## Determination of $\sin^2 \bar{\theta}_W$ from the Forward-Backward Asymmetry in $p\bar{p} \rightarrow Z \,^0 X \rightarrow e^+ e^- X$ Interactions at $\sqrt{s} = 1.8$ TeV

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 (Received 15 May 1991; revised manuscript received 19 August 1991)

An analysis of the forward-backward asymmetry in  $Z^0$  decays using data from the Collider Detector at Fermilab at  $\sqrt{s} = 1.8$  TeV yields  $A_{FB} = [5.2 \pm 5.9(\text{stat}) \pm 0.4(\text{syst})]\%$  and  $\sin^2\bar{\theta}_W = 0.288 \pm 0.002(\text{syst})$ .  $\pm 0.002(\text{syst})$ .

PACS numbers: 13.38.+c, 12.15.Mm

In the standard model [1], the neutral current can be defined by a mixture of the weak isospin and electromagnetic currents with mixing angle  $\bar{\theta}_W$ :

$$J_{\mu}^{\rm NC} = J_{\mu}^{I_3} - \sin^2 \bar{\theta}_W J_{\mu}^{\rm EM} \,. \tag{1}$$

The weak component of the neutral current violates parity, and leads to a charge asymmetry in the decay angular distribution of the  $Z^0$ . This asymmetry depends on the relative magnitudes of the weak and electromagnetic components of the neutral current, and hence on  $\sin^2 \bar{\theta}_W$ . In this paper we present a measurement of the forwardbackward asymmetry in  $p\bar{p} \rightarrow Z^0 X \rightarrow e^+ e^- X$  events, which probes the neutral-current coupling to light quarks at the  $Z^0$  mass scale, and leads to a measurement of  $\sin^2 \bar{\theta}_W$ .

In hadronic collisions, direct  $e^+e^-$  pairs are produced (at lowest order) by the annihilation of a quark-antiquark  $(q\bar{q})$  pair via either a photon or a  $Z^0$ . The angular distribution for  $p\bar{p} \rightarrow Z^0 X \rightarrow e^+e^- X$  is expected to be asymmetric in  $\cos\theta$ , where  $\theta$  is defined to be the angle between the outgoing  $e^+$  and incoming  $\bar{q}$  in the rest frame of the  $e^+e^-$  pair. The forward-backward asymmetry is defined by

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} , \qquad (2)$$

where  $\sigma_F = \int_0^1 [d\sigma/d(\cos\hat{\theta})]d(\cos\hat{\theta})$  and  $\sigma_B = \int_{-1}^0 [d\sigma/d(\cos\hat{\theta})]d(\cos\hat{\theta})]d(\cos\hat{\theta})$ . The lowest-order cross section has the form [2]  $d\sigma/d(\cos\hat{\theta}) = A(1 + \cos^2\hat{\theta}) + B\cos\hat{\theta}$  and has contributions from photon exchange,  $Z^0$  exchange, and photon- $Z^0$  interference. The dominant contribution to  $A_{FB}$  near the  $Z^0$  resonance comes from  $Z^0$  exchange.

We present a measurement of  $A_{FB}$  and  $\sin^2 \bar{\theta}_W$  using data corresponding to an integrated luminosity of 4.1 pb<sup>-1</sup> from  $p\bar{p}$  collisions at  $\sqrt{s} = 1.8$  TeV collected with the Collider Detector at Fermilab (CDF). The CDF detector is described in detail elsewhere [3]. Briefly, scintillator hodoscopes (BBC) located on either side of the detector identify inelastic events. Time-projection chambers (VTPC) measure the position of the event vertex. A drift chamber surrounds the VTPC and measures the momentum of charged particles in a 1.4-T solenoidal magnetic field. Electromagnetic and hadronic calorimeters extend from  $-4.2 < \eta < 4.2$  in a projective tower geometry, where  $\eta \equiv -\ln[\tan(\theta/2)]$  [4]. A proportional chamber near shower maximum in the central  $(|\eta| < 1.1)$ electromagnetic calorimeter measures shower shape and position.

Each  $Z^0$  event must satisfy a trigger which requires a central electromagnetic cluster with (i) transverse energy  $E_T > 12$  GeV ( $E_T = E \sin\theta$ ), (ii) an associated track with transverse momentum > 6 GeV, and (iii) the ratio of hadronic to electromagnetic  $E_T$  in the cluster (Had/EM) < 12.5%.

We require each  $Z^0$  event to have one electron in the central region, where there is good momentum determination, so that the charge of at least one electron and the sign of  $\cos\theta$  can be determined, and a second electron with  $|\eta| < 3.5$ . Both electron clusters must be located away from calorimeter edges so that their energies are well measured. The event vertex is required to be within 60 cm in the z direction of the nominal interaction point.

One central electron is required to have (i)  $E_T > 15$  GeV and lateral energy sharing in the calorimeter towers consistent with an electron shower; (ii) the ratio of cluster energy to track momentum E/P < 1.5; (iii) a shower in the strip chambers with a profile consistent with an electron shower and centroid position within 1.5 cm in the  $\phi$  direction and 3.0 cm in the z direction of the extrapolated track; and (iv) isolation I < 0.1, where  $I = (E_C - E_T)/E_C$ ,  $E_C$  being the total transverse energy in a cone in  $\eta$ - $\phi$  space of radius 0.4 centered on the cluster.

The second electron is required to have (i)  $E_T > 15$  GeV; (ii) I < 0.1; (iii) if in the central region, E/P < 1.5; (iv) if in the plug region, Had/EM < 0.05, transverse energy profile consistent with test beam electrons, and a track in the VTPC; (v) if in the forward region, Had/EM < 0.05 and longitudinal energy profile consistent with test beam electrons.

We take as our  $Z^0$  sample the 252 events with 75 <  $M_{ee}$  < 105 GeV. The background from QCD processes is estimated from studies of invariant mass and isolation to be 7 ± 3 events. If this background is symmetric in  $\cos \theta$ , the observed asymmetry is reduced by 3% of itself. The background from  $\tau$  pairs is estimated from Monte Carlo to be less than 0.5 event. The background



FIG. 1. Angular distribution of electrons (a) before and (b) after acceptance corrections. The solid line is the result of the likelihood fit.

from W+jet  $\rightarrow ev$ +jet in which the jet fakes a second electron is estimated to be less than 0.4 event.

Because of QCD processes,  $Z^{0}$ 's are produced with momentum transverse to the beam direction,  $p\vec{7}$ . In this case, the p and  $\vec{p}$  are not collinear in the  $e^+e^-$  rest frame, and the quark directions are not the p or  $\vec{p}$  directions. We adopt the method of Collins and Soper [5] in which  $\cos\theta$  is measured with respect to the average of the p and  $-\vec{p}$  directions in the  $e^+e^-$  rest frame. This introduces a small,  $p\vec{7}$ -dependent smearing of the measured angular distribution. In this method, we assume all quarks are valence quarks, i.e., the q comes from the p and the  $\bar{q}$  from the  $\bar{p}$ . The small contribution in which the  $\bar{q}$  comes from the p (approximately 10% of the cross section) gives an asymmetry opposite to that from the valence quarks. This contribution is included in our calculation of  $\sin^2\bar{\theta}_W$  below. The angular distribution  $dn/d\cos\theta$  is plotted before acceptance corrections in Fig. 1(a). It has the predicted parabolic shape except at large  $|\cos\theta|$  where the electron  $E_T$  cut reduces the acceptance. Figure 1(b) shows a plot of  $(1/\sigma)d\sigma/d\cos\theta$  corrected bin by bin for acceptance, using the ISAJET [6] Monte Carlo and a simple detector simulation.

We use a logarithmic likelihood fit with the functional form  $P(\cos\hat{\theta}) = \frac{3}{8} (1 + \cos\hat{\theta}) + A_{FB} \cos\hat{\theta}$  to determine  $A_{FB}$ ;  $A_{FB}$  and its variance can be estimated from the event sample without explicit reference to the angular dependence of the acceptance, provided the acceptance is symmetric with respect to  $\cos\hat{\theta}$ . The fit yields  $A_{FB} = (5.0 \pm 5.9)\%$ , and is shown in Fig. 1(b). The photon- $Z^0$  interference term contributes an asymmetry of approximately -1.5% in our mass region. Using  $A_{FB}$  and the lowest-order cross section with MRSB [7] parton distribution functions we derive the lowest-order value  $\sin^2\hat{\theta}_W|_{10} = 0.231 \pm 0.017 (\text{stat})$  [8].

Systematic uncertainties on  $A_{FB}$  and  $\sin^2 \bar{\theta}_W$  come from several sources and are summarized in Table I. Bias in the fitting procedure is determined to be less than 0.26% by fitting many Monte Carlo data samples. Systematic uncertainties due to trigger, track reconstruction, and electron selection are determined by Monte Carlo calculation, assuming the maximum bias in the acceptance consistent with the measured inefficiencies. Varying the calorimeter energy scales by 5% in the Monte Carlo calculation changes  $A_{FB}$  by < 0.03%.

 $A_{FB}$  has contributions from background and higherorder QCD, QED, and weak processes. We subtract from  $A_{FB}$  the contributions from background and QCD processes, and then derive  $\sin^2 \bar{\theta}_W$  from the corrected  $A_{FB}$ .

Order- $\alpha_s$  QCD processes affect the measurement in two ways: The  $Z^0$  acquires a longitudinal polarization which modifies the symmetric part of the angular distribution, and the measurement of  $\hat{\theta}$  is smeared in events with large  $p_T^Z$ . We estimate the size of the QCD corrections by convoluting our measured  $p_T^Z$  spectrum [9] with a calculation of the angular distribution as a function of  $p_T^Z$ [10]. After background and QCD corrections, we find  $A_{FB} = [5.2 \pm 5.9(\text{stat}) \pm 0.4(\text{syst})]\%$ . The uncertainty on

	$\Delta A_{FB}$ (%)	σA <sub>FB</sub> (%)	$\Delta \sin^2 \bar{\theta}_W$	$\sigma \sin^2 \bar{\theta}_W$
OCD background	+0.14	0.06	-0.0004	0.0002
ln $\mathcal{L}$ fitter	0	0.26	0.0	0.0008
Electron trigger	0	0.23	0.0	0.0006
Track reconstruction	0	0.19	0.0	0.0005
Electron selection	0	0.10	0.0	0.0003
Energy scale	0	0.03	0.0	0.0001
OCD corrections	+0.09	0.09	-0.0003	0.0003
OED corrections			-0.0014	0.0014
Weak corrections			-0.0013	0.0002
Parton distribution			•••	0.0004

TABLE I. Corrections and systematic uncertainties for  $A_{FB}$  and  $\sin^2 \bar{\theta}_W$ .

the measured  $A_{FB}$  is dominated by statistics.

We make higher-order electroweak corrections in order to estimate  $\sin^2 \bar{\theta}_W$ . We integrate the order- $a^3$  QED cross section using a Monte Carlo and a simple detector simulation which accounts for hard photon bremsstrahlung [11]. Calculations of the virtual and soft photon corrections for  $e^+e^- \rightarrow q\bar{q}$  [12] are time reversed to obtain results for  $q\bar{q} \rightarrow e^+e^-$ , and are included in the Monte Carlo calculation. After QED corrections we find  $\sin^2 \bar{\theta}_W = 0.229$ . The uncertainty in the QED corrections to  $\sin^2 \bar{\theta}_W$  is 0.0014, estimated by varying the infrared cutoff. Including the order- $a^3$  weak corrections, we find  $\sin^2 \bar{\theta}_W = 0.228$ . The theoretical uncertainty on the weak calculations is 0.0002.

There is an uncertainty in the relative contributions of u valence quarks, d valence quarks, and sea quarks in the proton, giving an uncertainty on  $\sin^2 \bar{\theta}_W$  of 0.0004, determined by integrating the cross section with several different parametrizations [7,13]. We use the MRSB parametrization for the final result.

Our final result,

 $\sin^2 \bar{\theta}_W = 0.228 + 0.017 + 0.002 \text{(syst)},$ 

is in good agreement with the value  $\sin^2 \bar{\theta}_W = 0.24 \substack{+0.05\\+0.05}$ measured by the UA1 Collaboration [14] from the asymmetry in leptonic decays of the  $Z^0$ , with the values  $\sin^2 \bar{\theta}_W = 0.2291 \pm 0.0040$  measured by the ALEPH Collaboration [15],  $\sin^2 \bar{\theta}_W = 0.2309 \pm 0.0048$  measured by the DELPHI Collaboration [16], and  $\sin^2 \bar{\theta}_W = 0.230$  $\pm 0.004$  measured by the L3 Collaboration [17] from the  $Z^0$  mass and leptonic width, and with the value  $\sin^2 \bar{\theta}_W$  $= 0.233 \substack{+0.006\\-0.006}$  measured by the OPAL Collaboration [18] from a simultaneous fit to the leptonic cross sections and forward-backward asymmetries.

The parameter  $\sin^2 \bar{\theta}_W$  is measured from the asymmetry, and  $\sin^2 \theta_W |_{\text{mass}} = 1 - M_W^2 / M_Z^2$  [19] from the ratio of the W and  $Z^0$  masses. We use the standard model to convert our measurement of  $\sin^2 \bar{\theta}_W$  from the asymmetry into a measurement of  $\sin^2 \theta_W |_{\text{mass}}$  [20,21]. A plot of  $\sin^2 \theta_W |_{\text{mass}}$  derived from the asymmetry as a function of the top mass is shown by the solid line in Fig. 2, assuming a Higgs mass of 250 GeV (the Higgs mass dependence is small); the dashed lines indicate the  $1\sigma$  experimental uncertainty. Figure 2 also shows the  $1\sigma$  confidence region derived from recent  $Z^0$  mass measurements [22,23] as well as a direct measurement of  $1 - M_W^2/M_Z^2 = 0.232$  $\pm 0.008$  [24] determined from the CDF W and LEP  $Z^0$ masses.  $A_{FB}$  is not strongly dependent on the top mass; the top mass dependence in Fig. 2 comes from higherorder corrections in the standard model incurred in the conversion from  $\sin^2 \bar{\theta}_W$  to  $\sin^2 \theta_W |_{\text{mass.}}$ 

In summary, we have measured  $A_{FB} = [5.2 \pm 5.9 \text{(stat)} \pm 0.4 \text{(syst)}]\%$  after background and QCD corrections, and  $\sin^2 \bar{\theta}_W = 0.228 \pm 0.017 \text{(stat)} \pm 0.002 \text{(syst)}$  after background and radiative corrections. The systematic uncertainties are summarized in Table I. Our measurement of  $\sin^2 \bar{\theta}_W$  is consistent, both without and with order- $\alpha^3$  radi-



FIG. 2. The solid line shows the central value of  $\sin^2 \theta_W |_{\text{mass}}$  derived from the asymmetry as a function of the top-quark mass; the dashed lines indicate the  $1\sigma$  experimental uncertainty. The dot-dashed lines show the  $1\sigma$  uncertainty on  $\sin^2 \theta_W |_{\text{mass}}$  determined from  $Z^0$  mass measurements. At right is the CDF value from  $1 - M_W^2/M_Z^2$  [24].

ative corrections, with previous measurements of  $\sin^2 \bar{\theta}_W$ at the  $Z^0$  mass. Our measurement of  $\sin^2 \theta_W|_{\text{mass}}$  from the asymmetry is consistent with measurements of  $\sin^2 \theta_W|_{\text{mass}}$  from the W and  $Z^0$  mass ratio over a broad range of top-quark masses.

We thank the Fermilab Accelerator Division and the technical staffs of the participating institutions. We thank Y. Srivastava, C. Hill, and W. Bardeen for assistance. This work was supported by the Department of Energy, the National Science Foundation, the Istituto Nazionale di Fisica Nucleare, the Ministry of Science, Culture, and Education of Japan, and the A. P. Sloan Foundation.

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