Trapping and Cooling of Atoms in a Vacuum Perturbed in a Frequency-Dependent Manner

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We show that light-induced mechanical forces that act on atoms may be significantly enhanced and acquire novel physical character when the electromagnetic reservoir which mediates the atomic relaxation is colored (i.e., frequency dependent).

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Spontaneous decay of an atomic transition is usually assumed to be characterized by a single, excitation-field-independent rate, the Einstein coefficient. However, it has been suggested [1] and clearly demonstrated [2] that spontaneous emission rates may be substantially modified by environmental factors that lead to changes in the coupling of the emitting atom to the electromagnetic vacuum. Changes in this coupling have frequently been attributed to changes in the local density of electromagnetic modes. It has been pointed out recently that a number of features of spontaneous decay become *excitation-field dependent* if an atom's environment perturbs the vacuum in a frequency-dependent manner. Frequency-dependent (or colored) vacua may arise, for example, inside mode-degenerate optical cavities. Some of the effects of colored vacua include dynamical modifications of spontaneous emission [3], cavity-modified Lamb shifts [4], vacuum-field dressed-state pumping [5], and velocity-dependent spontaneous emission [6].

New physics has also been found to occur in the case of atoms coupled to *squeezed* vacua [7]. We note that squeezed vacua, in contrast to colored vacua (which passively arise owing to the presence of boundaries), must be dynamically created and maintained. Thus, it is not surprising that colored-vacuum effects have been studied experimentally while squeezed-vacuum effects remain only a theoretical possibility.

The purpose of this Letter is to demonstrate that the mechanical effects of light take on a dramatically different character in the presence of a colored vacuum. We find that light-induced forces and the atomic velocity ranges over which they are effective can, in colored vacua, be much larger than the corresponding free-space quantities. The second result implies that atom traps with large capture velocities may be achievable. Simultaneously, the capability to cool to temperatures well below the Doppler limit, already present in free-space cooling techniques, persist in the colored-vacuum case. Our results directly relate to cavity QED studies [8] since intracavity vacua are frequently colored.

Traditionally, the light-induced mechanical forces [9, 10] are divided into two types. One is referred to as the radiation pressure force which arises from directional absorption and spatially dispersed reemission of light. The other is the dipole force which is related to the intensity gradient of the light field and the in-phase part of the atomic dipole. We are interested here in the effect of colored vacua on forces of the second type.

Using a dressed-atom model, Dalibard and Cohen-Tannoudji [11] showed that dipole-type forces can act to damp the motion of free-space atoms moving along a one-dimensional standing-wave optical field tuned somewhat above the atomic resonance. An intuitive understanding of this damping was offered in terms of a *Sisyphus-type* effect that arises from the tendency of free-space spontaneous emission processes to rearrange dressed sublevel populations as the atoms move so that the transfer of atomic kinetic energy into electromagnetic-field energy is favored.

We have generalized the formalism and physical picture developed by Dalibard and Cohen-Tannoudji to account for the presence of a colored vacuum. We find that colored vacua can dramatically enhance the Sisyphus-type effect or lead to its existence in situations where it is inoperative in free space. For example, a large Sisyphus effect may exist even for *resonantly* driven atoms in a colored vacuum. We note also that Sisyphus-type effects play a vital role in the recently observed polarization-gradient cooling and trapping [12] and hence these effects may be similarly enhanced and modified in the presence of colored vacua.

Consider a two-level atom, with ground state $|0\rangle$ and excited state $|1\rangle$, moving along a one-dimensional standing wave whose frequency ω is (to simplify analysis) in perfect resonance with the rest-frame atomic transition frequency ω_0. The Hamiltonian of such a system, written in the rotating frame and in the rotating-wave approximation, is given by

$$H=\frac{\hat{p}^2}{2m}+\frac{\Omega(\hat{z})}{2}(\sigma^\dagger+\sigma)\,, \tag{1}$$

where $\hat{z},\hat{p}$ denote atomic position and momentum operators, $\Omega(\hat{z})=\Omega\cos(k\hat{z})$ is the $\hat{z}$-dependent Rabi frequency, and $k=2\pi/\lambda$ is the wave vector of the laser field. The operators $\sigma=|0\rangle\langle 1|$ and $\sigma^\dagger$ denote standard lowering and raising atomic operators. In using the Heisenberg equations to determine the atomic behavior, we use the semiclassical approximation and substitute mean values for corresponding quantum-mechanical operators, i.e., $\hat{z}=\langle\hat{z}\rangle$

$=z$, $\hat{p}=p$. We introduce dressed-atom states [13]

$$|\pm\rangle=(1/\sqrt{2})(|0\rangle\pm|1\rangle)\,, \tag{2}$$

which, in the semiclassical approximation, are the energy eigenstates of the coupled bare atom plus field system. Note that for resonant excitation the composition of the dressed states is spatially independent. Note also that the energy ordering of the dressed-state sublevels varies with position, as can be seen in Fig. 1.

The dipole force acting on the atom is given by [11]

$$f(z)=-\frac{\hbar}{2}\frac{\partial\Omega(z)}{\partial z}[P_+(z)-P_-(z)]\,, \tag{3}$$

where $P_\pm$ are the z-dependent populations of the dressed states. It can be shown that the populations obey the following rate equations:

$$\dot{P}_\pm=-\Gamma_{\pm\mp}P_\pm+\Gamma_{\mp\pm}P_\mp\,. \tag{4}$$

In free space and for resonant excitation, $\Gamma_{+-}=\Gamma_{-+}=\Gamma/4$, where Γ is the Einstein decay rate. This equality implies a uniform distribution of population among the dressed-state sublevels that is independent of the atomic position and velocity. Hence, by inspection of Eq. (3), the atom experiences no net damping force.

However, in the presence of a colored vacuum, the decay rates must be multiplied by factors proportional to the electromagnetic mode density at the respective transition frequencies, i.e., $\Gamma_{ab}=\Gamma_{ab}^{\text{free}}\rho(\omega_{ab})$ with $a,b=\pm$.

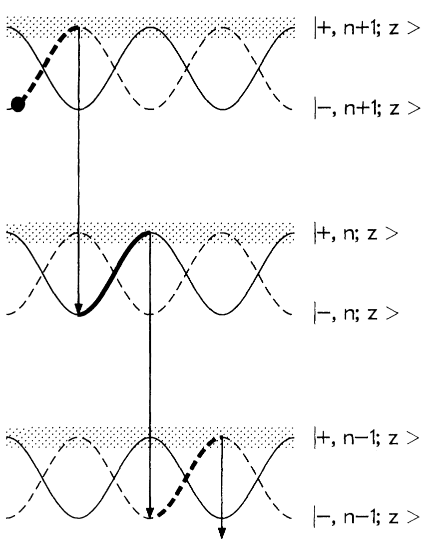

FIG. 1. Spatial dependence of the dressed-atom energy eigenstates for a two-level atom driven by a resonant, standing-wave field. The photon occupation number is denoted by n. The solid (dashed) curves represent the $|+\rangle$ ($|-\rangle$) states. The shaded area indicates the region of enhanced spontaneous emission arising from the presence of the colored vacuum. The bold line represents the trajectory of an atom (represented by the dot) moving from left to right through the standing wave.

Owing to the spatial dependence of the dressed-state transition frequencies under standing-wave excitation, coupling to a colored vacuum will generally lead to spatially dependent dressed-state transition rates and, hence, equilibrium dressed-state populations. As a result, otherwise nonexistent damping forces arise.

For convenience, we assume that the colored-vacuum density of modes, $\rho(\omega)$, has the following form:

$$\rho(\omega)=1+\frac{\alpha\Gamma_c^2}{\Gamma_c^2+(\omega-\omega_c)^2}\,, \tag{5}$$

where α denotes a spontaneous emission enhancement factor and ω_c is the cavity frequency. The mode density of certain optical cavities takes on approximately this form [4,5]. We shall limit our analysis to the case of strong enhancement factors, $\alpha\gg 1$, noting that α values on the order of unity have already been reported and means of achieving significantly higher values appear feasible [14]. We further assume that $\omega_c=\omega_0+\Omega$ so that the atom-vacuum coupling is largest at the maximum frequency of the upper-to-lower dressed-state transition frequency (see Fig. 1).

In such a case, the dressed-state populations fulfill

$$\frac{dP_\pm}{dt}=-\Gamma_{\text{pop}}(P_\pm-P_\pm^{\text{st}})\,, \tag{6}$$

where $\Gamma_{\text{pop}}=\Gamma_{+-}+\Gamma_{-+}$,

$$\Gamma_{\pm\mp}=\frac{\Gamma}{4}\left[1+\frac{\alpha\Gamma_c^2}{\Gamma_c^2+\Omega^2(1\mp\cos kz)^2}\right]\,, \tag{7}$$

and where the steady-state populations are given by $P_\pm^{\text{st}}=\Gamma_{\mp\pm}/\Gamma_{\text{pop}}$.

The above equations can be solved numerically to determine the detailed properties of the damping force in the presence of a vacuum colored as described above. Before doing so, however, we point out that the basic physics operative here is quite simple. In the absence of spontaneous decay, atoms moving through the standing wave remain in the same dressed-state sublevel. The energy of the dressed state rises and falls indicating a nondissipative interchange of atomic kinetic and potential energies. Under the influence of the colored vacuum of Eq. (5), spontaneous decay is present and occurs most rapidly on transitions of frequency ω_c. Such decay preferentially moves atoms passing the standing-wave's antinodes from the higher-energy dressed-state sublevel to the lower-energy one. Provided that the atoms pass through each antinode slowly enough so that they experience an upper-to-lower dressed-state transition, one finds, as shown in Fig. 1, that the atoms are forced to perpetually climb against a locally increasing potential and therefore dissipate kinetic energy. This damping turns out to be fully effective as long as the atomic velocity does not exceed

$$v_{\text{cap}}\simeq\alpha\Gamma\epsilon/k\,, \tag{8}$$

where $\epsilon=(2\alpha)^{1/4}(\Gamma_c/\Omega)^{1/2}$, and ϵ/k is the effective distance over which the colored vacuum increases the upper-to-lower dressed-state-sublevel transition rate from $\Gamma/4$ to approximately $\alpha\Gamma/4$. For atomic velocities up to v_{cap}, every antinode passage is most likely accompanied by an upper-to-lower dressed-state-sublevel transition. Note that the capture velocity v_{cap} for cooling in a colored vacuum is not bound by atomic constants and can increase without limit as the enhancement factor is increased. Hence, colored vacua may dramatically enhance the performance of standard atomic traps.

We have solved Eq. (6) utilizing the underlying spatial periodicity of the dressed-state populations and by assuming (as in Ref. [11]) that velocity changes accruing during an atomic relaxation time are sufficiently small to allow us to replace the time derivative of Eq. (6) with $v\,d/dz$. We have calculated the quantum average of the dipole force acting on the atom by substituting the expressions for the position-dependent populations into Eq. (3). Finally, we have calculated the spatially averaged force $\bar{f}$ obtained by averaging over a spatial period of the standing wave. In Fig. 2, we plot our numerical results for $\bar{f}$ as a function of velocity for various values of the cavity enhancement factor α. For comparison, we also included in Fig. 2 the velocity dependence of the force experienced by an atom driven by an off-resonant field in a free space (identical to Fig. 7, Ref. [11]). As already mentioned, the colored-vacuum-mediated damping forces are both larger and peak at higher atomic velocities than in the free-space case.

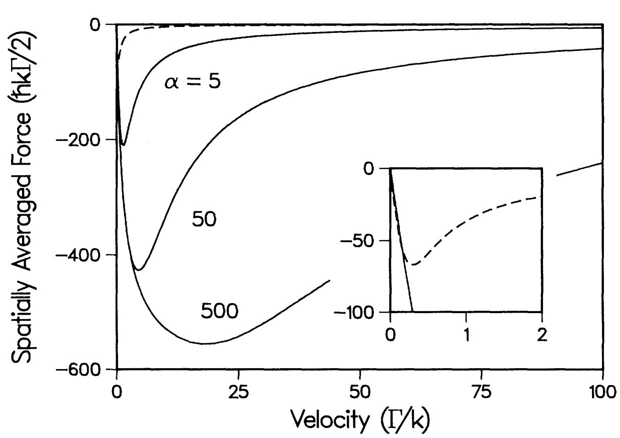

FIG. 2. Spatially averaged mechanical force experienced by a resonantly driven atom moving along a standing wave in a colored vacuum (solid lines) as a function of velocity for $\Omega=1000\Gamma$ and for various values of the enhancement factor α. The cavity linewidth is adjusted so that $\epsilon=1$ for each value of α, where $\epsilon=(2\alpha)^{1/4}(\Gamma_c/\Omega)^{1/2}$. Recall that the force vanishes when $\alpha=0$ for a resonant driving field. For comparison, the conventional force experienced by an atom in a flat vacuum ($\alpha=0$) driven by an off-resonant field ($\omega-\omega_0=200\Gamma$) is shown (dashed line). Inset: The velocity-dependent force at low velocities.

We now attempt to motivate the qualitative variation of damping forces with atomic velocity that is shown in Fig. 2. For slow atomic velocities ($kv/\Gamma<1$), the dressed-state populations at position z can be well approximated in terms of the steady-state populations at some previous position according to [11] $P_\pm(z)=P_\pm^{st}(z-v/\Gamma_{pop}(z))$. Expanding the time-lag expression in v we find that the spatial average of the dipole force is equal to $\bar{f}=-\gamma m v$. In the limit of large α the friction coefficient γ can thus be approximated by

$$\gamma\simeq\frac{\hbar k^2}{2m}\frac{\Omega}{\Gamma}\epsilon\,. \tag{9}$$

Note that the standard free-space friction coefficient for a resonantly driven two-level atom is equal to zero. Introducing a detuned free-space driving field whose Rabi frequency Ω exceeds the detuning Δ_1, one obtains a nonzero damping coefficient whose magnitude is on the order of $\hbar k^2\Omega/2\Gamma m$. Since we have found in our numerical simulations that $\bar{f}$ is optimized for $\epsilon\simeq1$, we see that, in the low-velocity limit, the damping force experienced by a resonantly driven atom in a colored vacuum is similar to that experienced by a nonresonantly driven atom in free space. The inset in Fig. 2 shows the spatially averaged damping forces for low-velocity atoms in both the free-space and colored-vacuum cases. We see that the slopes of the curves and hence the friction coefficients are nearly equal, in agreement with the above discussion.

For the case of intermediate velocities ($1<kv/\Gamma<2\alpha\epsilon$), the atom moves slowly enough to relax in the region of enhanced spontaneous emission, but does not have time to relax otherwise. In this case, the atom exhibits a maximal Sisyphus-type effect [11]; see Fig. 1. The mean dipole force $\bar{f}$ acts to provide damping and is given approximately by

$$\bar{f}=-\frac{\hbar k\Omega}{2}\frac{1+\exp(-2\pi\Gamma/vk)}{1+(2\Gamma/vk)^2}\,. \tag{10}$$

By inspection of Eq. (10), we see that for intermediate velocities the force achieves a value of the order of $\hbar k\Omega/2$. This force can be substantially larger than the free-space damping force.

For the case of high atomic velocities ($kv/\Gamma>2\alpha\epsilon$), the atom moves so fast that it experiences only the spatial average of the local decay rates [11]. Therefore, the force can be approximated as

$$\bar{f}=-\frac{\hbar\Omega\bar{\Gamma}_{pop}}{2v}\simeq-\frac{\hbar\Omega\Gamma}{2v}(1+2^{5/4}\alpha^{3/4}\epsilon)\,, \tag{11}$$

where the bar over the decay rate indicates that a spatial average over a period of the standing wave is to be performed. Note that the force is inversely proportional to v, indicating that the kinetic-energy loss per unit time is velocity independent in this regime (recall that the loss of kinetic energy of the atom passing the distance λ per unit time is given by $-v\bar{f}$). The α-dependent factor in Eq.

(11) may be quite large, so that $\bar{f}$ in the presence of the colored vacuum can become much larger than in free space.

One remaining question is what the low-temperature limit is for cooling of atoms in colored vacua. In free space, this limit is higher than the Doppler limit ($k_B T = \hbar\Gamma/2$, where k_B is Boltzmann's constant) because of the large quantum fluctuations of the dipole forces that give rise to diffusion of the atomic momentum [11]. In a colored vacuum, we have found that an atom experiences little diffusive heating when it is located in the vicinity of the minima of the potential-energy wells shown in Fig. 1. In particular, the dipole diffusion coefficient can be estimated at these locations (i.e., $z \simeq n\lambda/2 \pm \epsilon/2k$; $n=0, \pm 1, \pm 2, \ldots$) for an atom at rest and is given by

$$D_{\text{dip}} = 4\hbar^2 k^2 \Omega^2/\alpha^2\Gamma . \tag{12}$$

Therefore, the final temperature of the atom will be $k_B T = D_{\text{dip}}/m\gamma'$, where γ' is the local friction coefficient given by $\gamma' = \gamma/2\epsilon$ which is of the order of $\hbar k^2 \Omega/2m\Gamma$. Hence, cooling of atoms below the Doppler limit is possible for sufficiently large α, provided that $\Omega < \alpha^2\Gamma$. One should note that colored vacua allow for trapping of atoms in the potential minima under conditions when the final kinetic energy $k_B T/2$ does not exceed the potential-energy barrier $\Delta E \simeq \frac{1}{8}\pi\hbar\Gamma\sqrt{2\alpha}$. Thus, trapping will occur when $\Omega < \alpha^{5/2}\Gamma$. Therefore, if α is sufficiently large, both cooling and trapping of atoms beyond the Doppler limit should be possible in a colored vacua.

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