Dynamical Quasidegeneracies and Quantum Tunneling

In a recent Letter [1], Lin and Ballentine consider a driven anharmonic oscillator, with the Hamiltonian

$$H=0.5(p^2+x^4)-10x^2+10x\cos(\omega t). \tag{1}$$

They find that the tunneling rate between the two wells is much greater than the rate of ordinary tunneling, in the absence of driving. The purpose of this Comment is to explain that this effect is due to a dynamical symmetry of H, namely,

$$H(p,x,t)=H(-p,-x,t+\pi/\omega). \tag{2}$$

It follows from (2) that if $\psi(x,t)$ is a solution of the time-dependent Schrödinger equation, then so is $\psi(-x,t+\pi/\omega)$.

It was shown long ago by Heller [2] that discrete symmetries, which are irrelevant in classical mechanics, may have "surprising" quantum effects. For instance, classical trajectories can be trapped in symmetric regular regions of phase space, separated by chaotic regions. These chaotic regions thus behave as impenetrable barriers. Quantum mechanics, however, couples wave functions which describe equivalent motions on both sides of the "chaotic barrier." Motion is not trapped quantum mechanically [3]. This new type of quantum tunneling is dynamical in nature, and may be much faster than the ordinary quantum-mechanical tunneling through a static potential barrier [3].

Moreover, quantum levels corresponding to symmetric and widely separated regular tori appear as quasidegenerate doublets (or, generally, multiplets) [4] just as those corresponding to symmetric static potential wells. And, exactly as in the latter case, the time evolution of a wave function which is not initially symmetric will display quantum beats. This phenomenon occurs not only with static Hamiltonians, but also with "kicked" systems, whose time evolution is discrete. Typical quantum beats (called "collapses and revivals") are illustrated by Fig. 7 of Ref. [5] and Fig. 3 of Ref. [6]. These quantum beats disappear if the initial wave function is projected into each one of the invariant subspaces of the evolution operator, and each projection is considered separately [7].

The model considered by Lin and Ballentine [1] is slightly more complicated than the preceding ones, because the Hamiltonian (1) is time dependent. The discussion can, however, be simplified by defining a unitary evolution operator U such that

$$U\psi(x,0)=\psi(x,\tau), \tag{3}$$

where $\tau=2\pi/\omega$ is the period of the driving force. One can then restrict the discussion to the *discrete* time evolution generated by U (this becomes a "kicked" system, like those of Refs. [5] and [6]). The value of $\psi(x,n\tau)$, for any integer n, can be obtained by expanding

$$\psi(x,0)=\sum_j c_j v_j(x), \tag{4}$$

where the basis functions $v_j(x)$ satisfy

$$Uv_j(x)=e^{i\phi_j}v_j(x). \tag{5}$$

The quasienergies ϕ_j are real, and the eigenfunctions v_j are orthogonal. One then has

$$\psi(x,n\tau)=\sum_j e^{in\phi_j}c_j v_j(x), \tag{6}$$

which completely solves the problem.

The eigenfunctions v_j fall into two classes, odd and even under the symmetry (2). If they are localized in regular regions of the classical phase space, the corresponding quasienergies come as nearly degenerate doublets [4]. Consider, for example, a function $\psi(x,0)$ localized in the region labeled 1 in Fig. 1(b) of Ref. 1. Then, $\psi(-x,\pi/\omega)$ is localized in region 1′. Let us write

$$\psi(x,0)\equiv\tfrac{1}{2}[\psi(x,0)+\psi(-x,\pi/\omega)] + \tfrac{1}{2}[\psi(x,0)-\psi(-x,\pi/\omega)]. \tag{7}$$

The two terms in brackets in (7) belong to opposite symmetry classes and evolve, independently of each other, without any beats. It is only when their contributions are added, with phase factors $e^{in\phi_j}$ (with almost equal ϕ_j), that one obtains the quantum beats reported in Ref. [1].

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[1] W. A. Lin and L. E. Ballentine, Phys. Rev. Lett. **65**, 2927 (1990).
[2] E. J. Heller, Chem. Phys. Lett. **60**, 338 (1979).
[3] M. J. Davis and E. J. Heller, J. Chem. Phys. **75**, 246 (1981).
[4] O. Bohigas, S. Tomsovic, and D. Ullmo, Phys. Rev. Lett. **64**, 1479 (1990); **65**, 5 (1990).
[5] F. Haake, M. Kuś, and R. Scharf, Z. Phys. B **65**, 381 (1987).
[6] B. C. Sanders and G. J. Milburn, Z. Phys. B **77**, 497 (1989).
[7] A. Peres, in "Quantum Chaos," Proceedings of the Adriatico Conference on Quantum Chaos, Trieste, 1990, edited by H. Cerdeira and R. Ramaswamy (World Scientific, Singapore, to be published).

