Above-Threshold Laser Amplifier

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We have measured gain profiles by light-beating techniques for an argon-ion laser operated above threshold. The observed behavior is interpreted by developing a theory which incorporates four-wave mixing of signal and image frequencies. The gain shows the typical characteristics of a second-order phase transition. We also discuss the implications for fundamental laser noise, and predict strong correlations between the noise contributions from either side of the laser frequency.

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There is much current interest in the wide variety of phenomena that occur when light is injected into a laser. In appropriate conditions the injected light is amplified, and we refer to the system as an above-threshold laser amplifier. We show that study of the amplification characteristics provides substantial insight into the physics of the laser, including the nature of its fundamental noise: It is well known that the noise of a linear amplifier is intimately related to its gain [1]. Although the theory of laser noise has been extensively studied using a variety of theoretical techniques [2], there has, however, been surprisingly little work on the details of the above-threshold laser gain process, except in connection with studies of the injection-locking mechanism [3].

We report here heterodyne measurements of gain profiles for an argon-ion (Ar^+) laser operated as an above-threshold amplifier over a wide range of pumping rates. The measurements are made with injected signals that are sufficiently weak to ensure linearity of their amplification and to avoid injection locking or the instabilities that may be induced by stronger signals [4,5]. The data are interpreted in terms of a theory of the laser perturbed by an injected signal; a major conclusion of the work is the importance of interference between signal and image contributions in producing the observed heterodyne gain. Four-wave mixing of signal and image frequencies associated with modulation of the population inversion has been considered before [6], with applications to squeezed-light generation [7] and semiconductor lasers [8]. However, the latter show complicated gain characteristics which are difficult to interpret in terms of the basic physics of the laser. In comparison, the Ar^+ laser has several advantages that facilitate straightforward interpretation of the measurements. Rapid atomic relaxation rates ensure that the laser conforms to the well-understood adiabatic approximation. Spatial inhomogeneities, or hole-burning effects, are insignificant at moderate power levels. Furthermore, we use a novel technique for deriving the injected signal from the laser output, which eliminates the effects of slow jitter and allows measurements of high accuracy. The results clarify the underlying physics of the laser gain and noise mechanisms, and they provide clear evidence for the second-order nature of the laser phase transition at threshold [9].

Figure 1 shows the experimental arrangement, with a single mode of the double-ended argon-ion laser at 488 nm selected by means of an intracavity etalon. The laser cavity has a mode spacing of 156.1 MHz, and both mirrors have approximately 95% reflectivity. The laser output power can be varied over 2 orders of magnitude by adjustment of the discharge current. One output beam is routed through two acousto-optic modulators (AOM), which respectively produce a fixed shift of −80 MHz and a variable shift of $(80+\delta/4\pi)$ MHz in the beam frequency, where $\delta/4\pi$ can be varied over the range 0 to ± 5 MHz. The beam passes again through the modulators after reflection from a mirror, thus acquiring a total frequency shift $\delta/2\pi$ before it reenters the laser. The returning beam is attenuated by a factor of at least 10^5 to satisfy the small-signal requirements for linear amplification. The experiment is equivalent to one where the signal is produced by an independent source, but with the problem of jitter significantly reduced. The beams transmitted through and reflected from the cavity are examined by detectors A and B, with suitable attenuators to reduce the incident power level to ~ 1.5 mW. Each detector is thus exposed to a strong laser field at angular frequency ω_L, serving as a local oscillator, plus the weak transmitted or

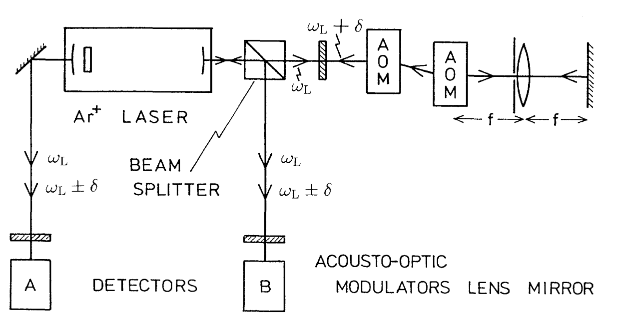

FIG. 1. The experimental arrangement.

reflected beam as modified by the above-threshold amplifier. The spectral response of the detectors, and the optical efficiency of the modulators, are uniform over the range 0–10 MHz.

Figure 2 shows gain profiles for the heterodyne beat signal in transmission. This gain is defined as the ratio of the *measured* beat signal power to that derived for the same input signal mixed, *before* reentering the laser, with an identical strength local oscillator. For each data set the laser discharge was well stabilized and its output power measured immediately before the beam splitter in Fig. 1. The gain was measured as a function of detuning $\delta/2\pi$ by spectral analysis of the detector output. Identical profiles were observed for positive and negative δ. The transmission gain profiles are Lorentzians whose widely differing bandwidths and peak gains are shown as functions of the laser pumping rate in Fig. 3. The profile narrows dramatically and the peak gain increases rapidly as threshold is approached from above. At the highest laser powers the width tends to twice the passive Fabry-Pérot cavity FWHM bandwidth Γ_0 and the gain approaches one-quarter. This behavior conflicts with the expectation from injection-locking studies [3] of a gain that always becomes very large at small detuning. The theory presented below resolves this paradox.

The corresponding gain profiles in reflection have the form of const ± Lorentzian. The constant is taken to be unity, appropriate to 100% reflection at large detuning, thus allowing accurate calibration of absolute gain. A flat frequency response at the output power $P=1.67$ mW and zero gain for $\delta\to 0$ at a 3 times greater power $P=4.92$ mW are particularly noteworthy features.

All of the above observations can be accurately described in terms of a theory [9,10] of the single-mode laser with an injected signal. The laser cavity is assumed to be symmetric with a mode of frequency ω_L in resonance with the atomic transition. The empty-cavity bandwidth Γ_0 equals $2\gamma_C$, where $\gamma_C/2$ is the internal field decay rate through each mirror. Then if $\gamma_\perp$ and $\gamma_\parallel$ are the atomic dipole and population inversion decay rates, g is the atom-field coupling, and D_P is the equilibrium population inversion in the absence of any cavity field, the laser equations in the presence of an injected signal are

$$\dot{\alpha}+(\gamma_C+i\omega_L)\alpha=gd+\gamma_C^{1/2}\beta_{\rm in}\,, \qquad (1)$$

$$\dot{d}+(\gamma_\perp+i\omega_L)d=g\alpha D\,, \qquad (2)$$

$$\dot{D}+\gamma_\parallel D=\gamma_\parallel D_P-g(\alpha^*d+\alpha d^*)\,. \qquad (3)$$

Here $\beta_{\rm in}$ is the injected signal field of frequency ω_S. We solve the laser equations correct to first order in $\beta_{\rm in}$, and it is necessary to include free-running, signal, and image contributions to the cavity field α,

$$\alpha=\alpha_L\exp(-i\omega_Lt)+\alpha_S\exp[-i(\omega_L+\delta)t]+\alpha_I\exp[-i(\omega_L-\delta)t]\,, \qquad (4)$$

where $\delta=\omega_S-\omega_L$ is the signal detuning. The collective atomic transition dipole moment d is represented by a similar expression, and there are also three contributions to the population inversion,

$$D=D_0+D_1\exp(-i\delta t)+D_1^*\exp(i\delta t)\,. \qquad (5)$$

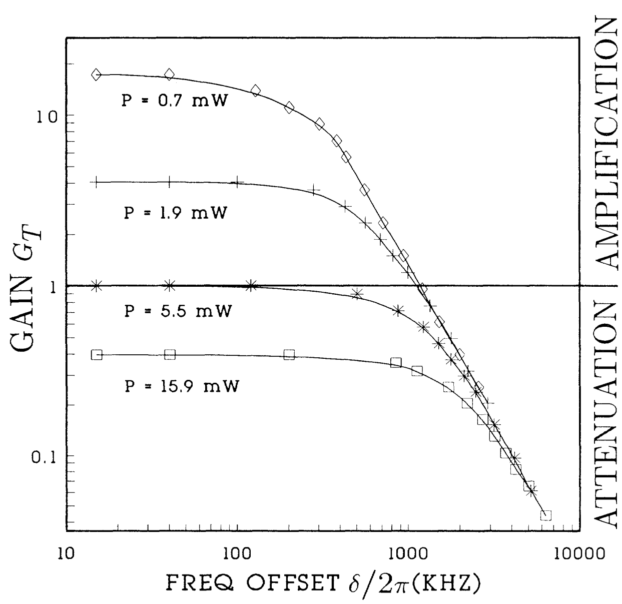

FIG. 2. Experimental profiles for heterodyne (beat) gain in transmission, G_T, at different laser powers P.

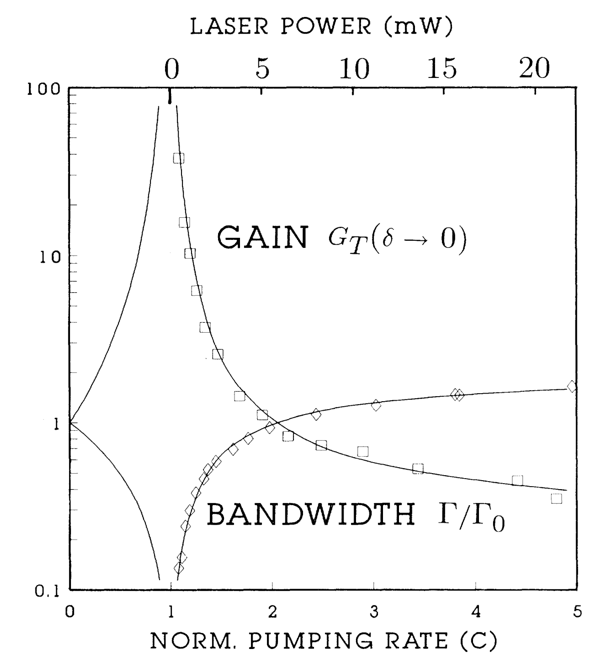

FIG. 3. Above-threshold amplifier gain and bandwidth as functions of laser pumping rate. □, gain measurement. ◇, bandwidth measurement. Continuous curves below threshold: theory (Ref. [12]). Curves above threshold: theory (this work).

The occurrence of both image and signal amplitudes α_I and α_S to first order in $\beta_{\rm in}$ is a signature of the nondegenerate four-wave mixing process that controls the amplification. These amplitudes are coupled by the D_1 modulation terms in the population inversion (5).

The above equations produce the usual mean photon number,

$$|\alpha_L|^2=(C-1)\gamma_\perp\gamma_\parallel/2g^2\,, \tag{6}$$

where $C=g^2D_P/\gamma_C\gamma_\perp$ is the cooperation parameter, or normalized pumping rate, equal to unity at threshold. The transmission *amplitude* gain is the ratio of heterodyne signals measured in the presence and absence of the amplifier,

$$g_T=\gamma_C^{1/2}(\alpha_L^*\alpha_S+\alpha_L\alpha_I^*)/\alpha_L^*\beta_{\rm in}\,. \tag{7}$$

Solution of the laser equations in the limit [11] $\gamma_\perp \gg \delta$ gives

$$g_T=i\gamma_C(\delta+iC\gamma_\parallel)/\{\delta^2+i\delta C\gamma_\parallel-2\gamma_C\gamma_\parallel(C-1)\}\,. \tag{8}$$

When the further limit $\gamma_\parallel \gg \delta,\gamma_C$ is also satisfied [11], the *intensity* transmission gain reduces to the Lorentzian form,

$$G_T=|g_T|^2=\gamma_C^2/\{\delta^2+[2\gamma_C(C-1)/C]^2\}\,. \tag{9}$$

The peak gain is accordingly $C^2/4(C-1)^2$, and the gain FWHM bandwidth is

$$\Gamma=4\gamma_C(C-1)/C=2\Gamma_0(C-1)/C\,. \tag{10}$$

These quantities are plotted as the continuous curves in Fig. 3, and they show excellent agreement with the measured points.

The corresponding below-threshold curves are obtained from the gain expression [12]

$$G_T=\gamma_C^2/\{\delta^2+[\gamma_C(1-C)]^2\}\,, \tag{11}$$

which is also readily derived from Eqs. (1) to (3). The expressions that give the peak gain and bandwidth above threshold are thus obtained by the replacement $C\to 1/C$ in the below-threshold expressions, together with an additional factor of 2 in the bandwidth expression. These features are in accord with the identification of the laser threshold as analogous to a second-order phase transition [9]. We believe that the measurements reported here provide the first experimental evidence for this behavior of the laser gain.

The individual signal and image field amplitudes in the limit $\gamma_\parallel \gg \delta,\gamma_C$ are

$$\alpha_S=-\frac{i\delta C-\gamma_C(C-1)}{i\delta C-2\gamma_C(C-1)}\,\frac{\gamma_C^{1/2}\beta_{\rm in}}{i\delta}\,, \tag{12}$$

$$\alpha_I^*=-\frac{\gamma_C(C-1)}{i\delta C-2\gamma_C(C-1)}\,\frac{\gamma_C^{1/2}\beta_{\rm in}\exp(-2i\phi_L)}{i\delta}\,, \tag{13}$$

where ϕ_L is the phase of α_L. It is seen that these amplitude vectors have almost equal magnitudes as $\delta\to 0$, and that a large cancellation occurs in the heterodyne gain of Eq. (7). Remarkably, the relative phases are such that the beat gain of Eq. (9) always equals the *difference* of the intensity gains calculated for the signal and image contributions individually, as is illustrated in Fig. 4 for $C=2$. Thus the signal and image gains both tend to infinity as $\delta\to 0$, while the net beat gain remains finite and well behaved. These features clearly resolve the injection-locking paradox mentioned earlier.

It is straightforward to calculate the gain profiles in reflection, and good agreement with experiment is again obtained. In particular the measured flat response (gain independent of δ) is correctly predicted at $C=\frac{4}{3}$, while the zero gain at zero detuning occurs for $C=2$. Since laser power is proportional to $C-1$, this agrees with the observation noted earlier that these two special cases occur at powers in the ratio 1:3.

Amplifiers necessarily contaminate the amplified signal with noise, whose minimum value is determined by very general considerations [1]. The gain measured in our experiments and represented by Eq. (9) thus implies a minimum amount of laser intensity fluctuation noise. The fundamental laser noise spectrum [13] indeed has the same Lorentzian form as the gain, and the magnitude of the noise exceeds the required minimum value. It is found that the above-threshold amplifier degrades the signal-to-noise ratio by a factor $C^2/4(C^2-C+1)$, which has a maximum value of $\frac{1}{3}$ for $C=2$. This factor can be increased to an optimal value of $\frac{1}{2}$ by the use of a nonsymmetric cavity. The close relationship of amplifier gain and noise, together with the importance of signal

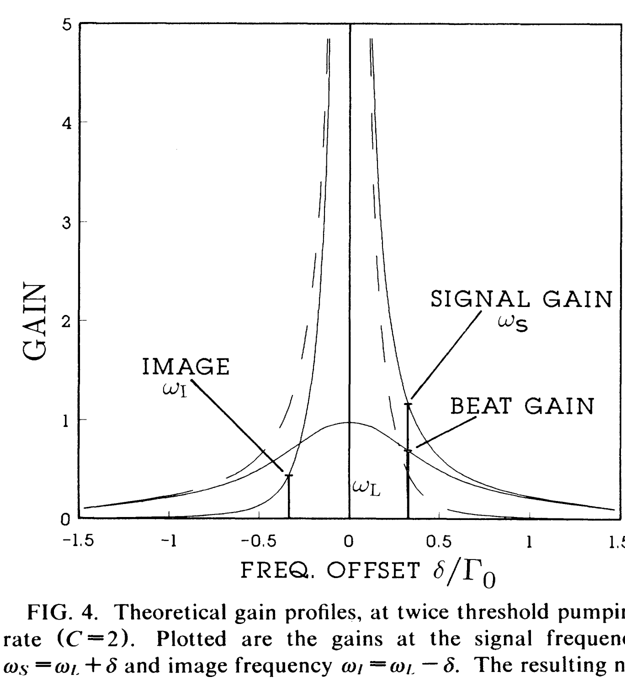

FIG. 4. Theoretical gain profiles, at twice threshold pumping rate ($C=2$). Plotted are the gains at the signal frequency $\omega_S=\omega_L+\delta$ and image frequency $\omega_I=\omega_L-\delta$. The resulting net beat is measured in the experiment.

and image correlations in determining the measured gain, established in the present work, suggest that strong correlations must exist between the noise contributions from either side of the laser frequency.

For lasers in which $\gamma_{\parallel} < \gamma_C$, for example CO_2, where the more general expression (8) must be used, the calculated gain shows a symmetrical double-peaked profile. This behavior is confirmed by observations with a CO_2 lidar system [14]. The amplifier noise [15] again has a similar spectral form to that of the gain.

In summary, we have demonstrated the importance of interference between signal and image contributions in the gain and noise properties of above-threshold laser amplifiers, with excellent agreement between experiment and theory. The technique for frequency shifting and reinjecting part of the laser output provides a powerful tool for laser diagnostics and it also has potential practical application to self-aligning lidar systems [16].

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