CP Asymmetries Induced by Particle Widths: Application to Top-Quark Decays

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An on-shell unstable particle produced in a tree-level process, such as W in $t \rightarrow qW$, induces a CP violating signal through its width. This effect arises from an Im(tree)×Re(loop) type of interference. Among $t \rightarrow qq'\bar{q}$ processes (e.g., $t \rightarrow bc\bar{b}$) proceeding either through the tree-level $t \rightarrow qW$ or the one-loop $t \rightarrow q' +$(virtual g) reactions, the CP asymmetry is largest for the case $t \rightarrow dc\bar{d}$ and is of $O(\eta/\lambda^2)$ in Wolfenstein's parametrization. The resulting effects are found to be small in the standard model, but are much larger than those obtained for radiative top-quark decays. We also briefly discuss B decays where such a mechanism may cause CP violation in, for example, $b \rightarrow se^+e^-$.

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Direct CP violation in the standard model (SM) [1] requires the interference between two phases, a Kobayashi-Maskawa [2] (KM) phase and a "strong" phase originating, for example, from an imaginary part of a loop diagram. One can therefore have a CP asymmetry arise from a Re(tree)×Im(loop) interference such as in [3] $b \rightarrow s+X$, or an Im(one loop)×Re(another loop) term as in [4] $b \rightarrow sl^+l^-$ where there is no tree-level graph. This is well known and has been explored in K and B physics, and has recently been applied in Ref. [5] to rare top-quark decays, where a loop×loop type of interference is discussed for $t \rightarrow u\gamma$. A CP asymmetry of $a \simeq 2\times10^{-3}$ for $m_t > 90$ GeV is found for $t \rightarrow u\gamma$, and a similar value holds for the more abundant loop process [6] $t \rightarrow ug$, which has a branching ratio [7,8] of $B(t \rightarrow ug) \simeq 2.5\times10^{-12}$. Since the relevant quantity for experimental observation is the CP asymmetry squared times branching ratio $(a^2 \times B)$, at least 10^{17} top-quark decays are required to observe CP violation from this reaction, if its origin lies within the SM. We discuss here a different mechanism for CP violation in heavy-quark decays which capitalizes on the finite width of one of the decay products. In this paper, we apply this observation specifically to top-quark decays, i.e., $t \rightarrow qW$, where the width of the W boson leads to CP violation effects. Although the resulting signal is much larger than in the radiative case, it is still small (in the SM) insofar as experimental detection is concerned. However, the predictions are clean and new physics scenarios could lead to much larger effects.

Let us consider a quark Q decaying into a three-particle final state, $p_1p_2p_3$, via two possible paths: (i) the tree process $Q \rightarrow p_1P$ with a real unstable particle P subsequently decaying into p_2p_3, and (ii) the loop process $Q \rightarrow p_2V$ where the virtual V couples to p_1p_3. Specifically for top decays, this includes the case $p_1 = b$, $P = W$, $p_2 = c$, $p_3 = \bar{b}$, and $V = g$ (i.e., $t \rightarrow bc\bar{b}$). Denoting the tree and loop amplitudes by A_1 and A_2, respectively, and assuming $|A_1| \gg |A_2|$, the CP asymmetry is then defined as

$$a = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}, \quad \Gamma = \int |A_1 + A_2|^2/dL , \tag{1}$$

with

$$a \simeq \frac{-2\,\mathrm{Im}(v_1 v_2^\dagger)}{\Gamma} \int \mathrm{Im}\hat{A}_1\, \mathrm{Re}\hat{A}_2\, dL , \tag{2}$$

where $\bar{\Gamma}$ is the width for $\bar{Q} \rightarrow \bar{p}_1\bar{p}_2\bar{p}_3$, and dL is the differential three-body phase space. By writing $A_i = v_i\hat{A}_i$ $(i = 1,2)$, the KM factors v_i have been separated out. In the general case A_2 is a sum of two terms [see Eq. (3) below], but for all practical purposes, we assume here that it is only one term that contributes. In addition, the tree contribution was taken as purely imaginary, i.e., $\mathrm{Re}\hat{A}_1 = 0$, and the loop×loop interference is neglected. Of course, designating A_1 as a "tree" should be considered somewhat of a mnemonic, since it is actually a loop diagram, having a bubble in the P propagator with an imaginary part. It is instructive to compare at this stage Eq. (2), which represents an Im(tree)×Re(loop) interference term, with the interference from Re(tree) ×Im(loop). The latter leads to a class of CP violation asymmetries in hadronic decays of the B mesons which have been well studied [3]. We clearly have here a mechanism which can occur only if one of the paths leading to the final state has a real unstable particle as an intermediate state.

Let us now specialize to $t \rightarrow qq'\bar{q}$ (e.g., $t \rightarrow bc\bar{b}, dc\bar{d}$) which comprise some of the three-body tree decays of the top quark. Since it is by now established that [9] $m_t > M_W$, the discussion above implies that there will be a CP asymmetry proportional to $\mathrm{Im}(t \rightarrow qW \rightarrow qq'\bar{q})$ $\times\mathrm{Re}(t \rightarrow q' + \text{virtual } g \rightarrow q'q\bar{q})$. Calculation of this interference is lengthy, but straightforward. Using KM unitarity and denoting momenta by the respective particle names, the loop amplitude can be written as

$$A_2 = \sum_{j=b,s} V_{tj}^\dagger V_{q'j}(G_j - G_d) , \tag{3}$$

where, suppressing the intermediate quark index,

$$G=\bar{u}(q)ig_s\gamma_a v(\bar{q})i\frac{-g^{\alpha\beta}}{(q+\bar{q})^2}\bar{u}(q')i\{[a_i(q+\bar{q})_\beta+a_2t_\beta+a_3\gamma_\beta]L+[b_1(q+\bar{q})_\beta+b_2t_\beta+b_3\gamma_\beta]R\}u(t)\,, \tag{4}$$

with $L,R=(1\mp\gamma_5)/2$ and g_s is the strong coupling constant. Each of the six form factors, a_i and b_i, is the sum of the contributions from ten one-loop Feynman diagrams. The exact expressions for these form factors are given in Ref. [8]. One then has to calculate the interference between the tree amplitude

$$A_1=-\frac{ig^2}{2}V_{tq}^\dagger V_{q'q}\bar{u}(q')\gamma_\mu Lv(\bar{q})\frac{-g^{\mu\nu}+(\bar{q}+q')^\mu(\bar{q}+q')^\nu/M_W^2}{(\bar{q}+q')^2-M_W^2+i\Gamma_W M_W}\bar{u}(q)\gamma_\nu Lu(t)\,, \tag{5}$$

and A_2, where $\Gamma_W=2.1$ GeV is the expected width of the W boson in the SM, and g is the weak coupling constant. The resulting trace is too long to be reproduced here, and will be symbolically denoted by T_j, where j indicates the virtual quark exchanged in the loop process. After multiplying by $\frac{4}{3}$ for color, inserting $\frac{1}{2}$ for initial spin, integrating over phase space, and dividing by the tree-level rate (in which we neglect $m_q,m_{q'}$ with respect to M_W) the CP asymmetry is given by

$$a=\frac{8g_sM_W\Gamma_W}{3\pi g^2m_t^6}\frac{1}{|V_{tq}^\dagger V_{q'q}|^2(1-x)(1+x-2x^2)}\int_{(m_{q'}+m_{\bar{q}})^2}^{(m_t-m_q)^2}dm_{q'\bar{q}}^2\int_{(m_{q'q}^2)_{\min}}^{(m_{q'q}^2)_{\max}}dm_{q'q}^2\frac{1}{(\bar{q}+q)^2}\frac{1}{(K^2+\Gamma_W^2M_W^2)}$$
$$\times\sum_{j=b,s}\mathrm{Im}(V_{tq}^\dagger V_{q'q}V_{tj}V_{q'j}^\dagger)[\Gamma_WM_W\mathrm{Re}(T_j-T_d)-K\,\mathrm{Im}(T_j-T_d)]\,. \tag{6}$$

Here $K=m_q^2+m_{q'}^2+2\bar{q}\cdot q'-M_W^2$, $x=M_W^2/m_t^2$, and the $\mathrm{Re(tree)}\times\mathrm{Im(loop)}$ contribution is included for completeness.

Because of the Glashow-Iliopoulos-Maiani mechanism [10], it is clear that $|T_b-T_d|\gg|T_s-T_d|$ and therefore we only keep $j=b$ in Eq. (6). Thus, among the six possible final states in $t\to qq'\bar{q}$, the two with $q=b$ will have a negligible asymmetry since $\mathrm{Im}(V_{tb}^\dagger V_{q'b}V_{tb}V_{q'b}^\dagger)=0$ for $q'=u,c$ and so only the very small T_s-T_d term contributes. This is rather disappointing since $t\to bc\bar{b}$ has the largest decay rate among all the candidate final states. However, this is not unexpected as usually one finds a small asymmetry for a large branching ratio, and vice versa. An exact calculation, without neglecting T_s-T_d, yields indeed an absurdly small asymmetry of $a\simeq10^{-11}$, for $t\to bc\bar{b}$ with $m_t=130$ GeV, while $B\simeq6\times10^{-4}$. The largest asymmetry is found for the process $t\to dc\bar{d}$, since, using Wolfenstein's parametrization [11] of the KM matrix, $\mathrm{Im}(V_{td}^\dagger V_{cd}V_{tb}V_{cb}^\dagger)\simeq\eta A^2\lambda^6$, and $|V_{td}^\dagger V_{cd}|^2\simeq A^2\lambda^8$, yielding an asymmetry which is proportional to the large factor η/λ^2 (recall $\lambda=0.22$). The more relevant quantity, $a^2\times B$, has also the largest KM factor of $A^2\eta^2\lambda^4$ for $t\to dc\bar{d}$. The second largest asymmetries are found for $t\to du\bar{d}$ and $t\to su\bar{s}$ with $a\simeq\eta$ and $a^2\times B\simeq A^2\eta^2\lambda^6$.

In Fig. 1 we present our results for the absolute value of the CP violation asymmetry as a function of m_t for the process $t\to dc\bar{d}$. We have used the central values of the ranges for the KM angles as given in Ref. [12], and $\delta_{13}=\pi/2$ in the notation of Ref. [12]. Since $B(t\to dc\bar{d})\simeq1.8\times10^{-6}$, therefore $a^2\times B\simeq6.5\times10^{-15}$ for $m_t=130$ GeV. Though we find here an improvement of almost 3 orders of magnitude in the number of events required to observe CP violation, as compared to radiative top decays, one is still far from approaching the number of t quarks anticipated at the Superconducting Super Collider or the CERN Large Hadron Collider.

Putting aside the small numbers for top decays, the new mechanism discussed here can be applied to other cases as well. Take for example $b\to sl^+l^-$. Here, there is no tree diagram and the CP asymmetry, from loop $\times$loop interference, is [4] at most of order 1%. However, one can reach the $b\to sl^+l^-$ final state (with $l=e,\mu$) through the tree process $b\to s\psi$ followed by $\psi\to l^+l^-$, where ψ stands for any of the $c\bar{c}1^-$ resonances [13]. Therefore, an $\mathrm{Im(tree)}\times\mathrm{Re(loop)}$ interference term, with $\mathrm{Im(tree)}\sim\Gamma_\psi$ can provide another source of CP violation. In addition, even for top decays, extensions of the SM may enhance the branching ratios of loop processes by 3-4 orders of magnitude [8,14], thus causing an increase of up to 2 orders of magnitude in the tree$\times$loop interfer-

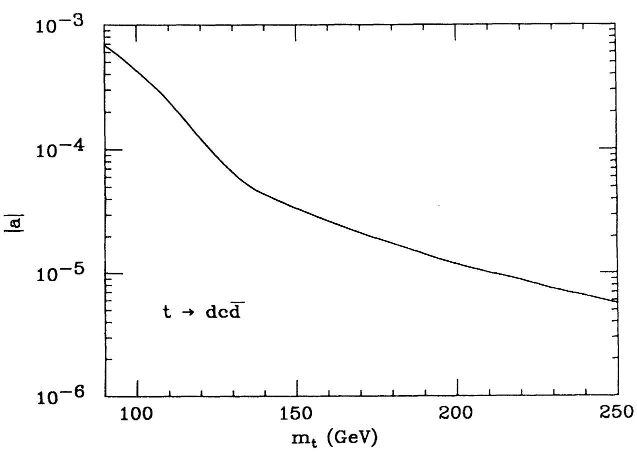

FIG. 1. The absolute value of the CP violation asymmetry in the standard model for $t\to dc\bar{d}$ as a function of m_t.

ence asymmetry. New phases from nonstandard couplings may enhance the asymmetry even more. Work along these lines is in progress.

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Note added.—After submission of the manuscript we became aware of previous work where particle widths are also suggested as a source for final-state phases in *CP*-odd phenomena [15]. However, they discuss it in a different context, namely, in production processes. The two works are therefore complementary.

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