Quantum Nondemolition Measurements of Photon Number by Atomic-Beam Deflection

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We present a new quantum nondemolition measurement scheme in which a measure of the photon statistics of a cavity field can be achieved by monitoring the deflection of atoms interacting with the field. Repeated measurements result in the collapse of the photon distribution in the field to a number state. Quantum jumps in the photon number are observed when the cavity field is coupled to an external reservoir.

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We wish to propose a new quantum nondemolition (QND) measurement of the photon number of a cavity field mode by measuring the deflection of atoms passing through the field. The measurement of a quantum observable of a system is accomplished by coupling it to a meter or probe. For an ideal measurement one requires that the system-probe interaction does not feed back noise into the variable to be measured and alter its dynamics. This type of measurement scheme in which the backaction noise arising from the probe coupling is avoided is known as quantum nondemolition [1].

It was pointed out by Braginsky, Vorontsov, and Khalili that in order to realize a QND energy measurement the system-meter coupling has to be proportional to the squared generalized coordinate of the system. Thus in order to measure the photon number of the electromagnetic field it is necessary to have an interaction Hamiltonian which has a quadratic dependence on the electric field. Such a coupling may be achieved in an optical four-wave mixing interaction, where a quadrature phase measurement of the probe beam gives information on the photon number of the signal beam [2]. QND measurements using this coupling have been made in optical filters [3].

Recently, Brune *et al.* [4] have proposed a measurement of the photon number in a microwave cavity by coupling the field to a beam of Rydberg atoms and measuring the atomic phase shift. Braginsky and Vyatchanin [5] have suggested using the Compton scattering of an electron in a dielectrical waveguide to measure the energy of the electric field.

In this Letter we wish to propose a new QND scheme based on measuring the deflection of ground-state atoms from the quantum field stored in a standing-wave optical cavity. Meystre, Schumacher, and Stenholm [6] have shown that the atomic-beam deflection by a light field is a sensitive function of the photon field statistics. We shall show that if the frequency of the light is sufficiently detuned from the atomic resonance and the cavity Q is sufficiently high the momenta of the deflected atoms constitutes a QND readout of the photon number observable.

In a recent experiment the momentum transfer to atoms by a standing-wave light field has been measured by Gould *et al.* [7]. However, clearly there are a number of requirements which must be considered for such a coupling to be QND. The atom must be prepared in the ground state so that new photons are not deposited into the system. In addition the atomic inversion must be negligible so that the atom does not exit in the excited state carrying off a quanta of energy. This requirement is that $|g|^2 n \ll \Delta^2 + \gamma^2$, where g is the single-photon Rabi frequency, n is the characteristic photon number, Δ is the detuning of the laser from the two-level transition frequency, and γ is the spontaneous emission rate. The number of spontaneously emitted photons from the atom while it is in the interaction region must be very small. This requirement is that $|g|^2 n \gamma t/(\Delta^2+\gamma^2) \ll 1$, where t is the interaction time. The atom must be in the field for a sufficient amount of time to see some interchange of photons between the counterpropagating waves, so that there is an appreciable probability of deflection. This requires that $|g|^2 nt/2\Delta = r$, where r is the characteristic number of $2\hbar k$ units of deflection. Finally, the interaction time must be sufficiently small that the transverse kinetic energy absorbed by the atom during the interaction can be neglected. This is known as the Raman-Nath regime [8] and requires $t < 2\pi/\omega_r$, where the recoil energy is $\hbar\omega_r = (2r\hbar k)^2/2m$, m is the atomic mass, and k is the wave number of the cavity mode. The above conditions may be satisfied in an experiment for the following parameters: $n = 10$ photons, $|g| = \gamma = 10^7$ s^{-1}, $\Delta = 5\times 10^8$ s^{-1}, and $t = 5\times 10^{-6}$ s. Such parameters are achievable with current technology, requiring the cavity mode to be tightly focused in the interaction region. The cavity quality must be sufficiently high that the cavity is repeatedly probed by a number of atoms in the mean lifetime of a cavity photon. The coupling between the cavity and an external reservoir may then be monitored through the evolution of the photon statistics via a series of quantum jumps to the equilibrium distribution.

We denote the boson annihilation operator for the field by a and the atomic inversion and atomic coherences by the Pauli spin operators σ_z, σ_+, and σ_-. Required also are the conjugate operators p and x describing the transverse momentum and position respectively of the center of mass. The Hamiltonian for the combined atom and radiation field system in the rotating-wave approximation and Raman-Nath regime can then be written as

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_0\sigma_z + \hbar(g\sigma_- a^\dagger + g^*\sigma_+ a)\cos kx\,. \quad (1)$$

Spontaneous emission has been neglected since we consider only the solution in the region of large detuning. This model also describes the interaction of slow Rydberg atoms with a micromaser field [9]. In this limit the resulting equations of motion can be derived from the *effective* interaction Hamiltonian

$$V_i = \hbar(|g|^2/\Delta)\sigma_z a^\dagger a(\cos 2kx + 1)\,, \quad (2)$$

in which neither the atomic inversion σ_z nor the photon number $a^\dagger a$ is altered by the evolution of the atom in the cavity. The strength of the perturbation is proportional to the intensity of the field. Note that the $\cos 2kx$ dependence of the Hamiltonian describes the momentum transfer $\pm 2\hbar k$; the momentum transfer $2\hbar k$, for example, corresponds to absorption of a photon from the $(+k)$ component, followed by induced emission into the $(-k)$ component of the standing-wave field. Since the possible momentum shifts are discrete multiples of $2\hbar k$, the final output momentum probability distribution is composed of a comb of images of the initial momentum distribution. In order to resolve these peaks, it is necessary to narrow the initial momentum spread so that $\Delta p < 2\hbar k$. In principle, if the initial momentum is sufficiently well defined, it is possible to detect spontaneous emission of the atom in the field. Such a photon, scattered out of the solid angle described by the cavity mode, would correspond to nonintegral $2\hbar k$ momentum transfer.

The probability of the atom exiting with momentum p after interaction time t is given by

$$Q(p,t) = Q(p,0) * \sum_r \left[\sum_n J_r^2\left(\frac{|g|^2 nt}{2\Delta}\right)P(n)\right] \times \delta(p - 2r\hbar k)\,, \quad (3)$$

where $*$ denotes momentum convolution, δ and J are the Dirac delta and Bessel functions, respectively, and $P(n)$ describes the photon statistics of the cavity. For simplicity, we have taken the Rabi frequency to be independent of time. The term in square brackets represents the probability that a momentum of $2r\hbar k$ will be transferred to the atom in the cavity. This is strongly dependent on the field photon statistics. For example, for a thermal field, where the maximum $P(n)$ corresponds to no photons in the cavity, the most likely momentum shift is zero. In the case of a coherent field in which the photon statistics are Poissonian, the cavity is well described as an atomic beam splitter in which the greatest probability lies in regions of nonzero atomic deflection. If the field is significantly amplitude squeezed so that the photon number distribution is much narrower than Poissonian, the output momentum distribution qualitatively approaches that generated by a number state. Highly phase-squeezed fields with greatly increased amplitude fluctuations exhibit a deflection probability similar to that of a field described by thermal statistics.

Since the position wave function at large distances from the interaction region is dependent on the statistics of the intracavity field, each atomic position measurement reduces the density operator corresponding to the field state. If the cavity lifetime is long compared to the time between atomic injections, Eq. (3) can be inverted and sufficient repeated measurements will eventually completely determine the photon statistics. However, if the cavity is not significantly coupled to any external reservoir, continual probing of the cavity will eventually result in the complete collapse of the field state to that of an exact number state corresponding to the absolute energy of the radiation within the normalization volume. We illustrate this effect by simulating repeated atomic position measurements and examining the residual density of states.

The procedure adopted is as follows. For simplicity we consider a monokinetic atomic beam in which the longitudinal velocity spread is small compared to the mean velocity. Based on an initial choice of field statistics, a particular momentum p_0 for an atom exiting the cavity is chosen. The diagonal elements of the field density matrix $P(n)$ are then altered by the back projection of the measurement, $P(n|p_0) = MP(p_0|n)P(n)$, where M is the normalization constant. The next momentum p_1 is then selected with this probability weighting for the statistics of the field, and the process is repeated. As the measurement proceeds the lack of knowledge, or entropy, of the state of the field $\sum_n P(n)\ln P(n)$ is reduced. Figure 1 il-

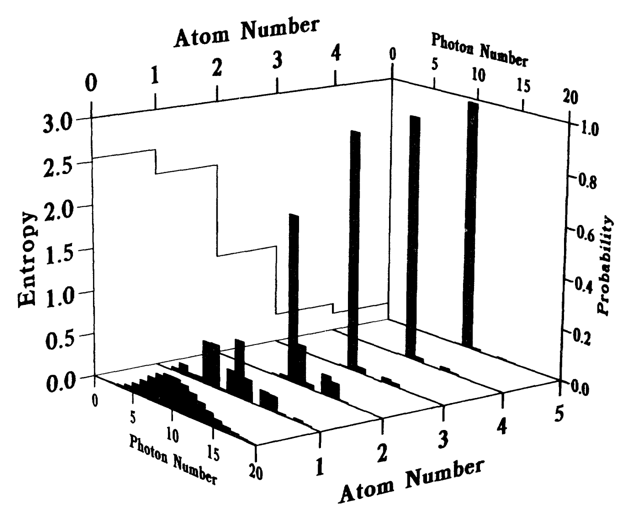

FIG. 1. Simulation of the collapse of the field density of states to a single photon number (for parameters see text). The vertical *probability* scale on the right corresponds to all six of the bar graphs. Projected on to the back wall is the entropy of the field state with scale denoted by the *entropy* vertical axis on the left. As information about the field is accumulated, the entropy is reduced.

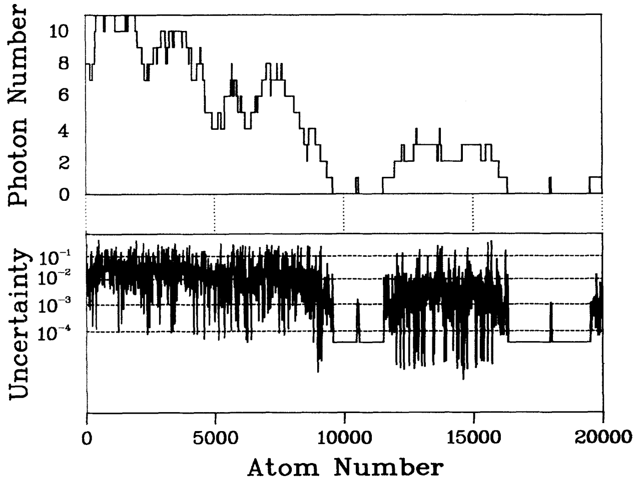

FIG. 2. Simulation of a cavity weakly coupled to a thermal reservoir of mean $\bar{n}=3$ where the decay rate of the cavity corresponds to 5000 atoms. The top graph illustrates the most likely photon number showing quantum jumps in the state and the relaxation of the cavity. The bottom graph displays the entropy measure of field uncertainty on a logarithmic scale. The vertical axis of this graph is labeled with the total possible residual probability not contained in the photon number state quoted in the top graph.

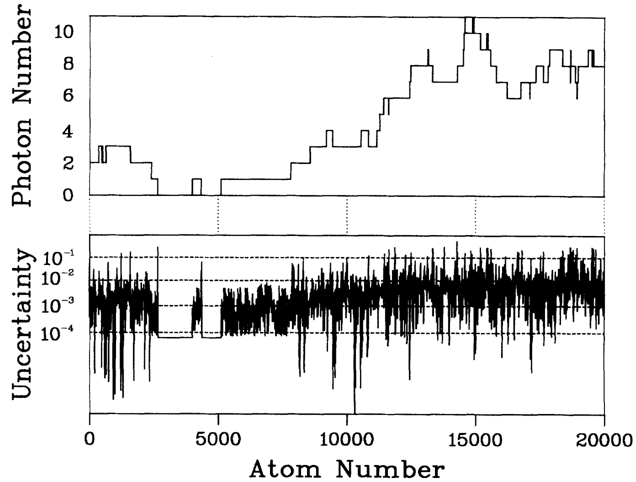

FIG. 3. Simulation of a cavity weakly coupled to a laser gain medium 10 times above threshold and with mean of 10 photons. The cavity field was initially described by a thermal distribution of mean of 3 photons. The cavity decay rate corresponded to 5000 atoms. The evolution of the photon number illustrates a possible trajectory for the growth of the field in the cavity.

lustrates a simulation of five probe atoms with a nonrelaxing field initially described by a coherent state with a mean of 10 photons. The atomic interaction parameters were $|g|^2t/2\Delta=50$. Each such simulation collapses the field to a single photon number which then does not change. The proportion of times that each number is selected is completely determined by the initial photon statistics. Note that the position measurement of each individual atom extracts partial information from the field, and it is only the cumulative information contained in the full measurement sequence which contracts the field to a well-defined number state. The final entropy can be used as an indicator of the quality of measurement. In the case of an appreciable longitudinal velocity distribution of atoms in the probe beam, it is necessary to select a velocity as well as exit momentum for each atom (see [4]). Such a simulation contains the same characteristics of projection to a single photon-number state after a number of probe atoms, but the velocity spread reduces the measurement quality and consequently retards the rate of entropy collapse. The collapse of the cavity field to a single photon number could be confirmed by measuring the deflection pattern of subsequent atoms which should conform to the deflection pattern of a number state.

Providing the cavity is only weakly relaxing, the repeated atomic measurements can still project the field into a well-defined photon number and it is then possible to observe quantum jumps in the state. Figure 2 illustrates this for a field weakly coupled to a thermal bath of mean $\bar{n}=3$. The field was initially described by a coherent state of mean 10, the decay rate of the cavity γ corresponded to 5000 atoms, and again $|g|^2t/2\Delta=50$ described the atomic interaction. As an indicator of the quality of measurement as the simulation evolved, we have illustrated the entropy of the field displayed on a logarithmic scale. The vertical axis is labeled by the residual uncertainty in the photon state. When the entropy is below the dashed horizontal line beside the 10^{-1} threshold, the corresponding photon number state displayed in the top graph must contain at least 90% of the probability. Despite the relaxation, the atomic measurements maintained for the most part one of the photon number probabilities at more than 0.9 over the initial decaying period, and over 0.99 once the cavity had assumed more completely the stationary statistics of the thermal bath at reduced photon number. The characteristics of the entropy measure during the simulation are very different when the most likely photon number in the cavity is zero from when it is not. In the case that $P(0)$ contains essentially all of the probability, the atom will almost always be undeflected. In this system, the relaxation of the cavity followed by the back projection of the measurement rapidly converges to stable photon statistics with no fluctuations in entropy. However, on the infrequent occasion that an atom is deflected the photon state must immediately jump since there is then absolutely no probability that the cavity is empty. Note that in the stationary state, the occupation times for the various photon numbers should be given completely by the photon statistics of the reservoir.

Figure 3 illustrates a similar simulation except the initial field was described by a thermal distribution of mean of 3, and the relaxation occurred through a laser gain medium 10 times above threshold and with a steady-state

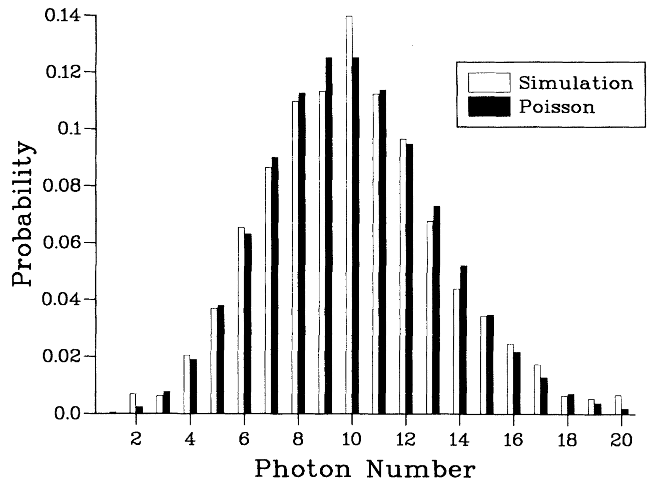

FIG. 4. Stationary photon statistics of a cavity coupled to a laser gain medium. We plot the proportion of times in which the simulation as in Fig. 3 occupied each number state. The number of atoms was 50000 and the cavity decay rate was set to 500 atoms. A comparison with a Poissonian distribution is given.

mean of 10 photons. The drift rate corresponded to 5000 atoms, over which time scale the growth of the field in the cavity is clearly displayed. Again the photon state was very well defined; for the most part one photon state contained more than 0.99 probability. Also note that the stationary statistics of the occupation times for each photon number should be the approximately Poissonian statistics of the above-threshold laser. This is more clearly illustrated in Fig. 4 in which the proportion of time for which each photon number state was occupied was calculated for the same laser but with 50000 atoms and a drift rate corresponding to only 500 atoms. The Poisson distribution is also displayed for comparison, with good agreement.

We have considered a QND coupling which allows the photon number of a cavity field to be monitored by measuring the momentum distribution of atoms scattered by the field. Repeated measurements result in the collapse of the photon distribution in the field to a number state. Quantum jumps in the photon number are observed when the cavity field is coupled to an external reservoir.

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[1] V. B. Braginsky, Y. I. Vorontsov, and F. Y. Khalili, Zh. Eksp. Teor. Fiz. **73**, 1340 (1977) [Sov. Phys. JETP **46**, 705 (1977)]; W. G. Unruh, Phys. Rev. D **18**, 1764 (1978); C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmerman, Rev. Mod. Phys. **52**, 341 (1980).

[2] G. J. Milburn and D. F. Walls, Phys. Rev. A **28**, 2065 (1983); N. Imoto, H. A. Haus, and Y. Yamamoto, Phys. Rev. A **32**, 2287 (1985).

[3] M. D. Levenson, R. M. Shelby, M. Reid, and D. F. Walls, Phys. Rev. Lett. **57**, 2473 (1986); N. Imoto, S. Watkins, and Y. Sasaki, Opt. Commun. **61**, 159 (1987).

[4] M. Brune, S. Haroche, V. Lefevre, J. M. Raimond, and N. Zagury, Phys. Rev. Lett. **65**, 976 (1990).

[5] V. B. Braginsky and S. P. Vyatchanin, Phys. Lett. A **132**, 206 (1988).

[6] P. Meystre, E. Schumacher, and S. Stenholm, Opt. Commun. **73**, 443 (1989).

[7] P. L. Gould, P. J. Martin, G. A. Ruff, R. E. Stoner, J. L. Picqué, and D. E. Pritchard, Phys. Rev. A **43**, 585 (1991).

[8] S. Stenholm, Rev. Mod. Phys. **58**, 699 (1986).

[9] S. Haroche, M. Brune, and J. M. Raimond, Europhys. Lett. **14**, 19 (1991); B. G. Englert, J. Schwinger, A. O. Barut, and M. O. Scully, Europhys. Lett. **14**, 25 (1991).