Geometrical Neutrino Mass Hierarchy and a 17-keV ν_τ

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We present an extension of the singlet majoron gauge model which realizes a novel scheme of geometrical neutrino mass hierarchy proposed recently by Glashow, wherein ν_e and ν_μ are Majorana particles with $m_{\nu_e} \approx m_{\nu_\mu} \approx 10^{-3}$ eV while ν_τ is a Dirac particle with a mass of 17 keV. Our model explains the solar-neutrino deficit via the Mikheyev-Smirnov-Wolfenstein mechanism and accounts for the recently reported anomaly in beta-decay spectra in a natural manner without any undesirable fine tuning of parameters. An interesting consequence of the model is that ν_τ is short lived with a lifetime of $\approx 10^{-3}$-10^{-1} sec.

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The first clue to physics beyond the standard model may very well come from the study of new as yet unknown properties of the neutrinos. Two most exciting results in this direction are (i) the deficit in the solar-neutrino flux observed in the chlorine [1], Kamiokande [2], and gallium [3] experiments and (ii) reports of an anomaly in the beta-decay spectrum first observed by Simpson [4] from studies of tritium and sulfur, and more recently by several other groups [5].

A natural and elegant explanation of the solar-neutrino deficit is the resonant conversion of ν_e produced in the solar core into ν_μ as it propagates through matter as discussed by Mikheyev, Smirnov, and Wolfenstein [6] (the MSW effect). The mass parameters for this scenario should lie in the range $m_{\nu_e} \approx m_{\nu_\mu} \approx 10^{-3}$ eV and ν_e-ν_μ mixing should be of the order of a few percent. On the other hand, if the observed anomaly in the beta-decay spectrum is genuine, it would require the existence of a neutral fermion with a mass of 17 keV and mixing angle with ν_e of order 10%. The simplest possibility is to assume that this neutral lepton is none other than the familiar tau neutrino. It must then be a *Dirac* particle in order to avoid conflict with double-β-decay experiments [7]. (A 10% admixture of a 17-keV *Majorana* neutrino in ν_e would imply an effective mass of 170 eV in double β decay to be compared with the experimental upper limit of a few eV.) A major challenge to the idea of a 17-keV ν_τ is to make it decay fast enough into invisible channels so as to avoid mass-density constraints of the Universe [8].

The above set of constraints on neutrino properties pose two theoretical challenges. The first is to construct a scheme which reproduces this seemingly bizarre hierarchy in neutrino masses while accommodating the constraints of mixing and lifetime. The second is to realize such a scheme within an extension of the standard model without unnatural fine tuning of parameters.

In a recent paper Glashow [9] has proposed an ingenious extension of the singlet majoron model [10] which answers the first challenge. In Glashow's scheme, the fermion spectrum of the standard model is extended to include right-handed neutrinos (ν_R), one per generation. The global $B-L$ symmetry is spontaneously broken as in the singlet majoron model [10] to generate a seesaw form for the neutrino mass matrix, i.e.,

$$M_\nu = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix}, \tag{1}$$

where m and M are 3×3 matrices. m is the Dirac mass matrix connecting ν_L to ν_R, and M is the Majorana mass matrix for the right-handed neutrinos induced by coupling to the majoron. Glashow's proposal is to require the matrix M to have rank 2, so that it has one zero eigenvalue, leading to one Dirac neutrino (identified with ν_τ) with $m_{\nu_\tau} \approx m_3$ and two Majorana neutrinos (ν_e and ν_μ) having generic seesaw masses $m_{\nu_i} \approx m_i^2/M_i$. The interesting point about this mass matrix is that if the Dirac masses $m_1 \approx m_2 \approx m_3 \approx 17$ keV, and the $B-L$ breaking scale is of the same order as the electroweak scale ($M \approx 300$ GeV), it will lead to the desired values for m_{ν_e}, m_{ν_μ} ($\approx 10^{-3}$ eV) and m_{ν_τ} (≈ 17 keV). In addition to these states, there are two heavy Majorana states with masses of order 300 GeV. Thus the masses obey a geometrical hierarchy with the ν_τ mass lying near the harmonic mean of the heavy and light Majorana masses. The cosmological constraint on ν_τ is avoided as it decays via majoron (χ) emission ($\nu_\tau \to \nu_i + \chi$).

The above pattern of mass eigenvalues and mixing is somewhat nontrivial to realize theoretically in a natural manner. For instance, (i) the value of 17-keV Dirac mass would ordinarily require a Yukawa coupling of order 10^{-7} and (ii) the absence of any significant generation dependence for Dirac neutrino masses, unlike that observed in the charged-fermion sector, will need an explanation.

In this Letter, we propose a renormalizable gauge model which realizes naturally such a scheme for neutrino masses. Our model is an extension of the singlet majoron model, where the Dirac mass matrix m of Eq. (1) arises only at the one-loop order. As a result, even with all Yukawa couplings of order 10^{-1}-10^{-2}, the induced Dirac masses can naturally be in the keV range. Furthermore, since these masses are induced through loops not involving the charged fermions, no significant generation dependence is expected. A key ingredient of our model is that the global U(1) symmetry is not generation blind. As a result the Majorana matrix M of Eq. (1) naturally has rank 2. An interesting consequence of our model is that ν_τ turns out to be short lived with a lifetime of $\approx 10^{-3}$–10^{-1} sec. This comes about as the decay amplitude for majoron emission ($\nu_\tau \to \nu_i + \chi$) goes as m/M rather than $(m/M)^2$ as in a class of singlet majoron models [11].

We shall work with the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group with the left-handed lepton doublets Ψ_{aL}, right-handed singlets e_{aR}, and right-handed neutrinos ν_{aR}, where $a=1,2,3$ is the generation index. A global U(1) symmetry is imposed which depends on the generation. Besides the standard Higgs doublet ϕ_S, the scalar sector of the model contains new particles, including a neutral singlet Δ carrying global U(1) charges. These particles are listed in Table I along with their U(1) charges. [Leptons not listed have zero U(1) charges.] The U(1) symmetry is anomalous.

The most general leptonic Yukawa coupling of the model invariant under the global U(1) is

$$\mathcal{L}_Y = g_a \bar{\Psi}_{aL} \phi_S e_{aR} + h_{ij} \bar{\Psi}_{iL} \tilde{\phi} \nu_{jR} + h_{i3} \bar{\Psi}_{iL} \tilde{\phi}' \nu_{3R} + h_{3i} \bar{\Psi}_{3L} \tilde{\phi}' \nu_{iR} + f \nu_{2R}^T C^{-1} \nu_{3R} \Delta + \text{H.c.} \qquad (2)$$

Here the index a runs over all three generations, whereas i,j take values 1 or 2. Without loss of generality we have chosen the charged-lepton couplings to be diagonal in flavor and defined the linear combination of ν_{1R} and ν_{2R} coupling to Δ simply as ν_{2R}.

The Higgs potential of the model contains, in addition to trivial invariants, the following terms:

$$V = \mu \phi^T i\tau_2 \phi' \eta_2^* + \lambda_1 \phi_S^T i\tau_2 \phi \eta_3^* \Delta^* + \lambda_2 \phi^\dagger \phi_S \eta_2^* \eta_3 + \lambda_3 \eta_1^* \eta_2 \Delta^2 + \lambda_4 \eta_1^* \eta_3 \Delta^{*2} + \text{H.c.} \qquad (3)$$

The parameters of the potential are so chosen that at the tree level, only the following fields have nonzero vacuum expectation values (VEV): $\langle \phi_S^0 \rangle = \kappa_S$; $\langle \Delta \rangle = v$. As a result, all charged fermions will acquire masses. The Majorana mass matrix for the right-handed neutrinos is given by

$$M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & fv \\ 0 & fv & 0 \end{pmatrix} . \qquad (4)$$

This matrix clearly has rank 2 as required to produce a Dirac ν_τ. The massless state ν_{1R} will be identified with $\nu_{\tau R}$. Note that this system is somewhat more economical than conventional seesaw matrices since M has just one parameter. All Dirac masses of the neutrinos are zero at this stage.

At the one-loop level, ϕ^0 and ϕ'^0 will acquire nonzero vacuum expectation values from the graphs shown in Fig. 1. These contributions are finite and can be estimated to be

$$\langle \phi^0 \rangle \equiv \kappa \simeq \frac{\lambda_2 \lambda_3 \lambda_4}{16\pi^2} \frac{v^4 \kappa_S}{m_\eta^2 m_\phi^2} , \qquad \langle \phi'^0 \rangle \equiv \kappa' \simeq \frac{\lambda_1 \lambda_3 \lambda_4}{16\pi^2} \frac{v^5 \mu \kappa_S}{m_\eta^4 m_\phi^2} . \qquad (5)$$

Assuming all the mass parameters ($\kappa_S, \mu, m_\eta, m_\phi$) to be of the order of the electroweak symmetry breaking scale (≈ 200 GeV) and λ_i to be in the range $(0.5-1) \times 10^{-1}$, we get $\kappa \approx \kappa' \approx 100$ keV–1 MeV. Once $\langle \phi^0 \rangle \neq 0$, $\langle \phi'^0 \rangle \neq 0$, nonzero Dirac mass entries will be induced in the matrix m of Eq. (1). If the Yukawa couplings h_{ab} are of order 10^{-1}–10^{-2}, these masses will naturally be in the keV

TABLE I. Global U(1) charges of leptons and the Higgs particles.

Particle	U(1) charge
ν_{1R}, ν_{2R}	+4
ν_{3R}	−5
Ψ_{3L}, e_{3R}	+9
$\phi = (\phi^+ \ \phi^0)^T$	+4
$\phi' = (\phi'^+ \ \phi'^0)^T$	−5
η_1^+	+1
η_2^+	−1
η_3^+	+3
Δ	+1

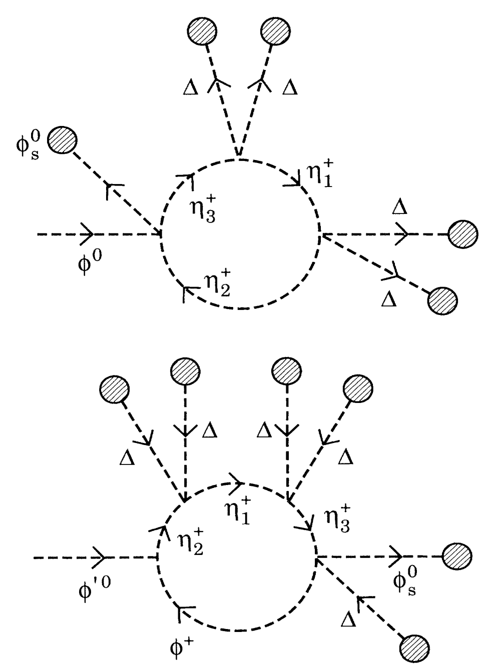

FIG. 1. One-loop diagrams generating finite vacuum expectation values for ϕ^0 and ϕ'^0.

range. All entries of m except the (3,3) entry are now nonzero, which we shall denote by m_{ab}. The first index corresponds to the left-handed neutrino and the second index to the right-handed neutrino.

It should be noted here that if all Yukawa couplings of ν_R (viz., h_{ab}) are of the same order, the Dirac masses of all three neutrinos will be of the same order, as hypothesized by Glashow.

Let us now turn to the diagonalization of the 6×6 matrix M_ν of Eq. (1) when the 3×3 submatrix M has rank 2 as in Eq. (4). It is convenient to write M_ν in the following block form:

$$M_\nu=\begin{pmatrix} A & B \\ B^T & H \end{pmatrix}. \tag{6}$$

Here A is a 4×4 matrix, B is 4×2, and the 2×2 matrix H given by

$$H=\begin{pmatrix} 0 & fv \\ fv & 0 \end{pmatrix} \tag{7}$$

is invertible. Since elements of A and B are of order κ, κ', while those of H are of order v, M_ν can be block diagonalized to order $(\kappa^2/v,\kappa'^2/v)$ by the unitary matrix

$$U=\begin{pmatrix} 1-\frac{1}{2}\rho\rho^T & \rho \\ -\rho^T & 1-\frac{1}{2}\rho^T\rho \end{pmatrix}+O\left(\frac{\kappa^3}{v^3},\frac{\kappa'^3}{v^3}\right), \tag{8}$$

where $\rho=BH^{-1}+ABH^{-2}$. The 4×4 light neutrino mass matrix is then

$$M_\nu^{\text{light}}=A-BH^{-1}B^T+O\left(\frac{\kappa^3}{v^2},\frac{\kappa'^3}{v^2}\right)$$

$$=\begin{pmatrix} 0 & 0 & 0 & m_{11} \\ 0 & 0 & 0 & m_{21} \\ 0 & 0 & 0 & m_{31} \\ m_{11} & m_{21} & m_{31} & 0 \end{pmatrix}-\frac{1}{fv}\begin{pmatrix} 2m_{12}m_{13} & m_{12}m_{23}+m_{13}m_{22} & m_{13}m_{32} & 0 \\ m_{12}m_{23}+m_{13}m_{22} & 2m_{22}m_{23} & m_{23}m_{32} & 0 \\ m_{13}m_{32} & m_{23}m_{32} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{9}$$

An inspection of Eq. (9) immediately leads to the conclusion that ν_τ is a pseudo-Dirac particle, with its main Dirac mass coming from the first matrix: $m_{\nu_\tau}\simeq(m_{11}^2+m_{21}^2+m_{31}^2)^{1/2}$. Since $m_{ab}=h_{ab}\kappa$ or $h_{ab}\kappa'$, with κ and κ' in the range 100 keV-1 MeV, this mass can naturally be of order 17 keV. ν_e and ν_μ masses arise from the second matrix of Eq. (9) and are seesaw suppressed. As an example, consider the following choice of parameters: $m_{11}=1.7$, $m_{12}=2.0$, $m_{13}=1.0$, $m_{21}=0.1$, $m_{22}=12.0$, $m_{23}=12.0$, $m_{31}=17.0$, $m_{32}=0.1$ (in keV) and $fv=300$ GeV. This yields $m_{\nu_e}=2\times 10^{-6}$ eV and $m_{\nu_\mu}=10^{-3}$ eV which are in the right range for the MSW mechanism for solar neutrinos to be effective. There is a tiny admixture of Majorana mass in ν_τ which splits the masses of ν_{τ_L} and ν_{τ_R}. For the choice of parameters above, this mass splitting is 4×10^{-7} eV.

The charged-current mixing angles $\theta_{e\tau}$ and $\theta_{\mu\tau}$ are determined essentially by the first term of Eq. (9): $\theta_{e\tau}\simeq m_{11}/m_{31}$, $\theta_{\mu\tau}\simeq m_{21}/m_{31}$. The ν_e-ν_μ mixing angle, on the other hand, receives comparable contributions from both terms. All these angles are arbitrary, and can be chosen to be at the few percent level. For the numerical example above, $\theta_{e\mu}=12\%$, $\theta_{e\tau}=10\%$, and $\theta_{\mu\tau}=1.8\%$.

As noted earlier, a 17-keV ν_τ should decay fast enough so as to avoid cosmological constraints on the mass density of the Universe. ν_τ decays in our model via majoron emission ($\nu_\tau\to\nu_i+\chi$), where the majoron χ is the massless Goldstone boson associated with the spontaneous breaking of the global U(1) symmetry. Let us now discuss the coupling of χ to fermions. The majoron field in our model is given by

$$\chi=N[\text{Im}\Delta+\beta_S\,\text{Im}\phi_S^0+\beta\,\text{Im}\phi^0+\beta'\,\text{Im}\phi'^0], \tag{10}$$

where

$$\beta_S=\frac{\kappa_S}{v}\left(\frac{5\kappa'^2-4\kappa^2}{\kappa_S^2+\kappa^2+\kappa'^2}\right),$$

$$\beta=\frac{\kappa}{v}\left[4+\frac{5\kappa'^2-4\kappa^2}{\kappa_S^2+\kappa^2+\kappa'^2}\right], \tag{11}$$

$$\beta'=\frac{\kappa'}{v}\left[-5+\frac{5\kappa'^2-4\kappa^2}{\kappa_S^2+\kappa^2+\kappa'^2}\right],$$

and N is the normalization constant. Since $\kappa,\kappa'\approx 100$ keV-1 MeV, while $\kappa_S,v\approx 200$ GeV, the constant $\beta_S\approx 10^{-10}$. The coupling of charged fermions such as e, u, and d to χ are proportional to $\beta_S m_f/m_W$. The coupling $g_{\chi ee}$ to the electron is then $\approx 10^{-16}$, which is consistent with all astrophysical constraints [12]. The induced coupling of χ to the electron by gauge-boson exchange at the one-loop level is even smaller.

The flavor nondiagonal couplings of the majoron to the light neutrinos, which determines the ν_τ lifetime, can be obtained by first applying the same unitary transformation U of Eq. (8) to the majoron coupling matrix, which in our model has entries in the off-diagonal block as well. In the basis where the light neutrino mass matrix is given by Eq. (9), we find the majoron coupling matrix to be

$$g_{\nu_i \nu_j \chi}=\frac{1}{v}\begin{pmatrix} 0 & 0 & 0 & 4m_{11} \\ 0 & 0 & 0 & 4m_{21} \\ 0 & 0 & 0 & -5m_{31} \\ 4m_{11} & 4m_{21} & -5m_{31} & 0 \end{pmatrix}+O\left(\frac{\kappa^2}{v^2},\frac{\kappa'^2}{v^2}\right). \tag{12}$$

It follows from Eqs. (9) and (12) that the unitary transformation which diagonalizes the light neutrino mass matrix will not simultaneously diagonalize the majoron coupling matrix. It is even more striking that the diagonalization of the leading Dirac mass term of Eq. (9) leaves behind an off-diagonal majoron coupling of order m/v. As a result, $g_{\nu_\tau \nu_e \chi} \simeq m_{\nu_\tau}/v$. The decay rate for $\nu_\tau \rightarrow \nu_e \chi$ is given by $\Gamma \simeq (81/32\pi)(m_D^3/v^2)\theta_{e\tau}^2$. For $\theta_{e\tau} \simeq 10\%$, and the U(1)-symmetry-breaking scale of the same order as the electroweak scale, the lifetime of ν_τ is 10^{-3}–10^{-1} sec, easily satisfying the cosmological constraints of mass density of the Universe [13].

Let us now turn to the implications of the model for nucleosynthesis. Since ν_τ in our model is a pseudo-Dirac particle, there can be ν_{τ_L}-ν_{τ_R} oscillation in the early Universe. Its strength depends on the ν_{τ_L}-ν_{τ_R} mass splitting δm. It has been shown in Ref. [14] that for $\delta m \lesssim 10^{-6}$ eV, ν_{τ_R} does not contribute as an extra species to nucleosynthesis. As shown in the earlier example, $\delta m \lesssim 10^{-6}$ eV can be satisfied in our model. The Higgs-induced four Fermi couplings of ν_{τ_R} are of order $G_F \times 10^{-3}$ (for $M_\phi \simeq 300$ GeV) so that they do not keep ν_{τ_R} in equilibrium during nucleosynthesis. On the other hand, if the lifetime for decay into majoron is of order 10^{-1} sec, due to time dilation effects, the lifetime in the rest frame of the Universe will be more than 10 sec. The majoron then does not contribute as an active additional species. We would then predict $N_\nu = 3$, consistent with the most recent analysis [15] which quotes $N_\nu \lesssim 3.3$. The nucleosynthesis constraint on δm implies that the oscillation length for a 1-GeV ν_τ beam in the laboratory will be about 1000 km and hence unobservable.

There are limits on m_{ν_τ} coming from supernova 1987A observations: $m_{\nu_\tau} \lesssim 28$ keV [16]. This bound is not violated by the 17-keV pseudo-Dirac neutrino. The new interactions of ν_{τ_R} present in our model are too weak (i.e., $G_F \times 10^{-3}$) to affect the neutrino signal from the supernova.

The rare process $\mu \rightarrow e\gamma$ will proceed in our model via ϕ^+ exchange. For the range of parameters we have chosen, the branching ratio is less than 10^{-12}, consistent with, but not too far from, the present experimental limit.

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[1] R. Davis *et al.*, in *Proceedings of the Thirteenth International Conference on Neutrino Physics and Astrophysics, Neutrino '88,* edited by J. Schneps, T. Kafka, W. Mann, and P. Nath (World Scientific, Singapore, 1989), p. 518.

[2] Kamiokande Collaboration, K. Hirata *et al.*, Phys. Rev. Lett. **63**, 16 (1989); **65**, 1297 (1990).

[3] V. N. Gavrin *et al.* (SAGE), in Proceedings of the Twenty-Fifth International Conference on High Energy Physics, Singapore, 1990 (to be published).

[4] J. J. Simpson, Phys. Rev. Lett. **54**, 1891 (1985); J. J. Simpson and A. Hime, Phys. Rev. D **39**, 1825 (1989); A. Hime and J. J. Simpson, *ibid.* **39**, 1837 (1989).

[5] B. Sur *et al.*, Phys. Rev. Lett. **66**, 2444 (1991); A. Hime and N. A. Jelley, Phys. Lett. B **257**, 447 (1991); I. Žlimen *et al.*, Phys. Rev. Lett. **67**, 560 (1991).

[6] S. P. Mikheyev and A. Yu Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[7] F. Avignone *et al.*, Phys. Rev. C **34**, 666 (1986); D. Caldwell, Nucl. Instrum. Methods Phys. Res., Sect. A **264**, 106 (1986); H. Ejiri *et al.*, Osaka University Report No. OULNS-90-21, 1990 (to be published).

[8] D. Dicus, E. Kolb, and V. Teplitz, Phys. Rev. Lett. **39**, 169 (1977).

[9] S. L. Glashow, Phys. Lett. B **256**, 218 (1991).

[10] Y. Chikashige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. **98B**, 265 (1981).

[11] S. L. Glashow, Phys. Lett. B **187**, 367 (1987). In the simplest version of the singlet majoron model, the leading $(m/M)^2$ contribution vanishes for the off-diagonal neutrino coupling of the majoron. See J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 774 (1982).

[12] J. E. Kim, Phys. Rep. **149**, 1 (1987).

[13] It is possible to construct an alternative model to realize Glashow's scheme for neutrino mass hierarchy by adding a second Higgs doublet ϕ_2 to the original singlet majoron model. With a discrete Z_2 symmetry under which ν_{3R}, ϕ_2, and Δ are odd, the Majorana mass matrix M will have rank 2. However, this scheme is highly unnatural, since the Dirac-type Yukawa couplings of ν_R will have to be tuned to 10^{-7}. In this case, the leading $(m/M)^2$ term in the off-diagonal $\nu_i \nu_j \chi$ coupling is nonvanishing as M has rank 2, leading to a ν_τ lifetime of $\approx 10^{10}$ sec.

[14] R. Barbieri and A. Dolgov, Phys. Lett. B **237**, 440 (1990).

[15] T. P. Walker, G. Steigman, D. N. Schramm, K. A. Olive, and H. S. Kang, Ohio State University Report No. OSU-TA-10/90 (to be published).

[16] R. Gandhi and A. Burrows, Phys. Lett. B **246**, 149 (1990); erratum (to be published); J. A. Grifols and E. Masso, Phys. Lett. B **242**, 77 (1990).