Chiral Symmetry in Hot QCD

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(Received 17 June 1991)

Recent lattice calculations of static hadronic screening lengths and the quark number susceptibility are interpreted in terms of a simple constituent quark model. The conclusion is that just above T_c the nature of the hadronic modes is rather well described in terms of elementary π and σ, as well as light quark, modes. Hence chiral symmetry in the absence of symmetry breaking terms is realized through massless quarks rather than through parity doubling.

PACS numbers: 12.38.Gc, 11.30.Rd

It is widely assumed that the QCD ground state breaks chiral symmetry spontaneously. Quark masses break the symmetry explicitly, leading to a small mass for the pion, the pseudo Goldstone boson of the broken symmetry. Furthermore the axial current is partially conserved, vanishing like the quark mass $m \sim m_\pi^2$. Effective Lagrangians that incorporate this scenario have been very successful in describing low-energy aspects of the strong interactions [1]. There is, however, no proof of chiral symmetry breaking in QCD. The strongest evidence probably comes from lattice calculations [2] which point towards a nonvanishing condensate $\langle \bar{\psi}\psi \rangle$ in the continuum limit. On the more qualitative level there is Casher's beautiful argument [3] which suggests that confining theories necessarily break chiral symmetry. In the limit of an infinite number of colors it was proved [4] that the *VVA* three-point function has a massless pole which, since only mesons contribute to leading order in $1/N$, was interpreted as being due to a massless pion. A similar argument can be made at any N invoking the so-called 't Hooft anomaly constraints [5] which again suggest the existence of a massless pion and hence chiral symmetry breaking —assuming confinement.

At finite temperature and/or finite density one expects the chiral symmetry to be restored beyond a certain temperature (density). One can ask then in what fashion the chiral symmetry will be realized in the symmetric phase. The standard scenario of deconfined quarks and gluons would suggest that there are massless quarks so that chiral symmetry is realized explicitly. There is a serious problem with such a scenario though which should also be kept in mind later on when we compare lattice data with a model featuring colored (constituent) quarks. If there is a gap in the phase boundary in the T-m plane —and recent Monte Carlo simulations suggest that this is so—then there should be a one-to-one correspondence between singularities in correlation functions in the symmetric and the broken phase [6]. If, in the symmetric phase, "real" quarks are the same as constituent quarks, then it ought to be possible to dissociate hadrons into quarks. As a matter of fact there is an argument due to Mueller [7] which suggests that there cannot be any (Coulombic) bound states at very high temperature. The correlation function of hadrons in this case have singularities which they do not have at zero temperature. Also, it is possible that QCD at high temperature can be reduced to an effective three-dimensional confining theory [6], which again makes it difficult to interpret effective models with colored quarks.

There is a different way to realize chiral symmetry if one assumes that the symmetric phase consists of hadronic modes rather than deconfined quarks [6]. This scenario consists in parity doubling the known spectrum of hadrons. In particular, the nucleon and its parity partner are *massive* and degenerate. The simplest way of seeing that this is consistent with chiral symmetry is by realizing that it is possible to write down a chirally symmetric mass term for parity-doubled spinors [8]. Alternatively, one can follow the group-theoretical arguments explained by McLerran [9]. In view of the standard lore about chiral symmetry breaking explained in the previous paragraph the notion of a confining yet chirally symmetric phase is somewhat strange. At large N, where both the concept of deconfinement and chiral symmetry restoration can be defined unambiguously, one can prove that $T_d \leq T_\chi$, i.e., that deconfinement happens before chiral symmetry restoration [10]. At finite N no definite conclusion can be reached however. The axial anomaly does not seem to put any constraints on the way chiral symmetry is realized at high temperature [11]. This means that the theory does not necessarily contain a massless excitation which, in the absence of spontaneous chiral symmetry breaking, one would identify with a massless quark. Hence the possibility of a confining but chirally symmetric phase is not ruled out *a priori.*

It was also suggested by DeTar [6] that numerical simulations of lattice QCD could shed some light on the issue of "pseudoconfinement" at high temperature, i.e., on the notion that only color-singlet hadronic modes propagate at large distances. By measuring the correlation function

$$C(z) = \sum_{x,y,t} \langle H(x,y,t;z)H(0)\rangle \sim e^{-M_H z} \tag{1}$$

of two hadronic operators in one of the spatial directions one obtains information about the *static* part of the real-time dispersion relation of the hadronic disturbance of the plasma created by the local operator H. The mass governing the exponential decay of $C(z)$ is called a screening mass while its inverse is called a screening length. Note that at least in principle it is possible to obtain the real-time dispersion relation from imaginary-time correlation functions by analytic continuation in the discrete Matsubara frequencies. In practice, of course, one is limited to measuring static correlations of the type in Eq. (1) and there are by now a number of such calculations. The first calculation was done by DeTar and Kogut [12] who found using $n_f=4$ flavors of Kogut-Susskind fermions that the screening masses are large (of order several times the temperature) and degenerate for the parity partners (π,σ), (ρ,A_1), (N,N^*). Furthermore, the pion mass above T_c scales linearly in the quark mass m—indicating that it is no longer a Goldstone boson. Similar results were also reported for two flavors [13]. Interestingly enough, almost *identical* results to those of Gottlieb *et al.* for the screening masses were obtained in the *quenched* ($n_f=0$) approximation at the same physical temperature [14]. There is also a recent calculation by the MT_c Collaboration [15] in which both quenched and unquenched data at rather weak gauge coupling are reported.

The fact that one finds hadronic modes with large parity-doubled masses seems to confirm the nontrivial nature of the hadronic plasma at large distances, including the realization of chiral symmetry by parity doubling. One must be careful, however, in interpreting these results. There is a rather trivial fact (also observed in Ref. [14]) which is the following: Meson and baryon correlation functions computed with free quarks give screening masses in the limit of vanishing quark mass which are essentially 2 and 3 times the lowest Matsubara frequency on the lattice, $\omega_0=\pi/n_t a=\pi T$, due to the antiperiodic boundary conditions used in the time direction. This might be a bit counterintuitive in the case of a meson which is a boson. This boson is, however, made up of two independently propagating fermions giving it twice the "mass" of its constituents. This mass receives perturbative corrections in a situation where perturbation theory is adequate. Large deviations indicate nonperturbative effects, i.e., the formation of a bound state. One should also note that it is difficult to differentiate between parity doubling and (almost) free quarks on the basis of the functional form of $C(z)$. (Using slightly more sophisticated tests for parity doubling [13] also does not help since those tests are based on properties of the quark propagator which are trivially satisfied in the case of free quarks.)

In Fig. 1, I present a compilation of screening masses which I have taken from the literature. Shown are π, σ, and nucleon masses which are the same for their parity partners. I have also shown the masses of mesons and baryons constructed from free quarks on the lattice, $M=n\,\mathrm{arcsinh}[\sin(\pi/n_t)]$, where $n=2$ for mesons and $n=3$ for baryons. The upper and lower lines correspond to $n_t=6$ and $n_t=4$ respectively. Using least-squares fits to the same functional form as used in the calculation of the QCD screening masses I obtained for the free meson and baryon masses $m_\pi/T=(5.8,6.5)$ and $m_N/T=(8.5,9.8)$ for $n_t=4$ and 6 (and $n_z=20$). The data at the three largest temperatures are from quenched calculations on lattices of time extent $n_t=4$ in Refs. [13] and [14] (MT_c Collaboration) where the gauge couplings were such that the temperatures are $3T_c/2$, $4T_c/2$, and $5T_c/2$. The data in the chirally broken phase are from $n_t=8$ lattices in Ref. [12] and correspond to $T/T_c=0.75$. The remaining data are taken from Ref. [11]. Their location in T/T_c is essentially a guess, since the gauge couplings there do not have any special value which allows for easy translation into those units. Also, there is no information about the zero-temperature spectrum at those values of the gauge coupling which would give the temperature in units of the ρ mass [16]. In the absence of a more sensible procedure, I have used the asymptotic scaling formula with a critical gauge coupling taken from the Columbia group [17]. For the qualitative argument that I am giving here this should suffice.

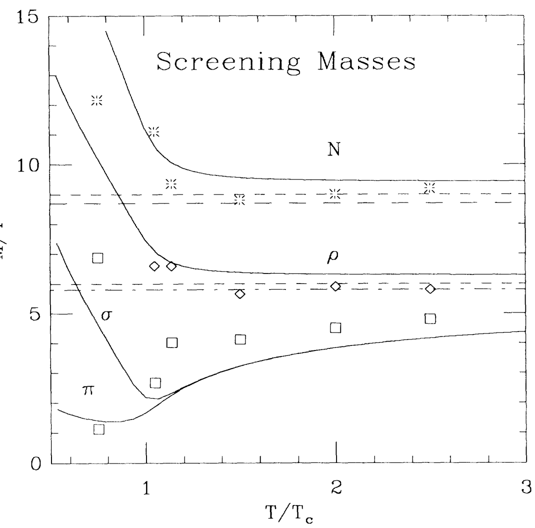

FIG. 1. The measured screening masses for (π,σ) (squares), ρ (diamonds), and (N^+,N^-) (bursts) along with the continuum σ model predictions (solid lines). The upper and lower dashed lines denote the masses of the free hadrons on an $n_t=6$ and $n_t=4$ lattice, respectively. The data points at the lowest temperature show the splitting of the (π,σ) in the broken phase.

Another interesting quantity obtained from lattice calculations is the baryon number susceptibility at zero chemical potential χ as a function of the temperature [18]. This quantity, which is defined as the derivative of the quark number density with respect to the chemical potential, is a direct measure of the mass of the lightest baryonic excitation since it tells us how difficult it is to add additional baryons to the system. Below T_c this quantity is essentially zero, because the baryons are massive, exponentially suppressing the susceptibility. For a gas of free massless quarks the susceptibility is just $\chi=n_f T^2$. Gottlieb *et al.* found that above T_c the susceptibility is large, $\chi a^2 \approx 0.22$ (a is the lattice spacing) compared to $\chi a^2=0.29$ for a gas of massless fermions on an $8^3\times 4$ lattice [the continuum value is $\chi/(4T)^2=0.125$]. Hence the baryonic excitations above T_c are light. Again, very similar results were also found in the quenched approximation [19].

In the remainder of this Letter I would like to discuss the data and interpret them in terms of a simple model which I believe captures the basic physics. It seems clear that ρ and nucleon screening masses are very well described in terms of simple propagation of nearly massless quarks. The large masses one sees are purely kinematic in origin and an artifact of the boundary conditions used in the imaginary-time formalism. On the other hand, it is equally clear that the π-σ masses are significantly smaller than those of free mesons. This, however, is hardly surprising. After all, the scalar and pseudoscalar modes are rather special as far as chiral symmetry is concerned: Approaching T_c from above, the pion is about to become a Goldstone boson whereas condensation is about to occur in the σ channel.

It is precisely the fact that the π is *not* very well described in terms of a simple quark model that has led to construction of the chiral quark model which has both quark and gluon as well as pion degrees of freedom [20]. One imagines that the constituent quarks inside a proton are essentially free and interact only weakly via gluons. Here I will explore whether a simple extension of this idea to finite temperature can describe what we have learned from the lattice. Whereas Manohar and Georgi [20] use a nonlinear σ model I will employ a linear model to describe the spontaneous breakdown of chiral symmetry. The reason is that one needs an explicit σ particle which in the nonlinear model has been integrated out. The model, then, is simply the old Gell-Mann–Levy model [21] where the fermion field is considered to be a constituent quark. In the following I will restrict myself to two flavors and I will drop all reference to the gluons which I assume are only weakly interacting with the constituent quarks. The Lagrangian is

$$\mathcal{L}=\tfrac{1}{2}[(\partial_\mu\sigma)^2+(\partial_\mu\boldsymbol{\pi})^2]+\frac{\mu^2}{2}(\sigma^2+\boldsymbol{\pi}^2)+\frac{\lambda}{4!}(\sigma^2+\boldsymbol{\pi}^2)^2 + c\sigma+\bar{\psi}[\partial\!\!\!/+g(\sigma-i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})]\psi\,. \qquad (2)$$

The analysis of the model at finite temperature is quite standard [22]. Since we are interested in comparing with the Euclidean lattice theory, all computations are done in the imaginary-time formalism. After shifting the σ field by its expectation value v one obtains for the zero-temperature, tree-level masses $m_\psi=gv$, $m_\pi^2=\mu^2+(\lambda/6)v^2$, and $m_\sigma^2=\mu^2+(\lambda/2)v^2$. This along with the relation $c=f_\pi m_\pi^2$ which follows from PCAC (partial conservation of axial-vector current) and the fact that $v=f_\pi$ fixes all the parameters in terms of physical quantities. I used $m_\pi=140$ MeV, $m_\sigma=600$ MeV, $m_\psi=300$ MeV, and $f_\pi=94$ MeV. By demanding that the shifted σ field has no expectation value, v picks up a temperature dependence through the finite-temperature parts of the tadpole diagrams at the one-loop level. If the explicit symmetry breaking term is absent there is a phase transition at T_c where $v=0$ and the symmetry is restored. In the following, since $c\neq 0$ the symmetry is strictly speaking never restored. Finite-temperature meson masses were computed in the following way: The counterterms at zero temperature were fixed by demanding that the inverse propagator satisfy $D^{-1}(p)=m^2$ for p close to $p=0$; this way the finite-temperature mass shift is simply obtained from $\Sigma_\beta(0)$ where $\Sigma_\beta(p)$ is the temperature-dependent part of the self-energy. The mass so defined is related but not identical to the mass obtained from the pole in the propagator. For the qualitative discussion that follows, this does not matter however. The one-loop temperature-dependent masses such defined are easily computed employing the "Saclay trick" explained by Pisarski [23]. All integrals reduce to simple one-dimensional integrals over Fermi or Bose-Einstein distribution functions.

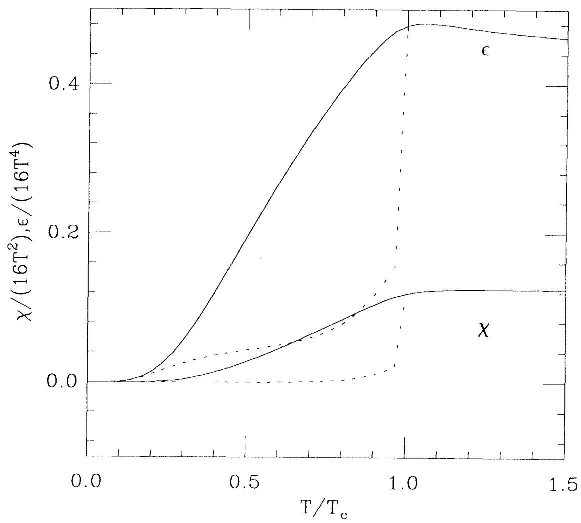

FIG. 2. Quark number susceptibility and energy density of π's, σ's, and constituent quarks in the σ model. The solid curves correspond to a gas of quarks below and above T_c whereas the dashed curves correspond to a gas of nucleons in the broken phase.

In Fig. 1 the solid lines represent the σ model masses as a function of the temperature. The ρ and nucleon masses plotted there are simply multiples of $gv(T)+\pi T$ which is the ("tadpole") constituent quark mass if the momentum-dependent part of the quark self-energy due to π and σ emission is dropped. These self-energy corrections are expected to contribute a mass of order $O(gT)$ for large T. The qualitative features of the lattice data are nicely reproduced by the simple continuum model. The quark number susceptibility is shown in Fig. 2. The plot was obtained by using the one-loop thermodynamic potential [24] whose temperature-dependent part is given by

$$\Omega(T,\mu,v)=(VT)\int\frac{k^2dk}{2\pi^2}\{-4N_c[\ln(1+e^{-\beta(E_q-\mu)})+\ln(1+e^{-\beta(E_q+\mu)})]+\ln(1-e^{-\beta E_\sigma})+3\ln(1-e^{-\beta E_\pi})\}\,, \qquad (3)$$

where $E_i = (m_i^2 + k^2)^{1/2}$. Ω depends on the vacuum expectation value v of the σ field through the masses m_i. Thermodynamic quantities like the susceptibility or energy density are obtained by substituting for v the value $v(T)$ where the full effective potential has a minimum. Using the constituent quarks produces the solid line which is much smoother than the data of Ref. [18]. (I have not included them in the figure because of the large finite-lattice-spacing corrections to the magnitude of the susceptibility.) If, on the other hand, one uses nucleons in the broken phase the dashed line is obtained. One might motivate such a procedure by invoking confinement in the broken phase. In the symmetric phase I have used the mass and degeneracy factor associated with quarks of course. Just below the transition it might be possible to describe quantities such as the energy density as a gas of massive constituent quarks rather than a gas of mesons and baryons (such a description makes intuitive sense since one can imagine the hadrons to overlap significantly near T_c). In this case there are matching degrees of freedom below and above the chiral transition which is different from the scenario described in a recent paper by Brown *et al.* [25]. The energy density calculated from Eq. (3) is also shown in Fig. 2. One sees a sizable change of the energy density but again the curve is smoother than the data [26]. The dashed line shows the energy density when nucleons are used in the broken phase.

It must be emphasized that the fact that quenched and unquenched simulations give very similar results points towards the fact that the features seen in the screening masses and the baryon number susceptibility are largely just consequences of symmetry, which in practice translates into properties of the quark propagator in a background gluon field. The data also tell us that π and σ are not well described in terms of weakly interacting quarks. The model has the correct symmetry built in and also treats these two modes as elementary excitations. (There is the obvious issue of double counting π's and σ's which I ignore. This issue is better discussed within a nonlinear model [20].) It is interesting to speculate on what the relevance of all this might be for the real-time hadronic excitations of the high-temperature QCD plasma. In this respect the model presented here paraphrases earlier work by Hatsuda and Kunihiro [27] in the context of the Nambu-Jona-Lasino model. There the authors found evidence for σ and π modes of finite width in addition to a quark-antiquark continuum. At the tree level the "constituent" quarks are massless in the symmetric phase. In higher orders the quarks pick up thermal masses, i.e., the pole in the quark propagator moves off the light cone while remaining chirally symmetric [28]. Hence there is no need for parity doubling. The fact that $m_\pi = m_\sigma$ is just a consequence of chiral symmetry. Monte Carlo simulations show [26] that the matter contribution to the energy density—a short distance quantity—is well described by a gas of weakly interacting massless quarks in the symmetric phase. The above discussion suggests that this might be the case at large distances as well, modulo the special status of the π-σ modes. To summarize, the simple constituent quark model described here nicely describes features of chiral symmetry restoration observed in lattice Monte Carlo calculations.

I would like to thank S. Aoki, G. Brown, M. Creutz, C. DeTar, S. Gottlieb, T. Hatsuda, S. Kahana, L. Trueman, E. Shuryak, and especially R. Pisarski for interesting discussions. Many thanks also to D. Toussaint who made some unpublished data available to me. This manuscript has been authored under Contract No. DE-AC02-76-CH00016 with the U.S. Department of Energy.

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