Spectral Analysis of the von Kármán Flow Using Ultrasound Scattering

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We report experimental evidence that the scattering of sound waves by a fluid flow may be used, as a nonintrusive and nonlocal method, to characterize the space-time structure of the flow. The experiment has been performed using, as a test flow, the von Kármán vortex street behind a cylinder at a low Reynolds number (Re=50). The results are in good qualitative agreement with earlier experiments on the von Kármán vortex street and with recent theoretical developments on sound-velocity interaction.

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The propagation of sound waves in moving fluids has been the subject of many experimental and theoretical papers [1,2]. An important issue in these studies is to understand whether the perturbations produced in the sound wave by the fluid motion may be used to measure relevant features of the flow. In a recent paper Lund and Rojas [3] have shown theoretically that ultrasound scattering may be used as a spectral probe of the spatial vorticity distribution, in the same way as light and neutron scattering are used in solid-state physics. Their result is rather similar to the one of Kraichnan [2] but it has the advantage of relating directly the scattered pressure with the spatial Fourier transform of the vorticity field, a quantity that plays a very important role in laminar and turbulent fluid flows [4]. The usual Fourier-transform properties imply that a good spectral resolution (i.e., in $\mathbf{k}$ space) is obtained to the detriment of spatial resolution. The method we propose here is thus quite different from other local techniques (i.e., in $\mathbf{x}$ space), among which are hot-wire anemometry and laser Doppler velocimetry [4–8].

Let us recall briefly the most important hypothesis used by Lund and Rojas to obtain their results. A plane sound wave, with velocity $\mathbf{V}_{\rm inc}$ and pressure $p_{\rm inc}(\mathbf{x},t)=P_0 \times\cos(\mathbf{K}\cdot\mathbf{r}-2\pi\nu_0 t)$, is incident on a target vorticity distribution $\boldsymbol{\omega}=\boldsymbol{\nabla}\times\mathbf{u}$, where $\mathbf{u}$ is the speed of the flow under study. The main hypotheses used in [3] are the following:

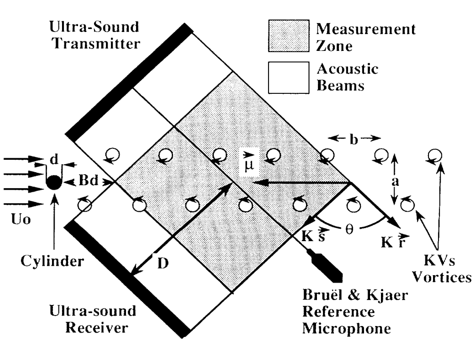

FIG. 1. Schematic diagram of the experimental apparatus, and schematic drawing of the von Kármán vortex street.

(a) $|V_{\rm inc}|\ll|u|\ll c=2\pi\nu_0/K$, where c is the speed of sound; (b) the time scale of the flow is much longer than ν_0^{-1}; and (c) the sound wave does not perturb the fluid flow.

All three hypotheses are satisfied if the flow under analysis is not in the high-Mach-number region. The coupling between the vorticity field and the sound wave comes from the nonlinear term $\mathbf{u}\cdot\boldsymbol{\nabla}\mathbf{V}_{\rm inc}+\mathbf{V}_{\rm inc}\cdot\boldsymbol{\nabla}\mathbf{u}$ in the Navier-Stokes equation [2,3]. Using the first Born approximation for scattering, it is possible to compute the pressure $p_{\rm scat}$ of the scattered wave as a function of the vorticity distribution only [3]. At a large distance $\mathbf{D}$ from the interaction region and scattering angle θ (see also Fig. 1), one gets for the Fourier transform in time $P_{\rm scat}$ of $p_{\rm scat}$ the following equation:

$$P_{\rm scat}(\mathbf{D},\nu)=P_0\frac{\nu i\exp(i\mathbf{K}\cdot\mathbf{D})}{c^2|\mathbf{D}|}\frac{\cos\theta}{1-\cos\theta}\times(\hat{\mathbf{r}}\times\hat{\mathbf{s}})\cdot\boldsymbol{\Omega}(\boldsymbol{\mu},\Delta\nu)\,, \tag{1}$$

with

$$\Delta\nu=\nu-\nu_0\,, \tag{2}$$

$$\boldsymbol{\mu}=(2\pi/c)(\nu\hat{\mathbf{r}}-\nu_0\hat{\mathbf{s}})\,, \tag{3}$$

where $\hat{\mathbf{r}},\hat{\mathbf{s}}$ are unit vectors in the incident and scattered directions, respectively, and $\boldsymbol{\Omega}(\boldsymbol{\mu},\Delta\nu)$ is the Fourier transform in space and time of the vorticity field $\boldsymbol{\omega}(\mathbf{r},t)$.

In order to check Eq. (1), we report in this Letter an experiment on ultrasound scattering by a fluid flow in the case of the von Kármán vortex street (KVS) that develops behind a cylinder [4] placed in a uniform velocity flow field. The KVS appears when the Reynolds number $\mathrm{Re}=U_0 d/\eta$ exceeds the critical threshold $\mathrm{Re}_c=45$ (U_0 is the mean flow velocity and η is the kinematic viscosity of the air). Line vortices parallel to the cylinder are emitted at the Strouhal frequency $f=(0.21\mathrm{Re}-4.5)\eta/d^2$, and their speed is $U_1=0.91U_0$. The distance $b=U_1/f$ between two corotating vortices (see Fig. 1) may be taken as constant although it slightly increases with the distance x from the cylinder. The choice of KVS has been made mainly because this instability generates a regular array of vortices which we expect to behave, for the incident wave, like a diffraction grating, producing max-

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imum scattering only along preferential directions. Furthermore, this flow has been well characterized by previous experiments and simple models may be found in the standard literature [4,9,10].

A schematic diagram of our experimental setup is shown in Fig. 1. A 50-cm×50-cm wind tunnel, 3 m long, is used to produce an air wind of speed of U_0, with a turbulent component, measured with a TSI hot-wire anemometer, less than 0.5%. A cylinder of diameter d is installed in the center of the wind tunnel. The vortices produced behind the cylinder pass through a sound beam whose distance B_d from the cylinder can be changed. The angle between the direction of the vortex propagation and that of the incident and scattered directions is $90° - \theta/2$. With such a geometry we probe the flow field at length scales corresponding to wave vector $\boldsymbol{\mu}$ whose components are

$$\mu_x = -2K\sin(\theta/2) = -4\pi(\nu_0/c)\sin(\theta/2)$$

and $\mu_y = 0$. It is thus simple to probe different length scales in the flow by simply changing ν_0 or θ. The transmitter and the receiver of sound are of the Sell type and have been constructed as described in [11]. They both have a 16-cm×16-cm area and respond from 5 to 100 kHz within 10 dB as we have checked using a calibrated BK4138 microphone. Two waveguides are placed in front of the receiver and transmitter to improve respectively the directivity in the detection and the divergency of the incident beam. The receiver is placed at a distance $D=1$ m from the interaction region of the KVS with the incident wave. The BK4138 microphone is installed at $\theta=0$ to measure the incident wave amplitude. The amplitude of the incident sound wave is $P_0 \simeq 1$ Pa which corresponds to $V_{\rm inc} \simeq 0.2$ cm/s. With such an arrangement we are almost in the far-field approximation; the measured diffraction angle is about 3° at a frequency of 40 kHz. All experiments were performed with a constant scattering angle equal to $\theta = 60°$. The output of the receivers are sent to an HP 3565 spectrum analyzer.

In order to probe the characterization length scale b of the KVS, we have tuned the emitter frequency ν_0 to ν_1 such that $|\mu_x| = 4\pi(\nu_1/c)\sin(\theta/2) \simeq 2\pi/b$ and recorded the time spectrum of the scattered pressure about ν_1. In Fig. 2 we report the modulus of the Fourier transform in time of $P(\boldsymbol{\mu},\Delta\nu)/P_0$ measured under the following experimental conditions: $d=0.3$ cm, $U_0=26$ cm/s, $B_d=0.5$ cm, $\theta=60°$, and $\nu_0=16.5$ kHz. In Fig. 2(a), the time spectrum has been obtained with the cylinder mounted perpendicular to the plane $(\hat{\mathbf{r}},\hat{\mathbf{s}})$. We see that, besides the peak at ν_0 (mainly due to diffraction effects), there is another peak at $\Delta\nu = -10.3$ Hz. This peak corresponds exactly to the Strouhal frequency f of the vortex street, as we have checked with hot-wire measurements. The sign of the frequency shift is that of the scalar product $\boldsymbol{\mu}\cdot\mathbf{U}_1$ (we have indeed verified that changing $\boldsymbol{\mu}$ to $-\boldsymbol{\mu}$ yields a positive frequency shift). We have varied ν_0 from 10 to 50 kHz but we did not observe any change in the peak position at $\Delta\nu = -f$ within our 30-mHz resolution. We interpret this result as a Doppler effect $(2\pi\Delta\nu = q_x U_1)$ where the sampled scattering wave vector q_x is constant. Indeed, for Reynolds numbers close to Re_c (here Re=52), the KVS is expected to be strictly periodic [9,10] in space with period b resulting in the existence of a fundamental mode $q_1 = 2\pi/b$ corresponding to a resonant frequency (where the scattered amplitude is maximum) $\nu_1 = cq_1/4\pi\sin(\theta/2)$ and eventual harmonics $(q_n = nq_1,\ \nu_n = n\nu_1)$. Because of diffraction effects on the incident sound wave the same q_1 can be probed within a range of frequencies ν_0 such that

$$\frac{cq_1}{4\pi\sin(\theta/2+\alpha)} \le \nu_0 \le \frac{cq_1}{4\pi\sin(\theta/2-\alpha)}\,, \tag{4}$$

where α is the maximum angle of diffraction.

Figure 3 shows the amplitude of the scattered peak as a function of ν_0. The symbols with connecting lines correspond to the experimental data. The bold curves are computed using in Eq. (1) the analytical model for the KVS in Ref. [9] together with a $\sin(x)/x$ shape for the sound beams (we have independently measured the diffraction profile). It is possible to show that the diffraction and the diffusion of the vortices are responsible for the broadening and the angular term of Eq. (1)

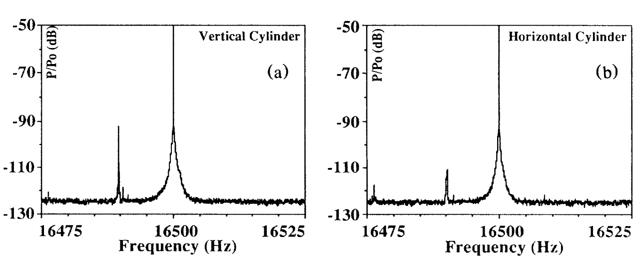

FIG. 2. Normalized modulus $P(\boldsymbol{\mu},\nu)/P_0(\nu_0)$ of the Fourier transform of the scattered pressure with the following experimental conditions: $d=0.3$ cm, $B_d=0.5$ cm, $\nu_0=16.5$ kHz, and $U_0=26$ cm/s. (a) Cylinder perpendicular to the scattering plane $(\hat{\mathbf{r}},\hat{\mathbf{s}})$. (b) Cylinder parallel to the scattering plane.

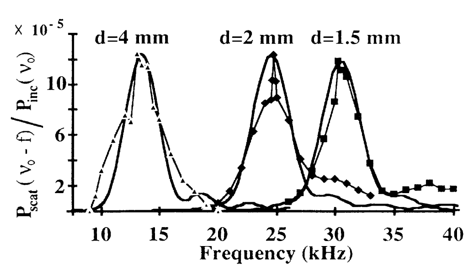

FIG. 3. Normalized modulus $P(\boldsymbol{\mu},\nu_0-f)/P_0(\nu_0)$ as a function of ν_0 at fixed scattering angle $\theta=60°$ for three different cylinders. Reynolds number is kept constant: $\mathrm{Re} \simeq 50$.

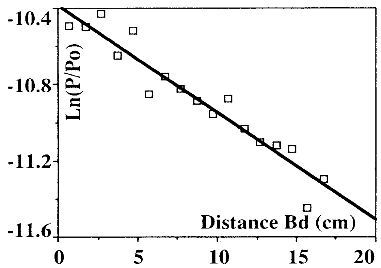

FIG. 4. Normalized modulus $P(\boldsymbol{\mu},\nu_0-f)/P_0(\nu_0)$ as a function of B_d with $d=0.4$ cm, $\nu_0=14$ kHz, and $U_0=20.5$ cm/s.

accounts for the asymmetry of the resonance curve. The experimental value of $(|P_{\rm scat}|/P_0)_{\max}\simeq 10^{-4}$ is in good agreement with the model. We have repeated the experiment for cylinders of various diameters and found that the spatial period b between the vortices is equal to $5.4d$, consistent with Ref. [7]. We did not observe a scattered signal at twice the fundamental value $\nu_2=33$ kHz, $q_2=4\pi/b$, due to the compensation effect of the counterrotating vortices on each side of the KVS.

In Fig. 2(b) the cylinder is mounted parallel to the plane $(\hat{\mathbf{r}},\hat{\mathbf{s}})$: The peak at -10.3 Hz is significantly reduced, in agreement with Eq. (1) which predicts that only the component of the vorticity perpendicular to the scattering plane contributes to the scattered pressure.

We have also measured the decay of the amplitude of $P_{\rm scat}$ as a function of B_d for the following experimental conditions: $d=4$ mm, $U_0=20.5$ cm/s, and $\nu_0=14$ kHz. Figure 4 shows the relative amplitude of the scattered peak at $\Delta\nu=-f$ as a function of the distance B_d in a log-linear scale. We expect an $\exp(-\mu_x^2 B_d\eta/U_1)$ decay law characteristic of the diffusion of vorticity [this behavior is readily obtained from the dynamical equation for Fourier modes of ω, $(\partial/\partial t+i\mu_x U_1+\eta\nu_x^2)\Omega_z(\mu_x,t)=0$, if three-dimensional effects are neglected]. The solid line shows the best linear fit, giving an experimental slope of $0.056\ {\rm cm}^{-1}$ in agreement with the 0.054-cm^{-1} theoretical value.

In conclusion, on the basis of our experiment, the interaction of a sound wave and fluid flow is coherent with a description in terms of sound scattering by the vorticity field of the flow in agreement with the calculation of Lund and Rojas [3]. The scattering wave vector can be easily adjusted by changing either the scattering angle or the frequency of the incident wave, providing a convenient way to probe the flow at desired length scales. Because of the frequency limitation of the spectrum analyzer (51 kHz range), we have not investigated length scales corresponding to the inner structure of a single vortex in the street. The main limitations to the actual experimental method are due to diffraction effects: First, these limit the spectral resolution as Fig. 3 shows; second, through the existence of the frequency line at the frequency of the incoming sound as in Fig. 2, they limit the dynamics of the measurement. We hope to improve both counts by the construction of larger sound transceivers and by numerical deconvolution of the data.

We believe that these results could have very useful applications to the study of turbulent flows. Because of the importance of spatial structures in these flows, $\Omega(\mathbf{k},\nu)$ is a quantity as relevant as $\Omega(\mathbf{x},t)$. If applicable, the method described here would yield a direct measurement of the vorticity field in $\mathbf{k}$ space.

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