattering chamber are covered with a thick layer of low-density absorber designed to minimize reflections. With nonabsorbing walls, resonances associated with wall-array reflections would exist, unnecessarily complicating the data analysis as well as calculations. As we shall see, experiments show that the system is an excellent approximation to the scattering of a cylindrical wave from an array of cylinders in free space. We have utilized cylinders with $\varepsilon=2$, 4, or 9. The loss tangents are quite low ($\varepsilon/\varepsilon_I \gtrsim 2\times10^3$).

Transmitted power as well as field distribution patterns were measured in a manner that will be described elsewhere in greater detail [18,19], but in brief there were three types of experiments. Type I is a transmission experiment where swept frequency modulated microwave power is incident via one tapered port and is detected at the other after having traversed the entire chamber. Type II is a transmission experiment where swept modulated microwave power is sent into the chamber via a small hole placed in the chamber coverplate at any desired location, and then detected at either of the two entry or exit ports. In type-III experiments the power is set at any desired fixed frequency, and may be sent into the chamber as in either type I or type II, but the purpose is to provide a spatial mapping of the electric field everywhere inside the chamber. This is accomplished by means of a tuned probe that is coupled nonperturbatively through any one of a lattice of small holes in the coverplate. The entire coverplate may be mechanically translated more than the distance between holes so that one can scan the entire chamber surface. The amplitude of the microwave field coupled into the tuned probe is measured by taking data at two phases 90° apart using a homodyne detector.

With no cylinders in the chamber the transmission coefficient is constant over any 5-GHz range to ±2 dB. We utilize the forward scattering from a single cylinder as a convenient technique to measure the ε of the cylinder, and also to verify the proper operation of the scattering chamber. In Fig. 2 we present a comparison of the calculated and measured frequency dependence of the ratio of the detected powers with and without a single cylinder of diameter 1.1 cm. The calculation was carried out exactly assuming that a 2D line source was irradiating an infinite cylinder in free space. We note the three Mie resonances which are very well fitted by assigning a value of $\varepsilon=9$. The agreement confirms that the absorbing walls of the scattering chamber are properly terminated.

In Fig. 3 we show the frequency dependence of the transmitted power for several type-I experiments for a 9×18 array of $\varepsilon=9$ cylinders with the same radius and spacing used in Fig. 1(b). Figure 3(a) corresponds to propagation of the incident wave along the [1,0] direction of the square lattice, Fig. 3(b) along the [2,1] direction, and Fig. 3(c) is along the [1,1] direction. In the three spectra it is clear that there is a sharp reduction, at a frequency close to 10 GHz, of about 30 dB in the power

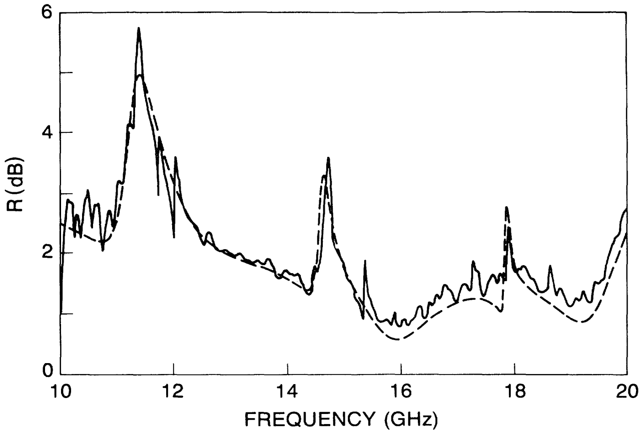

FIG. 2. Comparison of measured (solid line) and calculated (dashed line) forward scattering for a single cylinder of $d=0.56$ cm and $\varepsilon=9$. Both curves represent the ratio R in dB of the power with dielectric cylinder present divided by power with the cylinder absent.

transmitted. The transmitted energy remains at its low value for ≈ 1.5 GHz. The value of the transmitted power at the minimum is consistent with the noise level of the network analyzer. The position in frequency space and the width of the band is in excellent agreement with the gap predicted from the numerical results presented in Fig. 1(b). The data clearly demonstrate the existence of a "photonic gap" in this 2D system. There should also be a gap of about 1 GHz in width centered near 15 GHz. The data (Fig. 3) show a shoulder of 10–15 dB at the lower boundary of this gap. However, the transmission at higher frequencies is complicated and we do not have a quantitative understanding of it, although we suspect that it is related to appreciable mode conversion at the higher frequencies caused by slight tilts of the cylinders.

The experimental system is ideally suited for investigation of other interesting questions, such as the nature of isolated and overlapping defect states. We illustrate this application with data for the same configuration and orientation discussed in Fig. 3(a) where we have removed one cylinder from the center of the otherwise filled square lattice. In Fig. 3(d) we present the frequency dependence of the transmitted power for this defect configuration. When we compare the data of Figs. 3(a) and 3(d) we note the addition of a sharp transmission peak located well within the gap at 11.2 GHz. This new peak as we shall see is a consequence of a localized state. In additional experiments we have shown that one can readily demonstrate multiple defect states, follow their interactions, etc.

The verification of the localized nature of the state corresponding to the sharp transmission peak in the gap at 11.2 GHz is demonstrated by the data of Fig. 4. Here we present the spatial distribution of the power detected by the tuned probe as in a type-III experiment in a region of

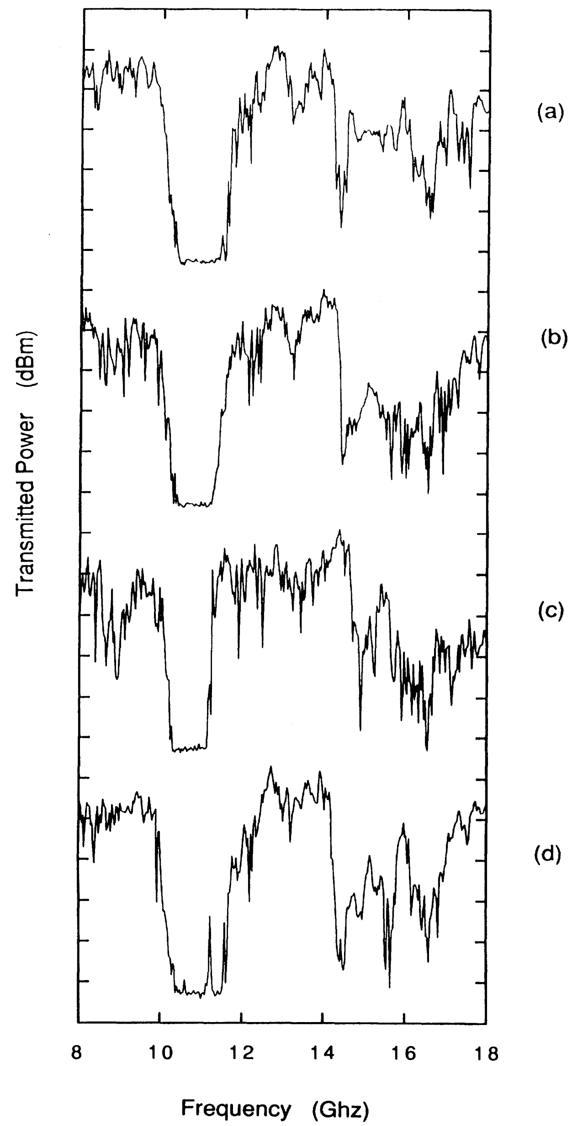

FIG. 3. Transmitted power as a function of frequency for type-I experiment for a 9×18 array of cylinders with $\varepsilon=9$, $d=1.27$ cm, $a=0.48$ cm. The wave is incident along (a) the [1,0], (b) the [2,1], and (c) the [1,1] direction. In (d) the orientation is as in (a) and a single cylinder has been removed from the center of the array. Note the extra sharp peak in the gap located at 11.2 GHz signifying a localized defect mode. Each vertical division is 5 dB.

7.6 cm$\times$7.6 cm centered about the missing cylinder. The mode is excited (at the peak frequency) via an open coaxial line inserted in a small hole drilled through the bottom plate of the chamber (i.e., as in a type-II experiment). The field is sampled by the probe every 0.254 cm. The original data consisted of a 30×30 array of points. The data displayed in Fig. 4 consist of a 30×117 array of points. The extra points were obtained by smooth interpolation of the original data and are included to clarify

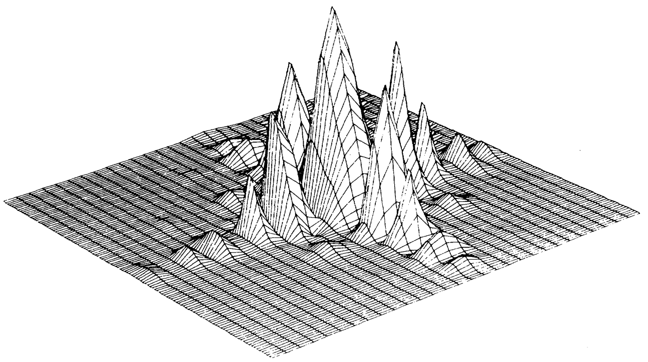

FIG. 4. The spatial distribution of the power detected by a tuned probe which is used to scan over a 7.6 cm×7.6 cm region centered about the "defect" created by removing a single cylinder from an array similar to that used to generate the data of Fig. 3 but with $d = 1.59$ cm.

the presentation. The observed pattern is not sensitive to the position of the input coaxial line within the mode region. The oscillations represent the interference or standing-wave pattern of the electric energy density generated by the defect scattering. Since the magnetic energy density is zero at the peaks of the electric energy density, the overall envelope of the peaks is some rough approximation to the surface of total energy density in the mode. This envelope clearly demonstrates the rapid falloff of energy density near the defect site, as expected for a localized defect mode. From a logarithmic plot of the values of the electric energy density peaks along a transverse cut through the center, we find that the decay is exponential, with a decay length of approximately $1.6d$.

We have presented theoretical and experimental results on a physically interesting, 2D model system, which demonstrate the existence of a photonic band gap and localized defect states. Having established by comparison of theory and experiment that the behavior of this system is relatively ideal, it is clearly possible to extend these measurements to other regimes of physical interest, such as interacting defects and random scatterers [16,17]. One may also address interesting questions such as the behavior of defect states as a function of the strength of the defect. Our results for these experiments will be reported elsewhere.

We would like to thank P. Fleury and D. Hamann for interesting discussions, and T. Kovacs and John Armstrong for many technical contributions. We thank Fritz Mikadeit for his assistance in taking data. This work was supported at UCSD by NSF Grants No. DMR-86-23858 and No. DMR-89-15815.

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