Optical Ramsey Spectroscopy in a Rotating Frame: Sagnac Effect in a Matter-Wave Interferometer

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A calcium atomic beam excited in an optical Ramsey geometry was rotated about an axis perpendicular to the plane defined by the laser beams and the atomic beam. A frequency shift of the Ramsey fringes of several kHz has been measured which is proportional to the rotation frequency of the apparatus and to the distance between the laser beams. The results can be interpreted in three equivalent ways as the Sagnac effect in a calcium-atomic-beam interferometer: in the rotating frame of the laser beams either along straight paths or along the curved trajectories of the atoms, or in the inertial atomic frame.

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Matter-wave interferometry with neutral atoms holds the promise of becoming a powerful tool for high-precision experiments (see, e.g., Refs. [1] and [2]) for several reasons: The high mass of atoms as compared with neutrons or electrons leads to a high momentum, equivalent to a small de Broglie wavelength, even at moderate velocities. Furthermore, compared with neutrons, atomic beams of high flux are easy to obtain in the laboratory and, compared with electrons, they are not sensitive to electric fields. Consequently, great effort has been expended recently to provide the necessary means for the realization of atomic-beam interferometry. Several methods to split or to reflect matter waves coherently have been investigated, e.g., diffraction of atoms by a transmission grating [3], by a double slit [4], by a standing light wave [5], or by the evanescent part of totally reflected light [6]. Recently, one of the authors [7] has pointed out that the interaction of an atomic beam with four traveling waves in a Ramsey excitation geometry [8–10] can be considered as an atomic-beam interferometer, which should be sensitive enough to detect frequency shifts induced by rotations or gravitational fields. A phase shift due to rotation, known as the Sagnac effect for light waves, occurs also for electrons [11] and neutrons [12]. It is proportional to the energy of the particle, to the enclosed area, and to the rotation frequency of the interferometer. Here we report the first demonstration of this effect for atoms. It is observed by a shift of the Ramsey fringes when a calcium atomic beam is excited by two pairs of separated counterpropagating laser fields and when the atomic-beam apparatus is rotating around an axis perpendicular to the atomic and the laser beams. Such a shift is furthermore of considerable interest in case optical Ramsey excitation is utilized in an optical frequency standard [13], because of non-negligible frequency offsets depending on the geographical latitude and on the orientation on Earth.

The experiments used a thermal beam of calcium atoms with a collimation angle of $\alpha \cong 3\times10^{-3}$ rad. The atoms passed sequentially two pairs of equally spaced traveling laser wave fields (Fig. 1) of a high-resolution dye-laser spectrometer [14]. Two "cat's-eye" retroreflectors were employed to obtain four parallel laser beams with the first two beams running in the same direction and the second pair counterpropagating with respect to the first pair. The intercombination transition $^3P_1$-$^1S_0$ of ^{40}Ca ($\lambda = 657.46$ nm) was used for these investigations because of the long lifetime ($\tau \cong 0.4$ ms) of the excited $^3P_1$ state.

The whole atomic-beam apparatus was mounted on a rotational stage and could be rotated around a vertical axis. Electrical power and signals, cooling water, and laser light were transferred via long cables, tubes, and a

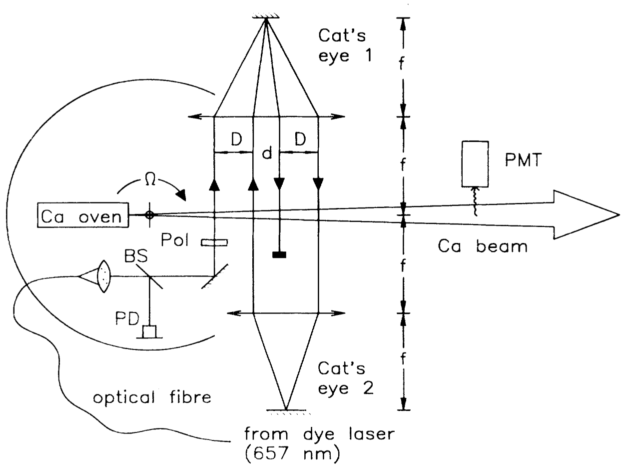

FIG. 1. Experimental setup to record optical Ramsey fringes in a calcium atomic beam by means of four traveling waves in a rotating system (see text).

polarization-preserving optical fiber. The twisting of these connections limited the number of possible rotations to about one turn. To avoid fluctuations of the polarization and power of the laser beams at the interaction zones during the rotation, the birefringent axis of the fiber was aligned at 45° with respect to a linear polarizer (see Fig. 1) and the power was stabilized.

Optical Ramsey fringes were observed as a function of laser frequency by monitoring the fluorescence emitted by the decay of the $^3P_1$ level about 20 cm behind the fourth interaction zone by means of a photomultiplier (see Fig. 2). The spectra of Fig. 2, curves *a*, *c*, and *e*, were recorded with the apparatus standing still, whereas the apparatus was rotated clockwise (CW) and counterclockwise (CCW) for the spectra of Fig. 2, curves *b* and *d*, respectively. Each spectrum shows a superposition of two recoil components separated by 23.1 kHz. The raw data signals are composed of Ramsey fringes superimposed on the dip of the corresponding saturated absorption. In order to remove the saturated-absorption profile from the fringe signal, a quadratic background was subtracted to obtain the spectra shown in Fig. 2. The spectra of Fig. 2, curves *a*–*d*, were recorded consecutively within about 5 min. During this time the frequency of the dye laser was drifting by about 1 kHz. The Ramsey fringes recorded during rotation are shifted by about ± 8 kHz, depending on the rotational direction. The observed results can be explained by atomic interferences in a rotating frame [7].

The calcium beam enters the first of the four interaction zones with the laser field (see Fig. 3) in the ground state $|a,0\rangle$. For detection by their fluorescent decay, we are interested in those atoms in the excited state $|b,m\rangle$ after the fourth zone (m denotes the number of photon momenta transferred to the atom). In the first (and subsequently in each other zone) the matter wave is coherently split into two partial waves with an internal state $|b\rangle$ or $|a\rangle$, respectively. When the atom is excited or returns back to the ground state, the momentum of the atom $\mathbf{p}_0$ is changed by the momentum of the photon $\hbar\mathbf{k}$. The deflection after the emission or absorption process therefore leads to a small diffraction angle $\theta = \hbar k/p_0 \approx 22$ μrad for a mean velocity of the calcium atoms of $v = 700$ m/s. There are two different possibilities (marked by the two trapezia of Fig. 3) where two different partial waves are combined in the final state after the fourth interaction zone. Each trapezium corresponding to one recoil component can be regarded as a Mach-Zehnder interferometer where the matter wave is split into two partial waves in the first laser field (beam splitter). Subsequently one partial wave is deflected in zones 2 and 3 and recombined again in zone 4 with the other one.

As in any other interferometer there are two exit ports where the calcium matter waves leave the interferometer either in the excited state (port I) or in the ground state (port II). The probability to find the matter wave at port I or II, respectively, contains a harmonically oscillating part depending on the phase difference $\Delta\phi$ of the two partial waves. The phase difference can be generated, e.g., by different path lengths, different frequencies of the partial waves, or phase shifts at the beam splitters. A rota-

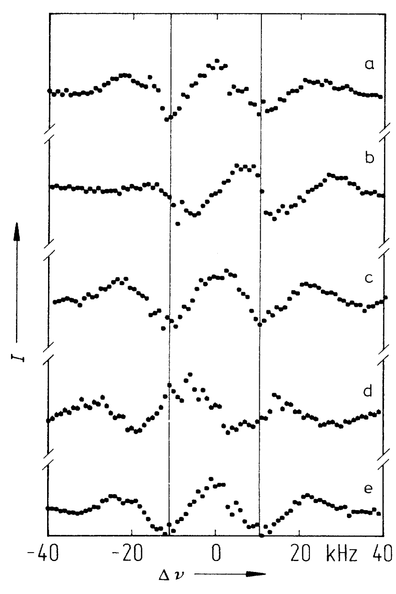

FIG. 2. Measured Ramsey fringes ($2D = 26$ mm, $d = 33$ mm) with the apparatus standing still ($\Omega = 0$, curves *a*, *c*, and *e*) and for $\Omega = -0.09$ s^{-1} (curve *b*) and $\Omega = +0.09$ s^{-1} (curve *d*) show a shift of the recoil doublet, when the interferometer is rotated, as compared to the position of the recoil doublet at $\Omega = 0$ (vertical lines).

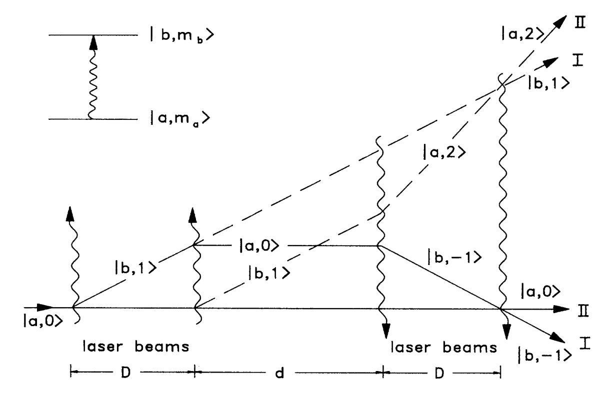

FIG. 3. Optical Ramsey excitation of an atomic beam by four traveling laser fields (see Fig. 1) interpreted as a matter-wave interferometer [7]. (Solid lines, high-frequency recoil component; dashed lines, low-frequency component; only traces leading to Ramsey resonances are drawn). In the first interaction zone the matter wave is coherently split into two partial waves with internal states $|a,m_a\rangle$ and $|b,m_b\rangle$ corresponding to energy levels *a* and *b*, respectively, and the number *m* of photon momenta transferred to the atom, leading to a spatial separation.

tion in the plane of the interferometer would also show up as a phase difference through the Sagnac effect. Provided the collimation angle of the atomic beam is sufficiently small, this probability could be measured simply by counting the atoms in port I or II. Since the calcium atoms in port I are in the excited state, we detect them by their fluorescent decay even if the two beams are largely overlapping.

A rigorous description of the interferometer can be obtained from the integral form of the Schrödinger equation for a two-level system, which provides all phase factors in a consistent and unified way. In the rotating frame with the laser beams at the rotation velocity Ω, the state of the system evolves like

$$|\psi(t)\rangle=|\psi^{(0)}(t)\rangle+\frac{1}{i\hbar}\int_{-\infty}^{t}dt'\exp\left[\frac{i}{\hbar}\int_{t'}^{t}\mathbf{\Omega}\cdot\mathbf{r}_{\rm op}\times\mathbf{p}_{\rm op}dt_1\right]\exp\left[-\frac{i}{\hbar}\left(\frac{p_{\rm op}^2}{2M}+H_0\right)(t-t')\right]V(\mathbf{r}_{\rm op},t')|\psi(t')\rangle\,, \tag{1}$$

where $\mathbf{r}_{\rm op}$, $\mathbf{p}_{\rm op}$, H_0, and V are, respectively, the position, momentum, internal Hamiltonian, and electric dipole interaction operators, and where $|\psi^{(0)}(t)\rangle$ is the solution in the absence of laser fields.

The laser fields put atoms with an initial momentum $\mathbf{p}_0$ in a superposition of wave packets of internal energy E_α ($\alpha=a,b$) and momentum $\mathbf{p}_0+m_\alpha\hbar\mathbf{k}$ (m_α integer). If we use these basis states, we infer the following rules from the rigorous equation:

(1) The phase factor corresponding to any segment along which the atomic wave function propagates freely during the time T is $\exp(iL_\alpha T/\hbar)$, where the Lagrangian L_α is given by

$$\begin{aligned}L_\alpha&=(\mathbf{p}_0+m_\alpha\hbar\mathbf{k})\cdot\mathbf{v}_\alpha-(E_\alpha+\tfrac{1}{2}Mv_\alpha^2)\\&=-E_\alpha+\tfrac{1}{2}Mv_\alpha^2\,.\end{aligned} \tag{2}$$

Here $\mathbf{v}_\alpha=(\mathbf{p}_0+m_\alpha\hbar\mathbf{k})/M$ is the atomic velocity in the inertial frame where the atom is freely falling. A relativistic generalization would be $L_\alpha=-(Mc^2+E_\alpha)(1-v_\alpha^2/c^2)^{1/2}$.

(2) The phase factor corresponding to each interaction zone (i) is given by the laser-field phase taken at the corresponding space-time point $(\mathbf{r}_i,t_i)$:

$$\exp(i\epsilon\omega t_i+i\epsilon'\mathbf{k}\cdot\mathbf{r}_i)\,, \tag{3}$$

where $t_{i+1}=t_i+T$ and $\mathbf{r}_{i+1}=\mathbf{r}_i+\mathbf{v}_\alpha T$, and where the choice $\epsilon,\epsilon'=0,\pm1$ is made in order to satisfy energy and momentum conservation.

(3) The phase factor corresponding to the rotation for the segment which follows the interaction zone (i) is

$$\exp[(i/\hbar)\mathbf{\Omega}\cdot\mathbf{r}_i\times(\mathbf{p}_0+m_\alpha\hbar\mathbf{k})T]\,, \tag{4}$$

where $|\mathbf{r}_i\times(\mathbf{p}_0+m_\alpha\hbar\mathbf{k})T/M|$ is twice the area swept by the atomic motion along the segment. If $\mathbf{A}$ is the vector area of the interferometer, this will lead to the total phase shift $\Delta\phi=2M\mathbf{\Omega}\cdot\mathbf{A}/\hbar$ which is the classical result for the Sagnac phase in a matter-wave interferometer.

Rules (2) and (3) have been derived by computing the matrix element of the rotation operator which appears in Eq. (1), along the straight trajectory (unperturbed by rotation) and to first order in $\mathbf{\Omega}$. Equivalently, one can use this rotation operator to shift the spatial coordinates of the interaction point. To first order in $\mathbf{\Omega}$,

$$\mathbf{r}_i'=\mathbf{r}_i+(\mathbf{\Omega}\times\mathbf{r}_i)T\,. \tag{5}$$

This point of view amounts to picking up the phase of the source term of Eq. (1) in the interaction region on the actual curved trajectory of the atoms submitted to the Coriolis force.

A third point of view is obtained by a frame transformation of Eq. (1) from the rotating frame to the inertial frame where the atoms are freely falling. In this frame the rotation term disappears from the atomic Hamiltonian but the phase of the laser fields has to be transformed from the rotating to the inertial frame and becomes rotation dependent. The three points of view are, of course, equivalent and lead to the following phase differences, when the previous rules are applied to both trapezia of Fig. 3:

$$\Delta\phi=2\pi\left[\Delta\nu\pm\frac{h\nu^2}{2Mc^2}+\Omega\frac{D+d}{\lambda}\right]\left(\frac{2D}{v}\right), \tag{6}$$

where the detuning term $\Delta\nu$ is responsible for the optical Ramsey fringes. These are displaced by the recoil shift ($h\nu^2/2Mc^2=11.6$ kHz in the case of the Ca line) with a $-$ sign for the trapezium represented by solid lines in Fig. 3 and with a + sign for the trapezium represented by dashed lines. This recoil doublet is visible in Fig. 2. From (6), we see that the Sagnac effect leads to a frequency shift

$$\Delta\nu_{\rm Sagnac}=\Omega(D+d)/\lambda\,. \tag{7}$$

We have measured the frequency shift for various angular velocities and two different separations for both CW and CCW rotations (see Fig. 4). The measured frequency shifts agree well with the ones derived from Eq. (7).

In this atomic interferometer, the respective roles of light and matter are exchanged in comparison with an optical Mach-Zehnder interferometer. The main difference is the fact that the beam splitters exchange energy as well as momentum. A closer analogy would be obtained with an optical interferometer using traveling-wave acousto-optic shifters as splitters. In such a case, if the mirror

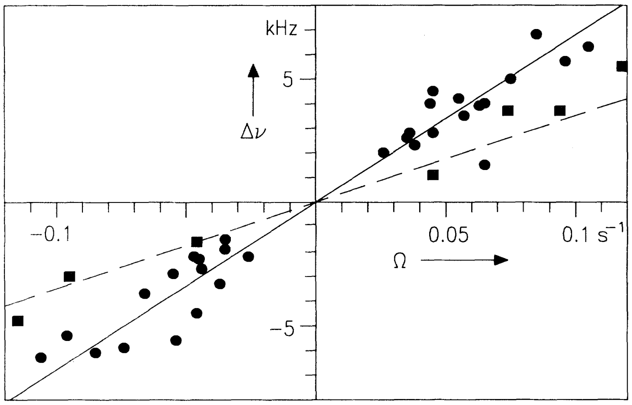

FIG. 4. Measured frequency shift of the optical Ramsey spectra for different angular velocities and two different distances $d+D$ between the laser fields (dots, $d+D=45$ mm; squares, $d+D=24$ mm) together with the expected result according to Eq. (7) (straight lines).

phase is changed by tuning the acoustic frequency, fringes will be obtained (however, without resonant character). This paper demonstrates that the matter-wave interferometer is highly sensitive to inertial fields which involve directly the external degrees of freedom of the atoms. Even higher sensitivity is envisaged with very slow atoms [Eq. (6)].

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Note added.— Kasevich and Chu [15] have demonstrated the acceleration of sodium atoms due to gravity in a matter-wave interferometer using stimulated Raman transitions.

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