Can the Sun Shed Light on Neutrino Gravitational Interactions?

A. Halprin and C. N. Leung

Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716

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We have examined the effects of a large gravitational field on the phenomenon of neutrino oscillations as contemplated in the Mikheyev-Smirnov-Wolfenstein mechanism. We find that the Sun's gravitational field would amplify any small breakdown in the universality of the gravitational coupling by many orders of magnitude. A breakdown of only 1 part in 10^{14} would still make the gravitational effect comparable to the conventional weak interaction. The differing energy dependences of the two level-crossing mechanisms can therefore be used as a very sensitive tool to test the conventional universality hypothesis.

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While we do not as yet have a universally accepted quantum theory of gravity, it is generally believed that at energies well below the Planck scale the effects of gravity on elementary particles can be approximated by a modification of the metric tensor. In particular, the Klein-Gordon equation for a scalar field $\Phi(x)$ becomes

$$\{[g_{\mu\nu}+h_{\mu\nu}(x)]\partial^\mu\partial^\nu+m^2\}\Phi(x)=0\,, \tag{1}$$

where $g_{00}=-g_{11}=-g_{22}=-g_{33}=1$ is the Minkowski metric and $h_{\mu\nu}$ is the modification due to gravity. In one formulation, the corresponding modification to the Dirac equation involves a vierbein, which is related to $h_{\mu\nu}$. However, if derivatives of the vierbein are ignored, which, as explained later, should be a good approximation for the situation we will discuss, then a Dirac particle will also satisfy the curved-space Klein-Gordon equation given above. Furthermore, it suffices to consider the Klein-Gordon equation for a Dirac particle since we will only be interested in effects which do not involve a spin flip. Thus, the neutrino fields $\psi(x)$ satisfy

$$\{[g_{\mu\nu}+Fh_{\mu\nu}(x)]\partial^\mu\partial^\nu+M^2\}\psi(x)=0\,, \tag{2}$$

where M is the neutrino mass matrix, and we have introduced a square matrix F to parametrize the universality of the gravitational interaction of neutrinos. We shall assume that F is diagonal in the neutrino mass basis and denote the diagonal elements in that basis by f_j. It is conventionally assumed that all f_j are equal (more precisely, $f_j=1$). In other words, the gravitational interaction of neutrinos is usually assumed to be universal. This universality is, of course, directly related to the weak equivalence principle (WEP). The WEP has been tested to about 1 part in 10^{11} for ordinary matter [1]. However, there exists no direct experimental limit to date on the universality of the gravitational couplings to different neutrino species [2].

In this Letter we propose the first direct test of the universality of the neutrino gravitational couplings. We will show that a violation of universality as small as 1 part in 10^{14} can be amplified by the Sun's gravitational field and lead to observable modifications of the Mikheyev-Smirnov-Wolfenstein (MSW) effect [3-5]. Alternatively, the experiment proposed here can be used to improve the current limit on the WEP by 3 orders of magnitude.

We shall entertain the possibility that f_j are different for different neutrino mass eigenstates and ask if and how well solar neutrino experiments can set limits on such a breakdown of universality. We restrict ourselves to the gravitational field produced by a static source. In the weak-field approximation and in the harmonic gauge, $h_{\mu\nu}$ can be written as

$$h_{\mu\nu}=2\phi(\mathbf{r})\delta_{\mu\nu}\,, \tag{3}$$

where $\phi(\mathbf{r})$ is the Newtonian gravitational potential which vanishes asymptotically. Following Ref. [6], we let the differing neutrino masses be reflected in their three-momenta rather than their energies and factor out a common time dependence to obtain the time-independent Klein-Gordon equation,

$$\begin{aligned}&\{E^2[1+4F\phi(\mathbf{r})]+\nabla^2-M^2\}\xi(\mathbf{r})=0\,,\\&\psi=\xi\exp(-iEt)\,,\end{aligned} \tag{4}$$

where $O(\phi^2)$ and $O(M^2\phi)$ terms have been neglected. We consider a spherically symmetric potential, and therefore it suffices to study only the radial motion for an S wave. Moreover, any reasonable macroscopic gravitational field will not vary appreciably on the scale of the neutrino's Compton wavelength (i.e., the derivative of the gravitational field will give us a factor of order $1/R_{\rm Sun}$, where $R_{\rm Sun}$ is the mean radius of the Sun), so that derivatives of $\phi(\mathbf{r})$ can safely be ignored. This is also why we may neglect derivatives of the vierbein when we obtain the Klein-Gordon equation for the neutrinos. We factorize the differential operator in Eq. (4), ignoring derivatives of ϕ, and take the outgoing wave to obtain the first-order differential equation

$$\left[-i\frac{d}{dr}-K(r)\right](r\xi)=0\,, \tag{5}$$

where

$$\begin{aligned}K(r)&=\{E^2[1+4F\phi(r)]-M^2\}^{1/2}\\&\cong E-M^2/2E+2F\phi(r)E\,.\end{aligned} \tag{6}$$

For simplicity we restrict our discussion to a two-flavor system, say e and μ, and introduce the quantities

$$\Delta m^2 = m_2^2 - m_1^2, \quad m_2 > m_1, \tag{7}$$

and

$$\Delta f = f_2 - f_1. \tag{8}$$

By custom, the mixing of this system is to be described by an "effective Hamiltonian" in which $r \to t$, and in which terms proportional to the identity matrix are dropped. From Eq. (6) we see that the effective Hamiltonian is given in the mass basis by

$$H_{\rm eff} = \begin{pmatrix} 0 & 0 \\ 0 & \Delta m^2/2E - 2\Delta f E \phi \end{pmatrix}. \tag{9}$$

The criterion for a level crossing to occur is therefore

$$\Delta m^2/2E - 2\Delta f E\phi \le 0. \tag{10}$$

Thus, gravitation can produce a level crossing if $\Delta f \neq 0$ and ϕ is sufficiently large.

The level-crossing condition for the conventional MSW effect is, for small vacuum mixing angle, given in terms of the Fermi coupling G_F and the electron density N_e by

$$\Delta m^2/2E - \sqrt{2} G_F N_e < 0. \tag{11}$$

We want to draw attention to the fact that no factor of E multiplies the Fermi coupling in this condition, while Eq. (10) has a factor of E multiplying ϕ. Consequently, the two level-crossing mechanisms can, in principle, be separated in experiments that carefully measure the solar neutrino energy spectrum. For a comparison of the strength of the gravitational effect with the weak-interaction MSW effect, we need only consider

$$\eta = |2\Delta f E\phi/\sqrt{2} G_F N_e|. \tag{12}$$

For the purpose of an order of magnitude estimate, we approximate the mass density within the Sun by $(m_p + m_n) N_e$ and the electron density by a uniform distribution, so that

$$\phi \sim 2\pi G_N (m_p + m_n) N_e (r^2/3 - R_{\rm Sun}^2), \quad r < R_{\rm Sun}, \tag{13}$$

where m_p and m_n are the mass of the proton and neutron, respectively. Since the relevant r is much smaller than $R_{\rm Sun}$, we find

$$\eta \sim 10^{+14} [E/(1\ {\rm MeV})] |\Delta f|. \tag{14}$$

Thus, a violation of gravitational universality at a level as low as 10^{-14} is as effective a level-crossing mechanism as the conventional MSW effect.

Although the gravitational effect can make the levels cross, it cannot induce crossings to take place, since it has been assumed diagonal in the mass basis. In order to induce a level crossing, we still have to appeal to the weak-interaction effect. Putting the weak and gravitational interactions together, the effective Hamiltonian can be written *in the flavor basis* as

$$H = \begin{pmatrix} \sqrt{2} G_F N_e + Q \sin^2\theta_v & Q \sin\theta_v \cos\theta_v \\ Q \sin\theta_v \cos\theta_v & Q \cos^2\theta_v \end{pmatrix}, \tag{15}$$

where

$$Q(r) = \Delta m^2/2E - 2\Delta f E \phi(r) \tag{16}$$

and θ_v is the vacuum mixing angle. The formulas for the conventional MSW mechanism are easily modified for the case at hand by the substitution $\Delta m^2/2E \to Q$. The mixing angle in matter, $\theta_m(r)$, is then given by

$$\cos 2\theta_m(r) = [\cos 2\theta_v - \lambda][1 - 2\lambda \cos 2\theta_v + \lambda^2]^{-1/2}, \tag{17}$$

where

$$\lambda = \sqrt{2} G_F N_e(r)/Q(r). \tag{18}$$

The resonance condition, i.e., the condition for θ_m to be maximal, is therefore given by $\sqrt{2} G_F N_e = Q$ for small θ_v, which is the level-crossing condition.

To illustrate the effects of a nonuniversal gravitational coupling, we consider the adiabatic approximation. The probability for a ν_e born in the vicinity of $r = r_0$ in the Sun to still be the same flavor at a large distance from the Sun, where the mixing angle is θ_v, is given by

$$P(\nu_e \to \nu_e) = [1 + \cos 2\theta_v \cos 2\theta_m(r_0)]/2, \tag{19}$$

where $\cos 2\theta_m(r_0)$ is given in Eq. (17). It should be noted that we can identify $\theta_m(\infty)$ with θ_v only because $\phi(\infty) = 0$. This survival probability depends on the solar density distribution. As before, we approximate the solar mass density by $(m_p + m_n) N_e(r)$, where $N_e(r)$ is the electron number density. However, instead of treating the electron density as a uniform distribution, we employ a more realistic approximation. In Fig. 1, the electron

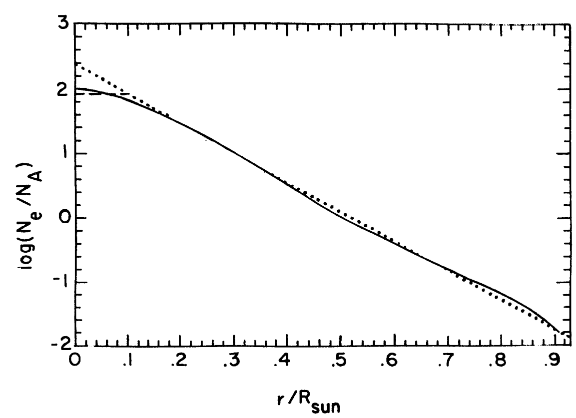

FIG. 1. Solid line: plot of the logarithm of the electron number density in the Sun as a function of the solar radius. N_A is Avogadro's number. Dotted line: exponential fit with the electron density distribution, Eq. (20). Dashed line: our approximation with the core density, Eq. (21).

density is shown as a function of the solar radius (solid line) [7]. According to Ref. [7], the distribution can be fitted with an exponential function (dotted line):

$$N_e(r) = 245 N_A \exp(-10.54 r/R_{\rm Sun})\ {\rm cm}^{-3}, \quad (20)$$

where N_A is Avogadro's number. This exponential fit introduces large errors in the solar core, where most of the neutrinos are produced. To compensate for this, we approximate the core density by a uniform distribution (as shown by the dashed line in Fig. 1), viz.

$$N_e(r) = \begin{cases} N_0, & r < r_c, \\ N_0 \exp[-(r-r_c)/L], & r > r_c, \end{cases} \quad (21)$$

where $N_0 = 85 N_A$ cm^{-3}, $r_c = 0.1 R_{\rm Sun}$, and $L = R_{\rm Sun}/10.54$. The corresponding gravitational potential is, for $r \le r_c$,

$$\phi(r) = 4\pi G_N N_0 (m_p + m_n)[r^2/6 - r_c^2/2 - r_c L - L^2]. \quad (22)$$

We use $r_0 = r_c = 0.1 R_{\rm Sun}$ in evaluating the survival probability in Eq. (19). In Fig. 2, we plot $P(\nu_e \to \nu_e)$ as a function of neutrino energy for $\Delta m^2 = 10^{-4}$ eV^2, $\sin 2\theta_v = 0.1$, and a range of Δf values. The values chosen for Δm^2 and $\sin 2\theta_v$ are consistent with the Homestake data [8] for $\Delta f = 0$ and satisfy the criteria for the validity of the adiabatic approximation. For $\Delta f \neq 0$, one sees a very marked change in the energy dependence of the probability even for $|\Delta f|$ as small as 10^{-14}. We hasten to add that the r dependence of the gravitational field complicates the analysis of the domain of validity of the adiabatic approximation [9], and some of our results may be an artifact of the approximation. A more careful study is in progress.

In summary, we find that a breakdown in the universality of the gravitational couplings of different neutrino species as small as 1 part in 10^{14} will have significant implications for the MSW effect. We further find that the energy dependence of the solar neutrino deficit can, in principle, be used to set an extraordinarily tight limit on such a possibility. The practicality of the procedure will be determined largely by the energy resolution of the solar neutrino detector and should be pursued vigorously. Finally, the effects discussed here are not limited to the Sun. In fact, the effects are stronger in more massive stellar objects, such as neutron stars and supernovae. The feasibility of detecting the effects from such objects is currently under study.

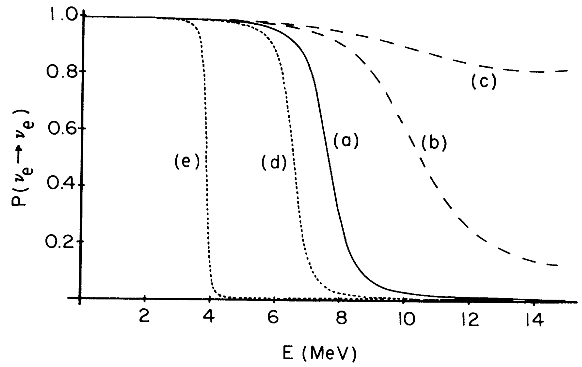

FIG. 2. Plot of the survival probability $P(\nu_e \to \nu_e)$ as a function of the neutrino energy for various values of Δf: curve a, $\Delta f = 0$; curve b, $2\Delta f = 10^{-14}$; curve c, $2\Delta f = 1.5 \times 10^{-14}$; curve d, $2\Delta f = -10^{-14}$; and curve e, $2\Delta f = -10^{-13}$. Δm^2 and $\sin 2\theta_v$ are chosen to be 10^{-4} eV^2 and 0.1, respectively.

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