Observation of Stochastic Resonance in Bistable Electron-Paramagnetic-Resonance Systems

L. Gammaitoni, M. Martinelli, and L. Pardi
Dipartimento di Fisica, Università di Pisa, I-56100 Pisa, Italy

S. Santucci
Dipartimento di Fisica, Università di Perugia, I-06100 Perugia, Italy

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Evidence of noise-induced effects in electron-paramagnetic-resonance (EPR) experiments is reported. The first observation of a stochastic resonance phenomenon in an EPR system operating in bistable conditions has been obtained. Experimental results indicate unambiguous effects on the frequency and phase of the response of the modulated bistable system in the presence of noise. Possible new progressions in the treatment of both theoretical and experimental stochastic resonance phenomenon are perceived.

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Increasing interest has been paid during the last years to noise-induced effects in nonlinear systems [1]. Laser systems operating in multistable conditions [2] and analog simulations with proper circuits [3] were the experimental tools used in the large majority of cases in investigating this research topic. The availability of electron-paramagnetic-resonance (EPR) systems working under proper conditions in bistable operation was demonstrated recently [4].

The dynamical behavior of modulated bistable systems in the presence of noise shows the peculiar phenomenon of the stochastic resonance (SR) [5]. The response of a bistable system to a stochastic driving force assumes a periodic character when a weak periodic force is added to the stochastic one. The extent of this character depends strongly on the strength of the stochastic drive. The periodic character of the response then shows a pronounced maximum for a noise-driven intensity related to the physical parameters of the system. This overall behavior is accounted for just as stochastic resonance. The SR phenomenon is observed in many cases by studying the trend of the power spectral density at the modulation frequency [5–12], even if other results have been presented that concentrate attention upon different physical observables such as the probability density of times of residence [9,11].

Experimental evidence of SR in optical systems (obtained with a bidirectional ring laser [8]) was reported in the pioneering work of McNamara, Wiesenfeld, and Roy. Further experiments based upon analog simulations have also been reported [7,9,11].

Here we report the first observation of stochastic resonance in EPR experiments. Evidence of the overall phenomenon is given and the most remarkable results are discussed in the light of the previous realization that EPR experiments can show instabilities and hysteresis phenomena [4]. The present results obtained with a standard widespread instrumentation strongly propose the EPR systems in bistable regimes as straightforward research tools and encourage following this route in the investigation of noise-induced effects in nonlinear systems.

The complete explanation of the bistable behavior of EPR systems is given elsewhere [4]; here we recall only the physical origin of the phenomenon. In standard EPR reflection spectrometers the paramagnetic sample is inserted in a resonant cavity; the cavity response is characterized by means of the trend of the reflection coefficient $\mathcal{R}(\nu)=P_r/P_i$ versus the frequency ν, with P_r power reflected from the cavity and P_i power impinging on the cavity itself. At resonant frequency of the cavity, $\mathcal{R}(\nu)$ has a minimum. A feedback circuit locks the frequency of the microwave source to such a minimum. Proper combinations of the spectrometer and sample parameters give, near the magnetic resonance, an additional minimum whose position is driven by the intensity of the magnetic field. In particular, the response of the cavity can assume a double-well symmetric shape. The multistable process is related to the switchings of the microwave source between the two wells, forced by the feedback system.

Experiments were performed with polypyrrole paramagnetic samples inserted in a paramagnetic resonance spectrometer. The sample met the requirements of an overall system bistable behavior [4]. Figure 1 shows the block diagram of the experimental setup used. The block S represents the microwave part of a standard EPR reflection spectrometer. The frequency ν of the source and the powers P_i and P_r are measured by the apparatus represented in block C. The signal S_D drawn from the detector feeds a proper feedback circuit (block F) consisting of a functional multiplier ($S_D\times$reference signal) and of a low-pass two-pole filter with a cutoff frequency $\nu_1=1/\tau$ fixed in our experiments at 30 Hz. The output of the feedback circuit [the error signal $\mathcal{E}(t)$] is used to automatically lock the electronically tunable microwave source to the frequency of the minimum of the spectrometer response. The only significant modification with respect to a standard EPR apparatus is the presence of a linear analog circuit Σ to add in the feedback path the noise $\mathcal{N}(t)$ and the deterministic low-frequency periodic

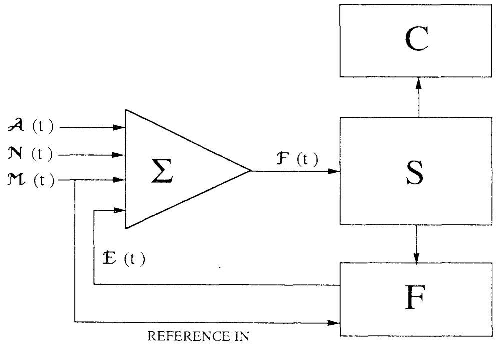

FIG. 1. The block diagram of the experimental setup.

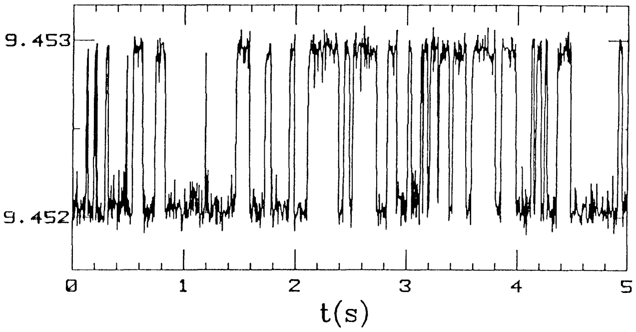

FIG. 2. Microwave source frequency ν time series. $\sigma=10.1\times10^{-3}$ V, ν is expressed in GHz.

signal $\mathcal{A}(t)$ with the standard high-frequency modulation $\mathcal{M}(t)$ and error signal $\mathcal{E}(t)$ used for automatic lock purposes. The result of this addition is the signal $\mathcal{F}(t)$ which is actually applied to the tuning port of the microwave oscillator.

A set of experiments implied the disconnection of the error signal from the source in order to measure the reflection coefficient $\mathcal{R}(\nu)=P_r/P_i$ in different conditions. In particular, we studied near magnetic resonance the trend of $\mathcal{R}$ as a function of Larmor frequency ν_0 and of the detuning $\nu-\nu_c$ with respect to the cavity resonant frequency far from magnetic resonance ν_c. Most of the experiments have been performed for $\nu_0=\nu_c$ (symmetric double wells).

Different parameters of the system can be used to monitor the bistable behavior: the most significant one is just the frequency. If the magnetic field (i.e., ν_0) is changed or an external signal is applied in $\mathcal{F}(t)$ the frequency of the microwave source $\nu(t)$ is forced to move near each minimum value or to switch between the values corresponding to the two different minima. The dynamic behavior of $\nu(t)$ is described by [13]

$$\ddot{\nu}+\frac{\dot{\nu}}{\tau}+\frac{\nu-\nu_c}{\tau^2}=\frac{\partial}{\partial\nu}\mathcal{U}(\nu,t)$$

with

$$\mathcal{U}(\nu,t)=\frac{K}{\tau^2}\{\mathcal{R}(\nu)+\nu[\mathcal{A}(t)+\mathcal{N}(t)]\}\,,$$

where K is a constant term related to the modulability of microwave source.

The same information content is reached if measurements of $\mathcal{F}(t)$ are performed, since $\mathcal{F}(t)$ is proportional to $\nu-\nu_c$.

Figure 2 illustrates a typical experimental result: the frequency ν of the microwave source is plotted versus time when $\mathcal{A}(t)=0$ and $\mathcal{N}(t)\neq0$. The stochastic force $\mathcal{N}(t)$ is an exponentially correlated Gaussian noise with finite variance σ^2 and correlation time $\tau=500$ μs. The bistable behavior of $\nu(t)$ is apparent. Frequency fluctuations around each stable equilibrium value can be observed too: The extent of these fluctuations depends on the specific settlement of the low-pass filter in F.

Figure 3 shows the same phenomenon observed in the signal $\mathcal{F}(t)$ in a set of experiments where both $\mathcal{A}(t)$ and $\mathcal{N}(t)$ are applied to the system. The signals $\mathcal{N}(t)$ applied in the different cases are shown together with the corresponding time series of $\mathcal{F}(t)$. The deterministic signal $\mathcal{A}(t)=\tilde{\mathcal{A}}\sin(2\pi\tilde{\nu}t)$ had in all cases the same amplitude $\mathcal{A}$ much lower than the value σ of $\mathcal{N}(t)$ and the frequency $\tilde{\nu}$ well inside the bandwidth of the low-pass filter.

We must remark that the gist of stochastic resonance phenomenon can be grasped in the observation of time

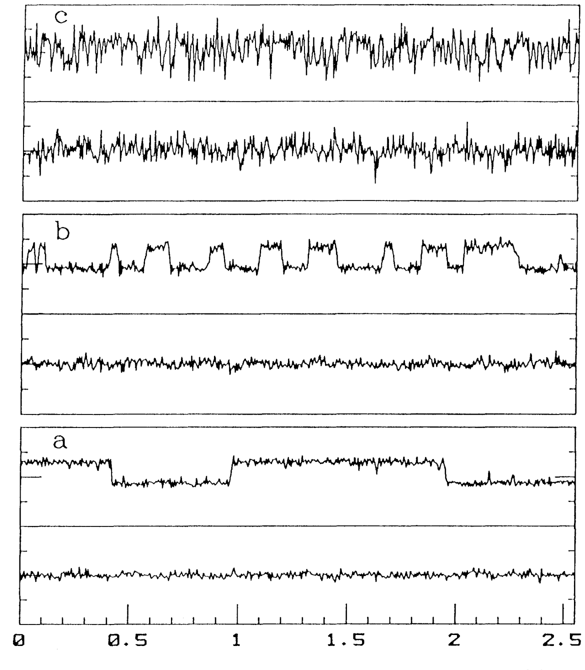

FIG. 3. Time series. Upper: system response $\mathcal{F}(t)$; lower: external noise $\mathcal{N}(t)$, for three different values of the noise standard deviation σ and the same $\mathcal{A}(t)=\tilde{\mathcal{A}}\sin(2\pi\tilde{\nu}t)$. $\tilde{\mathcal{A}}=4.5\times10^{-3}$ V, $\tilde{\nu}=3.9$ Hz and (a) $\sigma=9.3\times10^{-3}$ V, (b) $\sigma=13.4\times10^{-3}$ V, (c) $\sigma=32.9\times10^{-3}$ V. On the y axes small ticks correspond to 0.1 V; on the x axes the time is in seconds.

series of Fig. 3. Jumps between the two stable values of the signal $\mathcal{J}(t)$ occurred rarely and at random times when low values σ of $\mathcal{N}(t)$ were used [Fig. 3(a)]. When increased noise intensity was used [Fig. 3(b)] the jumps tended to occur periodically in time, driven by the periodic force $\mathcal{A}(t)$ as if a synchronization phenomenon happened. A further increase of noise intensity induced more frequent and random jumps in the signal $\mathcal{J}(t)$ that in a sense lost the synchronization to $\mathcal{A}(t)$.

In order to give a quantitative picture of the resonant character of the phenomenon we measured the power spectrum $F(\nu)$ of the signal $\mathcal{J}(t)$ for different values σ of the applied noise and fixed amplitude $\tilde{\mathcal{A}}$ and frequency $\tilde{\nu}$ of the deterministic signal. In particular, we determined in the profile of the power spectrum the component at frequency $\tilde{\nu}$ for each value of σ by means of a digital signal analyzer Data Precision model 6100. The spectrum $F(\nu)$ exhibits at frequency $\tilde{\nu}$ a very sharp peak with width comparable, within the experimental errors, with the one corresponding to the spectrum of $\mathcal{A}(t)$. Figure 4 reports the values, measured as a function of the noise variance σ^2, of the peak at $\tilde{\nu}$ of the quantity $F^*(\tilde{\nu})=F(\tilde{\nu})-B(\tilde{\nu})$, where $B(\tilde{\nu})$ denotes the spectrum background level in proximity of $\tilde{\nu}$. The trend of $F^*(\tilde{\nu})$ when the noise is increased shows a steep increase and pass through a maximum before following a slower decrease; this trend corresponds well with other descriptions of stochastic resonance phenomenon in different physical systems [8,9,11].

A suggestive confirmation of the interpretation in terms of synchronization effects is given in pictorial form in Fig. 5, where the time series of $\mathcal{J}(t)$ of Fig. 3(b) is directly compared with the deterministic signal $\mathcal{A}(t)$. The switching period of $\mathcal{J}(t)$ was found roughly coincident with the period of $\mathcal{A}(t)$; in addition, a nearly fixed phase relation ($\phi \approx \pi/4$) between the two signals was apparent.

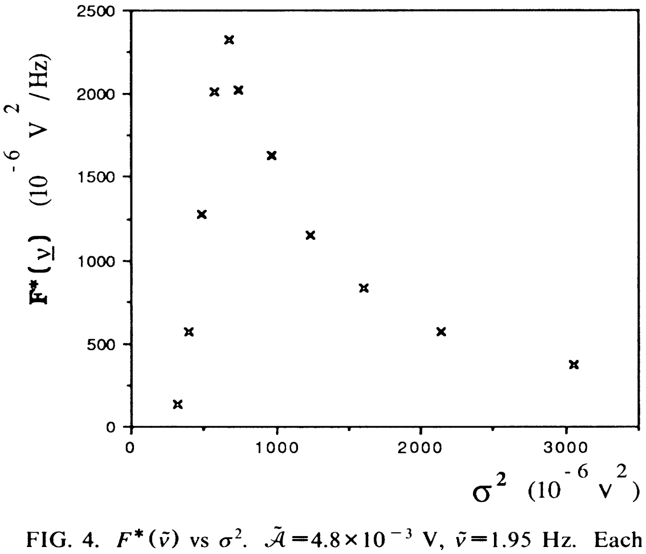

FIG. 4. $F^*(\tilde{\nu})$ vs σ^2. $\tilde{\mathcal{A}}=4.8\times10^{-3}$ V, $\tilde{\nu}=1.95$ Hz. Each power spectrum was averaged 200 times; the instantaneous bandwidth was $\Delta\nu=0.39$ Hz. Experimental errors were less than 5%.

We notice that a specific feature of EPR systems in the ambit we discuss here is the possibility of a direct and easy application of modulation and/or noise signals to drive the two-well reflection coefficient $\mathcal{R}(\nu)$. This is simply obtained by applying $\mathcal{A}(t)$ and $\mathcal{N}(t)$ to an additional couple of coils that modify the magnetic field and then affect the shape of $\mathcal{R}(\nu)$. This last technique discloses new experimental possibilities and allows a more direct interpretation of phenomena in terms of modulated potentials, as confirmed by preliminary results of experiments compared with the ones obtained with the former configuration.

A rapid analysis of results presented here demonstrates in the different cases good qualitative agreement with theoretical predictions: a deeper comparison is the matter for a forthcoming extended paper [13].

In conclusion, we obtained new evidence of the phenomenon of stochastic resonance by using a novel research tool, the EPR system operating in bistable condition, that appears as a real working physical system which is very promising in investigations of effects driven by noise in multistable modulated systems. In addition, the results obtained elucidate well that stochastic resonance implies a synchronization phenomenon in which not only the frequency but also the phase of the modulated system response is directly related to the modulation deterministic drive. We observed that the system response at the forcing frequency shows a specific phase relation with the periodic modulation. This relative phase takes the value of about $\pi/4$ when the response amplitude is at its maximum (resonance condition). This remark gives suggestions for more refined theoretical interpretations of the phenomenon and for new procedures in the experimental investigations, where cross-correlation techniques seem to acquire a peculiar weight.

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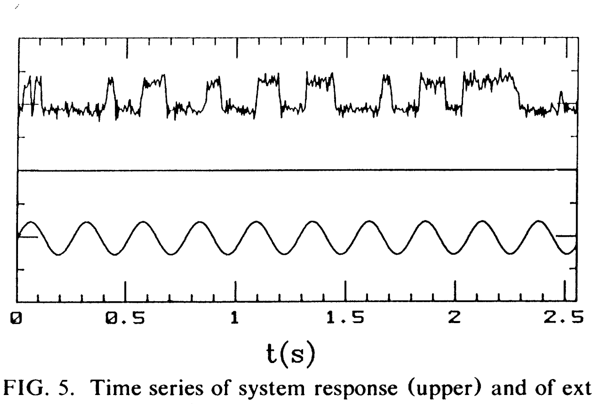

FIG. 5. Time series of system response (upper) and of external periodic force $\mathcal{A}(t)$. The values of parameters are the same as in Fig. 3(b).

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