Transverse Flow and Collectivity in Ultrarelativistic Heavy-Ion Collisions

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The detectability of collective transverse flow is discussed for CERN Super Proton Synchrotron energies in Pb+Pb collisions. Calculations with a quark-qluon string model suggest there will be measurable flow in Pb+Pb reactions but not in S+S reactions at 160 GeV. Calculations of transverse flow with a fluid-dynamic model show a strong dependence on the equation of state and that the amount of flow is significantly larger than in the string model.

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In the search for a quark-gluon phase of nuclear matter no unique signal has thus far been found. Proposed candidates for such a signal have tended to focus on a particular aspect of the reaction. Their ambiguous nature stems from both the complexity of a high-energy heavy-ion collision and from our poor understanding of the collision dynamics. It is therefore of great importance to analyze the overall features of the reactions, especially for the proposed Pb on Pb reaction at CERN Super Proton Synchrotron (SPS) energies. This reaction will provide the largest and most baryon-rich reaction volume attainable in the laboratory in the near future.

For nucleus-nucleus collisions at intermediate energies (20*A* MeV–2*A* GeV) the presence of a collective transverse flow has been well established in a series of experiments at the Bevalac [1]. As first predicted by models of compressional shocks [2,3], this has been demonstrated to be a reliable signal for some coherent form of matter at relativistic energies. An experimental detection of such dynamical behavior also at ultrarelativistic energies would clearly be of great importance for subsequent discussions of signals for a quark-gluon phase of nuclear matter. Furthermore, it could provide possibly the most direct measurement of the nuclear equation of state (EOS) and the transport properties of hadronic matter at high baryon and energy densities. At the CERN SPS at 60*A* and 200*A* GeV, transverse flow was seen neither in the asymmetric systems S+Au [4] and O+Au [5,6] nor in S+S experiments. However, the flow is expected to be stronger for a heavy, symmetric system as already seen at lower energies.

In a previous paper [7], we reported on the stopping power and the baryon and energy densities attainable for Pb on Pb collisions at 160 GeV/nucleon. For this reaction we found almost complete stopping in the quark-gluon string model and baryon-rich matter created with average energy densities of more than 10 GeV/fm^3 and existing for more than 1 fm/c in the center-of-mass frame. Thus, we concluded that the use of a one-fluid dynamical model is not unrealistic even at these energies for this reaction, in particular for the calculation of gross properties.

Here we continue the investigation initiated in Ref. [7] concerning the general features of collisions between heavy nuclei at ultrarelativistic energies. For the analysis of the transverse flow at these energies we will apply the quark-gluon string model (QGSM) [8–11] and the one-fluid dynamical model with an EOS for pure hadronic matter (HM) as well as an EOS including a first-order phase transition to a quark-gluon plasma (QGP) [12]. Observable effects of spherical and cylindrical collective flow were discussed already in the literature [13–15]; however, the resulting curved p_t spectra could be caused by other physical processes also. A sensitive method to analyze sidewards flow at relativistic energies is the relative transverse momentum analysis proposed by Danielewicz and Odyniec [16] for the Bevalac experiments. The transverse flow is expected to decrease with increasing energy and to be largest for symmetric heavy systems [17].

The QGSM is based on string phenomenology [18–21]. A detailed description and comparison of the QGSM with experimental data in a wide energy region can be found elsewhere [9–11]. At the energies discussed here the model does not treat gluons explicitly, but implicitly through their contribution to the string tension. The QGSM is a very detailed model describing hadron elastic and inelastic collisions as well as annihilation. Furthermore, the space-time picture of particle creation is simulated via string formation and their subsequent decay. In a microscopic picture the nuclei are initially described as a set of nucleons. We consider their quark-parton composition and describe the separate nucleon-nucleon collisions in a string picture. At a hadron-hadron impact one or more strings are formed. The leading partons of the string (quarks and diquarks) carry a given fraction (specified by a Monte Carlo process following appropriately chosen structure functions [10,22]) of the initial

TABLE I. Characteristic values of baryon transverse flow from experiments and the QGSM and fluid-dynamical models. At 1.8*A* GeV the QGSM underpredicts the transverse flow due to the absence of nuclear interactions. This effect should be less important at higher energies. At 10*A* GeV for Pb+Pb reactions both QGSM and RQMD predict an observable transverse flow. The one-fluid dynamical model overpredicts the transverse flow at Bevalac energies, and the same is also expected at higher energies.

	A in $(A+A)$	$E^{\mathrm{lab}}_{\mathrm{proj}}/A$ (GeV)	$p^x_{\max}/A$ (MeV/c)	$y_{\mathrm{c.m.}}$	F (MeV/c)	$\tilde{F}$
Bevalac expt.	40	0.4	70	0.44	160	0.37
Bevalac expt.	40	1.8	120	0.87	140	0.15
QGSM	32	1.8	~50	0.87	~59	0.054
RQMD ($b=3$ fm) [a]	208	10	~75	1.6	~45	0.033
RQMD ($\langle b\rangle$) [b]	208	10	~290	1.6	~180	0.134
QGSM ($b=4$ fm)	208	10	~125	1.6	~150	0.109
Hydro (QGP) [c]	32+208	14	~160	1.8	~88	...
Hydro (HM) [c]	32+208	14	~230	1.8	~127	...
Hydro (QGP) [c]	32+208	60	~200	2.4	~83	...
Hydro (HM) [c]	32+208	60	~270	2.4	~112	...
CERN SPS expt.	32	200	0	3.0	0	0.0
QGSM	32	200	0	3.0	0	0.0
QGSM	208	160	~50	2.9	~40	0.014
QGSM (no rescat.)	208	160	0	2.9	0	0.0
Hydro (QGP)	208	160	~200	2.9	~69	0.024
Hydro (HM)	208	160	~400	2.9	~138	0.047

[a]Reference [21].
[b]Reference [24].
[c]Reference [12].

momentum of a nucleon. The interaction of the strings is described approximately by the leading hadrons. After about 1 fm/c proper time the strings break up. After hadronization the newly formed secondary hadrons are allowed to rescatter. The main parameters of the model [7-11] are adjusted to describe $h+h$ [22] and $h+A$ [10] collisions. The model yields a generally good overall fit to most experimental data. The alteration of the space-time nuclear collision picture caused by an increasing incident energy is reproduced in our model by means of production and decay of strings and by rescatterings of secondary hadrons. The underlying EOS in the QGSM is not analyzed yet, but due to the involved scattering mechanism and the latent energy in the strings the EOS is not that of an ideal gas. At low energy the QGSM reduces to a standard cascade model without mean-field effects.

By analyzing events generated with the intranuclear cascade model, Danielewicz and Odyniec [16] found that the collective flow effects observed in the streamer-chamber data for ^{40}Ar(1.8 GeV/nucleon)+KCl were much stronger than predicted. Some typical Bevalac data on the flow are reported in Table I. Our string model underestimates the flow at these energies. As shown by Stöcker and Greiner [23], the reason that the intranuclear cascade model fails to reproduce the data is the neglect of mean-field effects. At higher energies we expect the influence of a mean field to be weaker, so we believe the QGSM will still underpredict the flow but will yield a better estimate than at low energies. We follow closely the experimental procedure used to determine the projected transverse momentum [16]. However, as the theoretical reaction plane is given by the impact parameter, we can also estimate whether it is experimentally feasible to determine the transverse flow, if present. The projected transverse momentum $\langle p_x(y)/a\rangle$ at given rapidity calculated in the model for ^{208}Pb+^{208}Pb at 10*A* GeV with impact parameter $b=4$ fm is of the order of 125 MeV/c. This is similar to the flow observed in the 1.8*A* GeV Bevalac experiments. The one-fluid dynamical model with a QGP EOS gives a maximum flow of the order 200 MeV/c for this reaction at 10*A* GeV.

The histogram in Fig. 1 shows the prediction of the QGSM for a small symmetric system ^{32}S+^{32}S at 160*A* GeV. At impact parameter $b=2$ fm there is no detectable flow, which is in agreement with experimental results [5,6]. However, as shown with full histograms in Fig. 2, it might be possible to detect flow for ^{208}Pb+^{208}Pb at 160*A* GeV. With the reaction plane defined as in experiments [16], at impact parameter $b=4$ fm, there is a maximum flow of the order 50 MeV/c at the end of the collision when almost all hadronic collisions are completed. The transverse flow does not change after 10 fm/c. The distribution of the angle $\Delta\phi$ between the theoretical reaction plane and the one calculated from the correlations between the particles, as one would do in an experiment, is wide (see Table II); the mean and the dispersion of the distribution is $\langle\Delta\phi\rangle=49°\pm41°$. This still provides a possibility for the experimental detection of the reaction plane.

In the QGSM the sidewards flow is a sole result of the

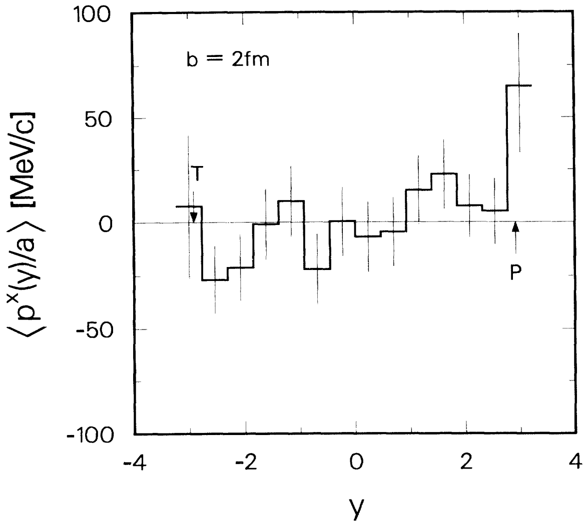

FIG. 1. Transverse flow $\langle p_x \rangle$ of scattered protons calculated in the quark-gluon Monte Carlo string model (QGSM) for the 160A GeV S+S collision at impact parameters $b=2$ fm. The labels P and T denote the original rapidities of the projectile nuclei.

rescattering of secondaries as is indicated by the dashed histograms in Fig. 2 which shows the predictions of the QGSM without rescattering: There is no flow in this case.

The one-fluid dynamical model was applied with two different EOS's: one with pure hadronic matter and one with a first-order phase transition from hadronic matter

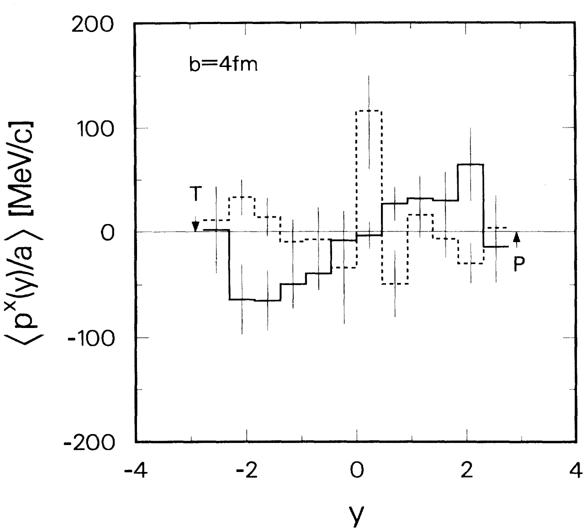

FIG. 2. Transverse flow $\langle p_x \rangle$ of scattered protons calculated in the quark-gluon Monte Carlo string model (QGSM) for the 160A GeV Pb+Pb collision at impact parameters $b=4$ fm. The labels P and T denote the original rapidities of the projectile nuclei. The dashed line indicates the results without the rescattering of secondaries for $b=4$ fm.

TABLE II. The distribution of the angle $\Delta\phi$ between the theoretical reaction plane and the experimentally detected reaction plane according to the QGSM for a sample of 40 Pb+Pb reactions at 160A GeV, and $b=4$ fm.

$\Delta\phi$	$\Delta N/\Delta\phi$
0°–45°	57.9
45°–90°	26.3
90°–135°	7.9
135°–145°	7.9

to a perturbative quark-gluon plasma. Previously it has been shown within fluid dynamics that both of the EOS's can produce flow for asymmetric collisions at 60 GeV/nucleon [12]; however, for pions the flow is probably not detectable. For $A+A$ systems the flow is smaller by a factor of 2 for the QGP EOS as compared to the hadronic EOS. The fluid-dynamical model predicts a decrease in the maximum of the flow with increasing energy as already seen for the experiments at lower energies [1]. Furthermore, the maximum is also sensitive to the chosen breakup time of the fluid and increases with time. With these complications in mind we show in Fig. 3 the $\langle p_x(y)/a \rangle$ for the two EOS at time 3.8 fm/c in the c.m. frame. The soft EOS of the QGP reduces the flow considerably but the maximum is still increasing for the times shown here. [At a later breakup time of $t=5.3$ fm/c the amplitude of $\langle p_x(y)/a \rangle$ is about 30% larger.] For the pure hadronic matter the reaction is faster and the flow is not increasing much for the times shown here. [At a later breakup time of $t=5.3$ fm/c the amplitude of $\langle p_x(y)/a \rangle$ is only 15% larger.]

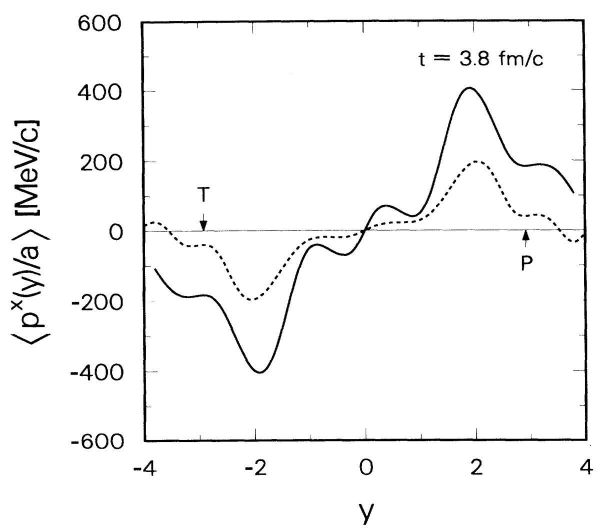

FIG. 3. Transverse flow $\langle p_x \rangle$ of protons with QGP (dashed line) and hadronic-matter (solid line) EOS, calculated in the fluid-dynamical model for 160A GeV at impact parameter $b=4$ fm.

Because of the experimental difficulties in obtaining $\langle p_x(y)/a\rangle$ accurately for the full rapidity range and particularly around the maximum one has introduced the quantity "flow," F, the slope of the function $\langle p_x(y)/a\rangle$ at center-of-mass rapidity to describe the collective flow: $F\equiv\partial\langle p_x(y)/a\rangle/\partial y|_{y=y_{\rm c.m.}}$. To investigate the systematics of the flow for different energies and different symmetric projectile-target combinations it is also useful to introduce the scale-invariant quantities [17]. It was shown that if we plot the dimensionless transverse flow, $\tilde{p}_x \equiv\langle p_x(y)/a\rangle/p^{\rm c.m.}_{\rm proj}$, versus $\tilde{y}\equiv y/y_{\rm c.m.}$, a universal transverse flow behavior can be obtained for many mass-beam-energy combinations. Consequently the scale-invariant flow, $\tilde{F}\equiv\partial\tilde{p}_x/\partial\tilde{y}$, provides a uniform description of the colliding system irrespective of mass or beam energy. Characteristic values of $\tilde{F}$ range from 0.0 to 0.4 in the Bevalac energy region; see Table I. In particular, many deviations from the fluid-dynamical scaling behavior indicate a drastic change in the reaction mechanism, for instance a phase transition. In Table I we show the results of such an analysis for ultrarelativistic energies. RQMD results are taken from [21]. RQMD and QGSM models are in agreement within the accuracy of the calculations.

We conclude that both string cascade models with proper treatment of rescattering and fluid-dynamical models predict observable collective transverse flow in Pb+Pb collisions at AGS and SPS energies. According to the QGSM the effect is on the borderline of detectability at SPS energies. It requires proton identification in a large solid-angle range. The high multiplicity of secondaries helps local equilibration and thermalization; however, the flow is too weak to be detected in light hadrons. The accurate measurement of the collective transverse flow is a direct evidence of collective behavior and a measure of the collective pressure.

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