Stringy Domain Walls and Target-Space Modular Invariance

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Based on the target-space modular invariance of the nonperturbative superpotential of the four-dimensional $N=1$ supersymmetric string vacua, we find topologically stable, stringy domain walls of nontrivial compactification modulus field configurations. They are supersymmetric solutions, thus saturating the Bogomolnyi bound. Their physical implications are discussed.

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Topological defects are known to be present in phase transitions triggered by spontaneously broken symmetries of a given theory. They can have very important physical implications, especially for cosmological consequences of the theory. It is thus interesting to explore the existence of these configurations in four-dimensional string vacua. Recently, cosmic string solutions were found in perturbative string theory [1]. We will present domain-wall solutions from nonperturbative potentials in four-dimensional string vacua [2].

In (2,2) and (0,2) superstring vacua in four dimensions with $N=1$ space-time supersymmetry there generically exist two types of moduli fields: the space-time dilaton and axion field S and the compactification dilaton and axion field T, which corresponds to the internal size of the compactified space [3]. The S and T fields have no potential at the string tree level as well as to all orders in string perturbation [4]. On the other hand, it is known that nonperturbative stringy effects like gaugino condensations [5] and axionic string instantons [6] give rise to nonperturbative superpotentials.

If one assumes that generalized target-space duality is an exact symmetry of string theory, the general form of the four-dimensional effective supergravity Lagrangian is required to be modular invariant [7,8]. The generalized target-space duality is characterized by the noncompact discrete group $PSL(2,\mathbb{Z})=SL(2,\mathbb{Z})/\mathbb{Z}_2$ specified by the linear fractional transformations $T\to(aT-ib)/(icT+d)$ with $a,b,c,d\in\mathbb{Z}$ and $ad-bc=1$. The fact that this is an exact symmetry even at the level of nonperturbative effects is supported by the genus-one threshold calculations [9,10] which in turn specify the form of the gaugino condensate [11-14]. In this Letter we shall therefore study stringy domain walls of $N=1$ supersymmetric four-dimensional superstring vacua by taking into account the modular-invariant superpotential of the T modulus field.

In fact, the physics of moduli fields is an intriguing generalization of the well-known axion physics [15] introduced to solve the strong CP problem in QCD. Below a scale f_a, the U(1) Peccei-Quinn symmetry is spontaneously broken and it is realized nonlinearly through a pseudo Goldstone boson, the invisible axion Θ. The low-energy effective Lagrangian of the invisible axion is described by

$$L_{\rm eff}=\frac{f_a^2}{2}(\nabla\Theta)^2+\frac{1}{4g^2}F^a_{\mu\nu}F^{\mu\nu a}+\frac{N_f}{32\pi^2}\Theta F^a_{\mu\nu}\tilde{F}^{\mu\nu a}\,. \qquad (1)$$

The nonlinearly realized Peccei-Quinn U(1) symmetry, $\Theta\to\Theta+\alpha$, where $\alpha=$const, is *explicitly* broken due to the nonperturbative QCD effects through the axial anomaly $\partial_\mu J^\mu_5=(N_f/32\pi^2)F^a_{\mu\nu}\tilde{F}^{\mu\nu a}$. Integrating out the instanton effects by a dilute instanton approximation [16,17] generates an effective potential proportional to $1-\cos N_f\Theta$ with degenerate minima at $\Theta=2\pi k/N_f$, where $k=0,1,\ldots,N_f-1$, thus breaking the original U(1) Peccei-Quinn symmetry down to a *discrete* subgroup $Z_{N_f}\in U(1)$. Note that, within the dilute instanton approximation, the form of nonperturbatively generated axion potential is completely determined by the invariance under the residual discrete subgroup Z_{N_f}. It is well known that these potentials lead to domain-wall solutions between adjacent vacua [18]. N_f domain walls meet at the axionic string.

In our case, the modulus field T possesses a nonlinearly realized noncompact symmetry $SU(1,1)/U(1)\approx SL(2,\mathbb{R})/U(1)$ to all orders in the sigma model perturbation. The real and imaginary parts of the modulus field T are nothing but the Goldstone bosons associated with spontaneously broken dilatation and axial symmetries, which are generalizations of the Peccei-Quinn symmetry in the invisible-axion physics. In a completely analogous way the world-sheet instanton effects [19] or space-time axionic instanton effects [6] break the above nonlinearly realized global symmetry to a discrete subgroup of it, $PSL(2,\mathbb{Z})$. Therefore, the superpotential should be a holomorphic function of T transforming covariantly under the above discrete subgroup. Indeed, it is known that the modular invariance puts strong constraints on the dynamical supersymmetry breaking and the stability of the vacua [8].

As an instructive and illuminating example we first consider a global supersymmetric theory by turning off gravity. The effective Lagrangian reads

$$L=\int d^2\theta d^2\bar{\theta} K(T,\bar{T})+\int d^2\theta W(T)+\int d^2\bar{\theta}\bar{W}(\bar{T})$$
$$=G_{T\bar{T}}|\nabla T|^2+G^{T\bar{T}}|\partial_T W(T)|^2 . \tag{2}$$

Here, $G_{T\bar{T}}\equiv\partial_T\partial_{\bar{T}}K(T,\bar{T})$ is the positive-definite metric on the complex modulus space and W is the superpotential, which has to be a modular-invariant (weight zero) form of PSL(2,$\mathbb{Z}$) defined over the fundamental domain $\mathcal{D}$ of the T field. The most general form of the superpotential is a rational polynomial $P(j(T))$ of the modular-invariant function $j(T)$ [20].

Since the symmetry group is a discrete group [21], it is natural to expect that the semi-positive-definite potential

$$V\equiv G^{T\bar{T}}|\partial_T W(T)|^2=G^{T\bar{T}}|\partial_j P(j)\partial_T j(T)|^2 \tag{3}$$

has a discrete set of degenerate minima, and thus there is a stable domain-wall solution (as suggested in [11]) [22], i.e., in this case the homotopy group $\Pi_0(\mathcal{M})$ is nontrivial. The term $|\partial_T j(T)|^2$ has two isolated zeros at $T=1$ and $T=\rho\equiv e^{i\pi/6}$ in the fundamental domain $\mathcal{D}$ for T [20]. Other isolated degenerate minima might as well arise when $|\partial_j P(j)|^2=0$.

We embed the domain wall in the (x,y) plane, which is thus perpendicular to the z direction. Then, the mass per unit area of the domain wall is

$$\mu\equiv\frac{E}{\int dx\,dy}=\int_{-\infty}^{\infty}dz[G_{T\bar{T}}|\partial_z T|^2+G^{T\bar{T}}|\partial_T W(T)|^2] . \tag{4}$$

We now look for static, spatially nontrivial field configurations that minimize the mass per unit area in Eq. (2). We can rewrite Eq. (4) as [23]

$$\mu=\int_{-\infty}^{\infty}dz\,G_{T\bar{T}}|\partial_z T-e^{i\theta}G^{T\bar{T}}\partial_{\bar{T}}\bar{W}(\bar{T})|^2$$
$$+2\,\mathrm{Re}(e^{-i\theta}\Delta W) , \tag{5}$$

where $\Delta W\equiv W(T(z=\infty))-W(T(z=-\infty))$. The arbitrary phase θ has to be chosen such that $e^{i\theta}=\Delta W/|\Delta W|$, thus maximizing the cross term in Eq. (5). Then, we find $\mu\geq K\equiv 2|\Delta W|$, where K denotes the kink number. Since $\partial_T W$ is analytic in T, the line integral over T is *path independent* as for a conservative force. The minimum is obtained only if the Bogomolnyi bound

$$\partial_z T(z)=G^{T\bar{T}}e^{i\theta}\partial_{\bar{T}}\bar{W}(\bar{T}(z)) \tag{6}$$

is saturated. In this case

$$\partial_z W(T(z))=G^{T\bar{T}}e^{i\theta}|\partial_T W(T(z))|^2 ,$$

which implies that the phase of $\partial_z W$ does not change with z. Thus, the supersymmetric domain wall is a mapping from the z axis $[-\infty,\infty]$ to a *straight line* connecting two degenerate vacua in the W plane. The domain wall is stabilized by the topological kink number $K=\pm 2\times|\Delta W|$. We would like to emphasize that this result is general; it applies to any globally supersymmetric theory with disconnected degenerate minima that preserve supersymmetry.

The above observation is neatly described by using the supersymmetry transformations of the moduli superfields. The moduli field Lagrangian possesses an enhanced $N=2$ space-time supersymmetry [24]. The nonrenormalization theorem of $N=2$ supersymmetry in turn guarantees no quantum correction to the mass density of the stringy domain walls [25]. This can be seen as follows. Denoting the "modulino" as χ, supersymmetry charges are

$$\mathbf{Q}_\alpha=(\gamma^\mu\chi)_\alpha\nabla_\mu T+\chi_\alpha G^{T\bar{T}}\partial_{\bar{T}}\bar{W}(\bar{T}) ,$$
$$\bar{\mathbf{Q}}_\alpha=(\bar{\chi}\gamma^\mu)_\alpha\nabla_\mu\bar{T}+\bar{\chi}_\alpha G^{T\bar{T}}\partial_T W(T) . \tag{7}$$

The anticommutator of supercurrents contains the Hamiltonian density and a total derivative, central charge term. The latter is nothing but the aforementioned kink number. We now introduce constant, chiral spinors $\epsilon_\pm$ of unit norm, $\bar{\epsilon}_\pm\cdot\epsilon_\pm=1$. Then, in the rest frame of domain wall, we find

$$\mu\mp K=\tfrac{1}{2}\int dz\,\bar{\epsilon}_\pm\{\mathbf{Q}_\alpha,\bar{\mathbf{Q}}_\alpha\}\epsilon_\pm\geq 0 . \tag{8}$$

Here, K denotes the kink number we derived in Eq. (5). Thus, the Bogomolnyi bound is achieved for $\delta_\epsilon\chi\equiv\epsilon\cdot\mathbf{Q}\chi=0$. It is identical to Eq. (6) in which θ is identified with the relative phase between ϵ_+ and ϵ_-.

We shall now illustrate the above points with the explicit choice for the superpotential $W(T)=(\alpha')^{-3/2}j(T)$, with α' being the string tension. As already discussed before, in the fundamental domain $\mathcal{D}$ the potential has two isolated degenerate minima at $T=1$ and $T=\rho\equiv e^{i\pi/6}$. At these fixed points, $j(T=\rho)=0$ and $j(T=1)=1728$. Therefore, the mass per unit area is $\mu=2\times 1728(\alpha')^{-3/2}$. Other cases can be worked out analogously and will be presented elsewhere [26].

Naive application of Eq. (5) implies that the domain-wall solution between the minima that are connected by the PSL(2,$\mathbb{Z}$) transformations has zero energy stored since W has the same value at those points. However, one can show that in the fundamental domain $\mathcal{D}$ there are always *at least* two degenerate minima with different values of the superpotential, and thus the energy density of the domain wall that interpolates between these two minima is *nonzero*. The energy density of the domain walls interpolating between the minima connected by the PSL(2,$\mathbb{Z}$) transformations are thus in turn determined by taking the path through all the minima in between. This adjusts the constant phase θ between the adjacent minima to maximize the cross term in Eq. (5). For example, for $W(T)=(\alpha')^{-3/2}j(T)$ the energy density stored in the domain wall that interpolates between $T=e^{i\pi/6}$ and $T=e^{-i\pi/6}$ is $\mu=2\times2\times1728(\alpha')^{-3/2}$.

We shall now study the case with gravity restored. For a nonvanishing superpotential, the $N=1$ supergravity action is described by a function

$$G(T,\bar{T})=K(T,\bar{T})+\ln W(T)+\ln\bar{W}(\bar{T}), \tag{9}$$

which should be modular invariant. Then, the scalar Lagrangian reads

$$e^{-1}L=-\mathcal{R}+G_{T\bar{T}}\nabla_\mu\bar{T}\nabla^\mu T+e^G[G^{T\bar{T}}|G_T|^2-3]. \tag{10}$$

Since $K=-3\ln(T+\bar{T})$ at tree level, the superpotential should transform as a weight -3 modular function under modular transformations [7,8]. The most general choice, nonsingular everywhere in the fundamental domain $\mathcal{D}$, is

$$W_{m,n}(T)=H_{m,n}(T)/\eta(T)^6,$$
$$H_{m,n}\equiv[j(T)-1728]^{m/2}\times j^{n/3}(T)P(j(T)),\quad m,n=\mathbb{R}^+. \tag{11}$$

Here, $\eta(T)$ is the Dedekind eta function, a modular form of weight $\frac{1}{2}$, and $P(j(T))$ is an arbitrary polynomial of $j(T)$. The potential is of the following form:

$$V_{m,n}(T,\bar{T})=\frac{3|H|^2}{(T+\bar{T})^3|\eta|^{12}}\left[\left|\frac{T+\bar{T}}{3}\left(\frac{\partial_T H}{H}+\frac{3}{2\pi}\hat{G}_2\right)\right|^2-1\right], \tag{12}$$

where $\hat{G}_2=-4\pi\partial_T\eta/\eta-2\pi/(T+\bar{T})$. In general the scalar potential (12) has an anti-de Sitter minimum with broken supersymmetry [8]. However, one can see that for $m\geq2$, $n\geq2$, and $P(j)=1$, the potential is semipositive definite with the two isolated minima at $T=1$ and $T=\rho$ with unbroken local supersymmetry just like in the global supersymmetric case [27].

We now minimize the domain-wall mass density. By the planar symmetry, the most general *static* ansatz for the metric [28] is

$$ds^2=A(|z|)(-dt^2+dz^2)+B(|z|)(dx^2+dy^2)$$

in which the domain wall is oriented parallel to the (x,y) plane. Using the supersymmetry transformation laws

$$\delta\psi_{\mu\alpha}=[\nabla_\mu(\omega)-\tfrac{1}{2}i\,\mathrm{Im}(G_T\nabla_\mu T)]\epsilon_\alpha+\tfrac{1}{2}(\sigma_\mu\bar{\epsilon})_\alpha e^{G/2},$$
$$\delta\chi_\alpha=\tfrac{1}{2}(\sigma^\mu\bar{\epsilon})_\alpha\nabla_\mu T-e^{G/2}G^{T\bar{T}}G_{\bar{T}}\epsilon_\alpha, \tag{13}$$

with commuting, covariantly constant, chiral spinors $\epsilon_\pm$, the Arnowitt-Deser-Misner mass density μ can be expressed as [29]

$$\mu\mp K=\int dz\sqrt{g}[g_{ij}\delta\psi^{\dagger i}\delta\psi^j+\tfrac{1}{2}G_T\bar{T}\delta\chi^\dagger\delta\chi]\geq0. \tag{14}$$

The i,j indices are for spatial directions. The minimum of the Bogomolnyi bound is achieved if Eq. (14) vanishes. Again, the stringy domain wall is stabilized by the topological kink number.

Unfortunately, the nice holomorphic structure of the scalar potential is lost. In other words, there is now a *holomorphic anomaly* in the scalar potential due to the supergravity coupling. This implies that the path connecting two degenerate vacua in superpotential space is *not* a straight line. In fact, one can understand the motion as a *geodesic* path in a nontrivial Kähler metric, and thus in $G(T,\bar{T})$. One can show (numerically) that in our example the path along the circle $T=\exp[i\theta(z)]$, with $\theta=(0,\pi/6)$, i.e., the self-dual line of $T\to1/T$ modular transformation, is the *geodesic* path connecting between $T=1$ and $T=\rho$ in the scalar potential space. Thus, we have again established the existence of stable domain walls. The superpotential is quite complicated, however, and we were not able to find an analytic solution of the domain walls. We will present the numerical solution in our forthcoming paper [26].

It is interesting to note that stringy cosmic strings [1] can be viewed as boundaries of our domain walls. Because the domain-wall number is two, the intersection of two such domain walls is precisely the line of stringy cosmic strings. On the other hand, such stable domain walls are disastrous from the cosmological point of view. One possible solution to this problem is that after supersymmetry breaking, the degeneracy of the two minima is lifted. In that case, the domain wall becomes unstable via the false vacuum decay [30].

In this Letter, we examined the stringy domain walls that appear generically for modular-invariant, four-dimensional $N=1$ supersymmetric string vacua. Since the solution is supersymmetric, there are *four-dimensional chiral* fermionic zero modes on the domain wall. These "chiral" fermionic zero modes imply that the domain walls are superconducting. Still, the domain walls remain anomaly-free [26]. Even though we have restricted ourselves to the T modulus field, one can generalize the argument to the S modulus field by extending the modular invariance to the S field as well [31]. This is a stringy generalization of the weak-strong-coupling duality [6,32]. In this case the components of S couple to the gauge fields like $1/g^2$ and Θ as in Eq. (1). For a U(1) gauge field, and the domain-wall background for S, these couplings can be interpreted as *spatially varying* effective dielectric constant and magnetic susceptibility ($\epsilon=\mu^{-1}=\mathrm{Re}S$). Also the *CP*- and *P*-violating background (depending on ImS) can induce Faraday rotation of chiral waves such as circularly polarized photons. Details containing the above issues will be reported in a separate publication [26].

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