Stabilizing High-Period Orbits in a Chaotic System: The Diode Resonator

E. R. Hunt

Condensed Matter and Surface Science Program and Department of Physics and Astronomy, Ohio University, Athens, Ohio 45701-2979

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The chaotic dynamics found in the diode resonator has been converted into stable orbits with periods up to 23 drive cycles long. The method used is a modification of that of Ott, Grebogi, and Yorke [Phys. Rev. Lett. **64**, 1196 (1990)]. In addition to stabilizing existing low-period orbits, the method allows making small alterations in the attractor permitting previously nonexistent periodic orbits to be stabilized. It is an analog technique and therefore can be very fast, making it applicable to a wide variety of systems.

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Recently, there have been considerable theoretical and experimental efforts to control chaos, that is, to convert the chaotic behavior found in many physical systems to a periodic time dependence. Ott, Grebogi, and Yorke [1] (OGY) proposed a general way to achieve this control using a feedback technique in which small, carefully chosen, time-dependent perturbations are made on one of the parameters of the system. Ditto, Rauseo, and Spano [2] implemented the OGY method in a periodically driven physical system, converting its chaotic motion into period-1 and period-2 orbits. Also, laminar flow has been produced in a previously unstable thermal convection loop by a thermostat-type feedback mechanism [3]. Spin-wave instabilities have been suppressed with a nonfeedback technique, the addition of a periodic field [4]. A nonfeedback procedure has also been shown to be feasible for the periodically driven pendulum [5] and the Duffing-Holmes oscillator [6]. None of these systems has been stabilized in a high-period orbit.

We have converted the chaotic dynamics found in a driven diode resonator system to a number of stable orbits with periods as long as 23 drive cycles. The technique used is a modification of the OGY method. Deviations of the chaotic variable within a specified window from a set point are fed back to perturb the controlling drive. In contrast to the OGY method, we allow fairly large perturbations which can change the chaotic system slightly, permitting high-period orbits. Our technique, which is completely analog, allows the system to find stable orbits by itself. In order to find these orbits one varies the set point, window, and amplitude of the feedback; and many different orbits automatically lock in. The technique is fast and could have a wide range of applicability.

It is pointed out in OGY that the presence of chaos, when controllable, can be advantageous. The orbit to be used in a given application can be chosen in order to maximize a system's performance. We have stabilized nineteen different orbits, all initially in the same chaotic regime. Furthermore, we can easily change from one orbit to another. The high-period orbits have the advantage that most regions of the attractor are visited. This is important because the different regions correspond to different physical states of the system, and one may want to sample as many of these as possible.

Ott, Grebogi, and Yorke [1] prescribe a method to transform a system initially in a chaotic state into a controlled periodic one. Their idea is that there exists an infinite number of unstable periodic orbits embedded in the attractor, and that only small, carefully chosen perturbations are necessary to stabilize one of these. Creating new orbits is purposely avoided by requiring that the perturbations be small. To obtain a period-1 orbit, a local map around the desired fixed point, including the stable and unstable directions, is constructed. When the chaotic variable is in the neighborhood of the fixed point, the perturbation is applied to a system parameter so that the next cycle (iterate) will fall on the stable manifold of the point. The chaotic variable will then move toward the fixed point in successive iterations. They verified their technique numerically using the Hénon map, extracting the period-1 orbit, which remained stable in the presence of added noise.

Ditto, Rauseo, and Spano [2] successfully used the OGY method in a physical system comprised of a gravitationally buckled magnetostrictive ribbon. They were able to achieve stable period-1 and period-2 orbits in the chaotic regime by making perturbations, limited to less than 9% in one of the system's available parameters. They found an approximate linear mapping function in the vicinity of the desired fixed point, and used this to calculate how much feedback to apply in order to move the fixed point into the neighborhood of the stable manifold. Because of experimental inaccuracies they could not get the system exactly on the stable manifold, and a new correction was applied each cycle. For period-2 the parameter was adjusted every other cycle.

By the very nature of chaos, high-period orbits are impossible to achieve by making only one correction in the long period. For example, our system has a largest Lyapunov exponent of approximately 0.4, allowing only five cycles on the average before any uncertainty in the chaotic variable increases tenfold. Thus any attempt to obtain a large-N periodic orbit must usually involve multiple corrections. Exceptions to this are possible when the period-N cycle is stable or nearly stable by itself. This situation occurs when one of the iterates falls near the extremum of the first return map.

The system used in this work is the diode resonator, which is composed of a p-n junction rectifier in series with an inductor. When driven with an increasing sinusoidal voltage, the system exhibits the classic period-doubling route to chaos [7]. It is a system well characterized by a two-dimensional map [8]. The peak forward current through the diode provides a convenient chaotic variable. Our resonator utilizes a 1N2858 diode and a 100-mH inductor with 25-Ω dc resistance, and is driven at 53 kHz.

Our method of control uses occasional proportional feedback. The peak current I_n is sampled, and if it is within a given window, the drive voltage is amplitude modulated with a signal proportional to the difference between I_n and the center of the window. If it is not within the window, no modulation signal is applied. The maximum correction is proportional to the size of the window and the system gain, both of which are adjustable. For low-period orbits the method is essentially equivalent to that of OGY. For longer orbits the perturbations can be large enough to alter the attractor slightly, so that a periodic orbit can exist where none did previously.

A block diagram of the system is shown in Fig. 1. The current through the resonator is converted to a voltage by the I/V device. If a peak current signal falls within the adjustable range of the window comparator, which is centered about zero, a trigger is generated for the timing circuit, which in turn generates pulses for the sample/hold (S/H) and the switch. The current peaks may be offset, so that control may be attempted for any amplitude. The deviation of the peak from zero is switched through an amplifier to become the control (feedback) signal, which amplitude modulates the signal generator. The switch can remain closed for a large fraction of a cycle; the remainder of the cycle is used by the S/H. The whole correction process takes less than 20 μs.

Stabilizing low-period orbits is very easy. For period-1, for example, one notes that the fixed point lies on the diagonal of the first return map. The window is opened around the point and the amplitude of the feedback is adjusted until locking occurs. However, it is not essential to know initially where the fixed points are. Scanning the adjustable controls quickly reveals many periodic orbits. Locked into period-1, the control signal is extremely small, corresponding to changes in the drive voltage of less than 0.5%. The technique is quite robust in that the drive voltage may be changed $\pm 10\%$ without losing control.

Figure 2 demonstrates the method for a controlled period-5 orbit. Figure 2(a) is a double exposure of the first return map of the current peaks in the chaotic regime with the five overexposed dots representing the controlled state. Note the small deviations of the dots from

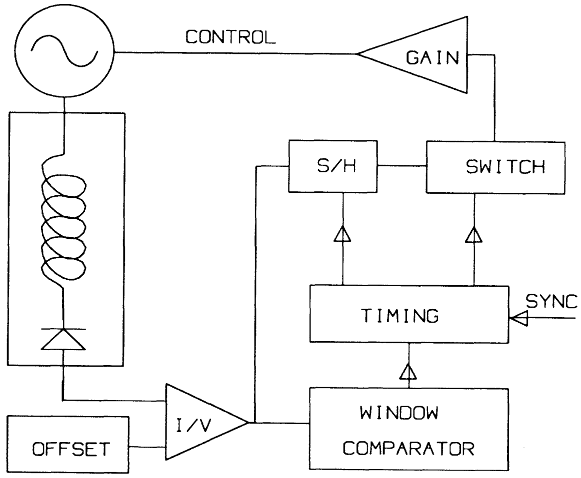

FIG. 1. Block diagram of the system. The diode resonator is boxed. The I/V is a current-to-voltage converter whose peak output is held by the sample/hold (S/H) if the peak is within the window. The held peak is switched and amplified to become the control or feedback signal, which amplitude modulates the signal generator.

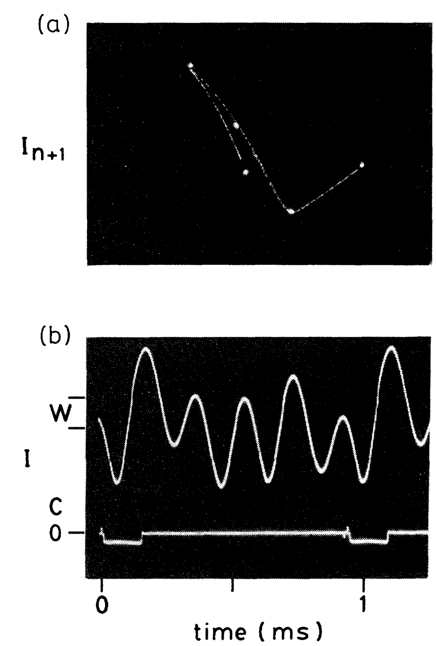

FIG. 2. (a) Double exposure showing the first return map (I_{n+1} vs I_n) and five overexposed dots representing the period-5 stabilized orbit (arbitrary scale). (b) Upper trace: The current through the resonator vs time. Also shown is the window (W). Lower trace: The control signal (C). Only the smallest peak is in the window.

the attractor showing the effect of the control signal. Period-5 is a stable orbit in the uncontrolled system. However, the drive voltage used here is about 5% less than that corresponding to the stable orbit which, therefore, would not exist without the feedback. Figure 2(b) shows the current through the resonator (upper trace) locked to period-5 by the feedback signal (lower trace). The window is designated by W. In this case the feedback signal lasts for only one drive period out of the five. The signal has a negative periodic component, which causes a 3% increase in the drive voltage. The feedback also has a fluctuating component (too small to be seen) which stabilizes the orbit.

A controlled period-21 orbit is shown in Fig. 3. The drive voltage is unchanged and the chaotic attractor is the same as for the period-5 case. As seen in Fig. 3(a), there are some deviations of the dots from the attractor, somewhat larger in this case due to the larger feedback signal. The upper trace of Fig. 3(b) shows the current through the resonator and the window used. The lower trace is the control signal showing six corrections in the 21 cycles. The largest correction shown here corresponds to a 9% modulation of the drive voltage. Some of the correcting perturbations could have no effect or even be opposite to those needed to control, but on the average they must stabilize the orbit or else these orbits would not be found. Note that the different regions of the attractor are represented by a number of dots.

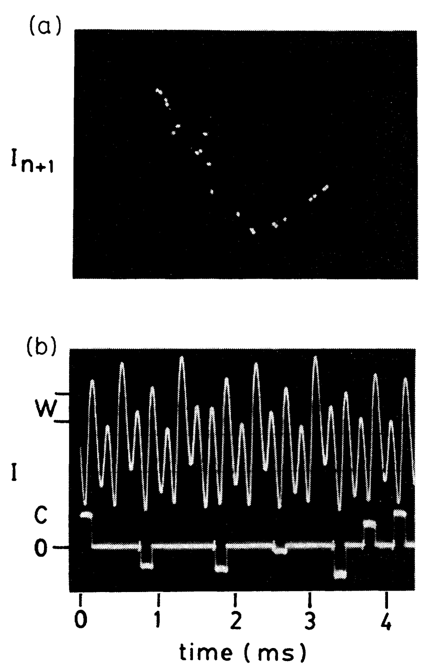

FIG. 3. (a) First return map with dots representing a period-21 orbit. (b) Upper trace: The current and the window used for this orbit. Lower trace: The control signal. The drive voltage is the same as that in Fig. 2.

By changing the level, width of the window, and gain of the feedback signal, we have obtained all periods up to 23 except 13, 14, 15, and 17. Perhaps these would require perturbations larger than we allow. Also, at just one level and gain, we have found periods of 2, 10, 12, 21, and 22 by simply adjusting the width of the window. At present we scan the level, width, and gain to stabilize the different periods. So far we have not found a systematic method to obtain a particular orbit. However, once the conditions are found for particular orbits, they are easily reproduced making it possible to rapidly switch between them.

In conclusion, we have demonstrated that many high-period orbits may be obtained in the chaotic regime of the diode resonator system. The technique involves proportional feedback to amplitude modulate the drive voltage. For low-period orbits only a small control signal is needed, in agreement with that expected from previous work. Larger perturbations, still less than 10% alter the attractor to some extent but can stabilize high-period orbits. These orbits visit most regions of the attractor. We feel that the versatility of the method more than offsets any disadvantage incurred by a small change in the attractor. The method should be applicable to a wide variety of systems. An important advantage is that it can be completely analog and, therefore, works at a fairly high speed.

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Note added.—After submission of this paper, Peng, Petrov, and Showalter [9] published a paper describing the control of a chaotic system by the same method as described here. Their system, which models a chemical reaction, is described by three coupled equations and a one-dimensional map. They stabilized period-1, -2, and -4.

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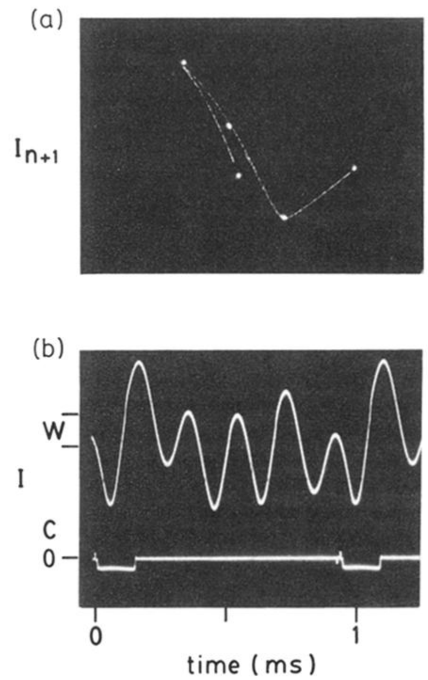

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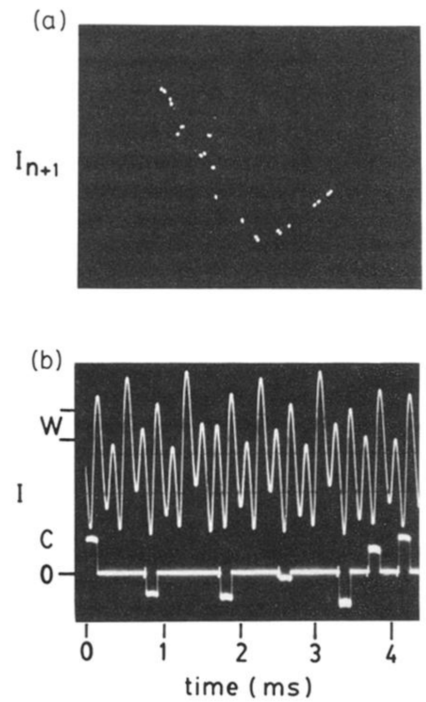

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