Drag Reduction in Turbulent Flows by Polymers

J. K. Bhattacharjee
Department of Physics, Indian Institute of Technology, Kanpur, 208016, India

D. Thirumalai
Institute for Physical Science and Technology and Department of Chemistry and Biochemistry, University of Maryland, College Park, Maryland 20742

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A hydrodynamic model for turbulence in dilute polymer solution is used to describe drag reduction. It is shown that flexible polymers lead to an enhancement of molecular viscosity at small length scales while at intermediate length scales the effective viscosity is decreased. The onset of drag reduction is predicted to occur only above a minimum value of the polymer concentration, c_m. A direct calculation of the dependence of c_m on the energy input per unit mass, ϵ, shows that $c_m \sim \epsilon^{-1}$. Comparing this result with the scaling theory of de Gennes allows us to conclude that the exponent describing the power-law dependence of polymer deformation on spatial scale is $\frac{4}{5}$.

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It is well known that when small amounts of flexible polymer are added to a turbulent flow the overall drag is reduced [1,2]. The reduction in drag, in comparison to the pure solvent, can often be a factor of 2 or larger. The degree of effectiveness in inducing drag reduction (inferred by measuring pressure loss in flows through a pipe, for example) depends on the nature of the added polymer molecules. However, the general characteristics of polymer-induced turbulent drag reduction are the following: (1) the existence of the minimum value of the Reynolds number below which there is no apparent effect; (2) the percentage drag reduction increases with concentration of the polymer c but appears to saturate at a high enough value of c; (3) for a turbulent flow in the presence of a wall the scaling of the velocity profile with the distance from the wall is unchanged even in the presence of the polymers.

The most commonly accepted explanation for drag reduction due to Lumley [1,3] is based on the notion of effective viscosity increase. He argued that the polymer molecules outside the viscous sublayer in a turbulent flow bounded by a wall can become greatly stretched. The stretching of polymer molecules is facilitated by large strain rates generated due to the turbulent flow. Thus outside the viscous sublayer the solution containing the stretched polymer molecules has a large increase in the effective viscosity. The increase in viscosity leads to a thickening of the viscous sublayer (more precisely the spatial extent of the buffer layer separating the viscous sublayer and the inertial sublayer becomes larger). The viscosity in the thickened laminar sublayer corresponds to that of the polymer solution, and is not very different from that of the pure solvent because the polymer concentration is quite low. The reduced velocity gradient due to the increased thickness of the viscous sublayer results in the reduction of the Reynolds stress and hence the drag.

Recent experiments [4,5] in which the polymer molecules were injected at the center of the pipe have shown that drag reduction is observed even in situations where the presence of the wall plays no apparent role. This observation led Tabor and de Gennes [6,7] to propose an alternative explanation based on the idea that polymer molecules at small length scales (or equivalently high frequency) should exhibit elastic properties even when the polymer concentration is very small. They argued that the viscous effects which play a crucial role in the Lumley picture are not at all relevant for the phenomenon of drag reduction. Some of the assumptions of the scenario due to de Gennes and Tabor have been questioned by Ryskin [8] who has used a yo-yo model for polymer dynamics to calculate certain characteristics of the polymer-induced drag reduction in turbulent flows.

In this Letter we introduce a hydrodynamic model to describe the turbulent flow in dilute polymer solutions. The advantage of our approach is that for this model systematic calculations are in principle possible. Thus the effect of flexible polymer molecules on turbulent flows can be studied without resorting to any unjustifiable assumptions. The underlying flow of the solvent in our model is elongated and we consider the fluctuations about this basic flow. The fluctuating velocity field satisfies the usual Navier-Stokes equation for an incompressible fluid augmented by the reaction of the polymer molecules on the solvent. In addition, a random force is added which mimics the mechanism maintaining the underlying flow, and provides the energy input required for maintaining the turbulent flow. For simplicity, we assume that the dynamics of the polymer molecules are adequately described by the Rouse model [9]. The description in terms of a more realistic model that incorporates both the excluded-volume effects and hydrodynamic interactions should not qualitatively change the nature of our results.

In momentum space the appropriate Navier-Stokes equation may be written as

$$\dot{v}_i(k)+\nu_0 k^2 v_i(k) = \tfrac{1}{2}\sum_{j,l,k} P_{jlk}(k)v_j(k)v_l(k-p)+f_i+F_i^P , \quad (1a)$$

where

$$P_{ijl}(k)=k_j P_{il}(k)+k_l P_{ij}(k) , \quad (1b)$$

$$P_{ij}(k)=\delta_{ij}-k_i k_j/k^2 , \quad (1c)$$

and ν_0 is the (unrenormalized) kinematic viscosity. The statistics of the random force is assumed to be that used by DeDominicis and Martin [10] and has the correlation

$$\langle f_i(k,\omega)f_j(k',\omega')\rangle = \frac{c_0}{k^{D-4+y}}\delta(k+k')\delta(\omega+\omega')P_{ij}(k) , \quad (2)$$

where D is the spatial dimension and c_0 is a constant. The model described by Eqs. (1) and (2) with $F_i^P\equiv 0$ is known to produce the scaling laws found by Kolmogorov [11] for the special choice of $y=4$ [12-14]. If the underlying flow is elongated and if the concentration of the polymers is small (i.e., the solution is dilute) then the reaction of the polymer on the solvent has the form [15,16]

$$F_i^P=-c\int_{-\infty}^{t}dt'v_i(k,t')\sum_{p=1}^{\infty}\exp[-2(t-t')/\tau_p]\sum_i k_i^2/(1-\gamma_{ii}\tau_p) , \quad (3)$$

where c is the concentration of the polymer molecules and τ_p ($\propto N^2/p^2$) is the relaxation time of the pth Rouse mode [9]. The condition of elongated flow requires $\gamma_{11}=-2\gamma_{22}=-2\gamma_{33}=\gamma$ and $\gamma_{ij}=\delta_{ij}$ for $i\neq j$. In order to display the results of the model we will assume for the present that $\gamma_{ii}\tau_p\ll 1$. Thus our model excludes the possibility of the coil-stretch transition induced by the flow, and all the consequences arise due to the polymer-flow interactions through the nonlinear terms in the theory. The assumption $\gamma_{ii}\tau_p\ll 1$ also removes the anisotropy from the polymer-flow interaction and the approximate form for F_i^P becomes

$$F_i^P=-ck^2\int_{-\infty}^{t}dt'v_i(k,t')\sum_{p=1}^{\infty}\exp[-2(t-t')/\tau_p] . \quad (4)$$

Our hydrodynamic model is governed by Eqs. (1), (2), and (4).

Before outlining the calculation for our model we briefly recall the relevant known results for the pure solvent case [12-14] which corresponds to setting $F_i^P\equiv 0$. The nonlinear term in Eq. (1) dresses the bare viscosity ν_0 to produce a renormalized viscosity ν_R, where $\nu_R=\nu_0+\Delta\nu$. This effective viscosity is the consequence of the Reynolds stress in the problem and can be interpreted as the turbulent drag. Our purpose will be to see how $\Delta\nu$ is affected by the presence of small trace amounts of polymers. For positive values of the exponent y of the forcing term [cf Eq. (2)], $\Delta\nu=\Gamma k^{-y/3}$, and $\Delta\nu$ dominates the bare viscosity for all spatial scales less than k_0^{-1}, where

$$\Gamma k_0^{-y/3}=\nu_0 . \quad (5)$$

Notice that for $y=4$ Eq. (5) gives rise to the intrinsic Kolmogorov scale. The amplitude Γ for the exact scaling law of Eq. (5) has been computed by Yakhot and Orzag [12,13] to lowest order in perturbation theory for the model of the randomly stirred fluid.

The zeroth-order Green's function $G_0(k,\omega)$ for our hydrodynamic model is given by

$$G_0^{-1}(k,\omega)=-i\omega+\nu_0 k^2+ck^2\sum_{p=1}^{\infty}\frac{1}{-i\omega+2/\tau_p} . \quad (6)$$

The hydrodynamic ($\omega\tau_p\ll 1$) pole corresponding to Eq. (6) occurs at $\bar{\nu}k^2$, where $\bar{\nu}=\nu_0+c\sum_{p=1}^{\infty}\tau_p/2$. This is the enhancement of the viscosity at short length scales due to the presence of the polymers. We will establish that when the nonlinear term is included, the effective viscosity in the inertial range is decreased. This decrease is directly caused by the existence of several relaxation times in the zeroth-order Green's function given by Eq. (6). The decrease in the effective viscosity is computed by rewriting Eq. (6) as

$$G_0(k,\omega)=\sum_{i=1}^{\infty}A_i/(-i\omega+\alpha_i) , \quad (7a)$$

where the roots α_i to lowest order in k^2 are

$$\alpha_1=\bar{\nu}k^2,\quad \alpha_{i+1}=2/\tau_i\quad (i=1,2,\ldots) , \quad (7b)$$

and the coefficients A_i are

$$A_1=1,\quad A_{i+1}=-\tfrac{1}{2}ck^2\tau_i\quad (i=1,2,\ldots) . \quad (7c)$$

If we consider scales larger than that determined by the time criterion of Lumley, only the first term in Eq. (7a) dominates. In this case the scaling law corresponding to the pure solvent case holds, and the polymer-enhanced viscosity $\bar{\nu}$ is renormalized to $\bar{\nu}_R=\bar{\nu}+\Delta\bar{\nu}$, with $\Delta\bar{\nu}\sim\Gamma_p k^{-y/3}$.

We can calculate the correction to $\bar{\nu}_R$ by considering the full Green's function, and treating the nonlinear term perturbatively. If we are interested in the inertial regime, $k\ll k_0$ with k_0 being given by Eq. (5), then the molecular contribution to the Green's function can be neglected. The response function in the inertial regime becomes

$$G^{-1}(k,\omega)\simeq -i\omega+\Sigma(k,\omega) . \quad (8)$$

The self-energy $\Sigma(k,\omega)$ is evaluated to one-loop order in a self-consistent fashion using the appropriate dressed propagator [17]. The result for $\Sigma(k,\omega)$ may be used to calculate the effective viscosity $\Sigma(k,0)\equiv\Sigma(k)$ to first order in c and we find [18]

$$\Sigma(k)=\left(\frac{2k^2}{D-1}\right)\int_{p+q=k}\frac{d^Dp}{(2\pi)^D}\frac{b(k,p,q)}{\Sigma(p)}\frac{1}{p^{D-4+y}}\left[\frac{1}{\Sigma(p)+\Sigma(q)}-\frac{c}{2}\sum_j\tau_j\left(\frac{p^{2-y/3}}{L(q)}+\frac{q^{2-y/3}}{L(p)}\right)\right], \tag{9}$$

where

$$L(q)=2/\tau_j+\Sigma(q) \tag{10a}$$

and

$$b(k,p,q)=\frac{(k\cdot q)^3}{k^4q^2}-\frac{(p\cdot q)(k\cdot p)}{p^2k^2}+\left(\frac{D-3}{2}\right)\left(1-\frac{(p\cdot k)^2}{p^2k^2}\right). \tag{10b}$$

It should be emphasized that Eqs. (9) and (10a) are only valid in the inertial regime, $k\ll k_0$. It is obvious that when $c=0$ one obtains the scaling result $\Sigma(k)\sim k^{2-y/3}$. The integrals proportional to c give the required correction to scaling. To obtain the leading correction we neglect the momentum dependence in $L(q)$ and write

$$\Sigma(k,0)=I_0-c\sum_j I_1(j). \tag{11}$$

Examination of the integrals I_0 and $I_1(j)$ shows that I_0 scales as $k^{2-y/3}$ and I_1 scales as k^{6-y}, and thus the zero-frequency self-energy can be written as

$$\Sigma(k,0)=\Gamma k^{2-y/3}[1-c\Gamma_1 k^{4-2y/3}],\quad\text{for } k\ll k_0, \tag{12}$$

where the amplitude Γ has been computed earlier to lowest order [12]. The dimensional constant Γ_1 is obtained from the ratio of integrals I_0 and $I_1(j)$ [18]. The equation for $\Sigma(k,0)$, valid only in the inertial regime $k\ll k_0$, allows us to identify an effective scale-dependent kinematic viscosity $\nu(k)=\Gamma k^{-y/3}(1-c\Gamma_1 k^{4-2y/3})$. It is clear that in the inertial regime $\nu(k)$ dominates over the usual viscosity $\bar{\nu}=\nu_0+\sum_{p=1}^{\infty}\tau_p/2$ as long as $y>0$. Equation (12) can be used to identify a spatial scale k_p^{-1} induced by the polymer-flow interaction

$$c\Gamma_1=k_p^{2y/3-4}. \tag{13}$$

Thus the polymer-induced effect will dominate the effective kinematic viscosity $\nu(k)$ in the inertial regime if k_p is smaller than the intrinsic Kolmogorov scale k_0 given by Eq. (5). This implies that $(c\Gamma_1)^{1/(2y/3-4)}<(\Gamma/\nu_0)^{3/y}$ or setting $y=4$ as required for the Kolmogorov cascade picture of turbulence

$$c>c_m\equiv\nu_0/\Gamma\Gamma_1. \tag{14}$$

The effect of addition of polymer leads to an increase in viscosity given by $\bar{\nu}$ at smaller scales, and a decrease in viscosity given by $\nu(k)$ at larger scales. It is the effective decrease in viscosity in the inertial regime, provided c satisfies Eq. (14), which leads to the phenomenon of drag reduction. Our hydrodynamic model predicts that unless the concentration of the polymer exceeds a threshold given by Eq. (14) there will be no reduction in the drag. This existence of c_m is in accord with the scenario due to de Gennes who arrived at the same conclusion by using a very different picture. It should be pointed out that Lumley's picture [1,19] leads to the concept of intrinsic drag reduction implying that one should observe drag reduction in turbulent flows even in limit of zero polymer concentration.

It is worth pointing out the reason for the effectiveness of polymer molecules, in contrast to other substances such as normal liquids, in inducing the reduction in drag in turbulent flows. This becomes transparent when the N dependence of the polymer-induced scale $r_p=(c\Gamma_1)^{3/2}$ [cf. Eq. (13)] is examined. It can be shown [18] that for Rouse model $\Gamma_1\propto N^4$ which leads to the conclusion that drag reduction is achieved provided $r_p\propto c^{3/2}N^6$ is greater than the viscous dissipation scale, $r_0=(\nu_0/\Gamma)^{3/4}$, at which the Kolmogorov cascade is truncated. Thus if N is small then the concentration of added substance has to be sufficiently large to satisfy the inequality $r_p>r_0$. In the extreme case of simple liquids, without a spectrum of relaxation times, i.e., $N=1$, the flow at such large concentration of the solute would no longer remain turbulent, and hence drag reduction cannot be obtained. Our theory also makes it clear that many other substances (without N being really large) can induce drag reduction and hence this phenomenon should not be restricted to polymeric systems alone. However, because of the strong N dependence of r_p it is likely that the most effective drag reducers, while still maintaining the practical requirement that the flow be turbulent [1,2], are polymeric systems.

The dependence of c_m on ϵ the energy input per unit time can be found by noting that $\Gamma\sim\epsilon^{1/3}$ and $\Gamma_1\sim\epsilon^{2/3}$ and consequently $c_m\sim\epsilon^{-1}$. Using a scaling picture, de Gennes determined that $c_m\sim\epsilon^{1-3/4v}$, where v was an undetermined exponent. Comparison of the results obtained using our model with the scaling picture of de Gennes allows for a direct determination of the exponent v. This leads us to suggest that $v=\frac{3}{8}$. The crucial assumption in the Tabor-de Gennes scenario is that in the passive range the polymer molecule is elongated. The elongation of the polymer with spatial scale was assumed to be $\lambda(r) = (r^*/r)^n$, where r^* is the scale where the characteristic hydrodynamic frequency equals the polymer relaxation rate and n, the undetermined exponent, is related to v by

$$v = (5n/2 + 2/3)^{-1}. \tag{15}$$

Using the value of $v = \frac{3}{8}$ obtained for our model yields $n = \frac{4}{5}$. This value of $\frac{4}{5}$ is smaller than that conjectured by de Gennes. This implies that the polymer molecule undergoes less elongation than anticipated by the Tabor-de Gennes scaling theory. Finally, the dependence of c_m on N for our model can also be obtained and we find that [18]

$$c_m \sim N^{-x} \epsilon^{-1}, \tag{16}$$

where $x=4$ for the Rouse model and $x \sim 3.6$ for the Zimm model for which the Flory value of the exponent v $(=\frac{3}{5})$ is used. It is perhaps due to the strong dependence of c_m on N that a direct experimental determination of c_m for polymers containing long chains is difficult. Nevertheless, as pointed out by de Gennes careful experiments with different values of N can be used to show the existence of c_m.

Our hydrodynamic model shows that the polymer molecules do not affect the Kolmogorov cascade until the spatial scale $r_p \sim k_p^{-1}$ [cf. Eq. (13)] is reached. It is the presence of this scale caused by the polymer-flow interaction that leads to a decrease in the effective viscosity in the inertial regime, and hence to a reduction in drag. The physical reason for the decrease in viscosity at intermediate length scales is due to the existence of several time scales inherent in the polymer molecule. Thus our analysis suggests that the phenomenon of drag reduction should be fairly general, and should be observable whenever a sufficient amount of solute which has a spectrum of relaxation times (e.g., many organic compounds) is added to a turbulent flow. The crucial requirement is that the polymer-induced scale $r_p \sim cN^x$ ($x = 3.6$ for real polymers) be greater than r_0, the intrinsic Kolmogorov scale. Thus while many substances with a spectrum of relaxation times may be capable of inducing drag reduction the strong N dependence of r_p suggests that long polymer chains are likely to be the most effective drag reducers [19].

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[1] J. L. Lumley, Annu. Rev. Fluid Mech. **1**, 367 (1969); J. Polym. Sci. **7**, 263 (1973).

[2] W. M. Kulicke, M. Kotter, and H. Grager, Adv. Polym. Sci. **89**, 1 (1989).

[3] J. Lumley, Phys. Fluids **20**, 564 (1977); J. L. Lumley and I. Kubo, in *The Influence of Polymer Additives on Velocity and Temperature Fields*, edited by B. Grampert (Springer-Verlag, Berlin, 1985), p. 3.

[4] W. L. McComb and L. Rabie, Phys. Fluids **22**, 183 (1979).

[5] H. W. Bewersdorff, Rheol. Acta **21**, 587 (1982); **23**, 552 (1984).

[6] M. Tabor and P. G. de Gennes, Europhys. Lett. **2**, 519 (1986).

[7] P. G. de Gennes, Physica (Amsterdam) **140A**, 9 (1986).

[8] G. Ryskin, Phys. Rev. Lett. **59**, 2059 (1987).

[9] P. E. Rouse, J. Chem. Phys. **21**, 1272 (1953).

[10] C. DeDominicis and P. G. Martin, Phys. Rev. A **19**, 219 (1979).

[11] See, for example, H. Tennekes and J. L. Lumley, *A First Course in Turbulence* (MIT, Cambridge, MA, 1972).

[12] V. Yakhot and S. Orzag, Phys. Rev. Lett. **57**, 1722 (1986).

[13] D. Ronis, Phys. Rev. A **36**, 3322 (1987).

[14] J. K. Bhattacharjee, Phys. Rev. A **40**, 6374 (1989).

[15] Y. Rabin, S. Q. Wang, and K. F. Freed, Macromolecules **22**, 2420 (1989).

[16] S. Q. Wang, Phys. Lett. A **125**, 208 (1987).

[17] K. Kawasaki, Ann. Phys. (N.Y.) **61**, 1 (1970).

[18] D. Thirumalai and J. K. Bhattacharjee (to be published).

[19] J. L. Lumley, in *Topics in Applied Physics*, edited by P. Bradshaw (Springer-Verlag, Berlin, 1976), Vol. 12, Chap. 7.