Chiral Properties of Dynamical Wilson Quarks at Finite Temperature

Y. Iwasaki, K. Kanaya, S. Sakai, and T. Yoshié
Institute of Physics, University of Tsukuba, Ibaraki 305, Japan
(Received 2 January 1991; revised manuscript received 12 June 1991)

Chiral properties of Wilson fermions in full QCD with two degenerate flavors at finite temperature are investigated by numerical methods. We show that when the quark mass is properly defined, for a given g (gauge coupling constant) and K (hopping parameter), its value is almost independent of whether the system is in the high-temperature phase or in the low-temperature phase. The temperature dependence of hadronic screening masses is consistent with the physical picture that both U(1) and SU(2) chiral symmetries are recovered in the high-temperature phase.

PACS numbers: 12.38.Gc, 11.30.Rd

In the Wilson formulation [1] of quarks on a lattice, chiral symmetry is explicitly broken by the so-called Wilson term which is introduced in order to avoid species doubling. Therefore it is not *a priori* obvious whether one is able to define a chiral limit for Wilson quarks, in particular, on a lattice with finite lattice spacing. Only after the existence of a chiral limit is established, irrespective of the phase of a quark-gluon system, does it become possible to ask how chiral symmetry of the quark-gluon system is realized.

In a previous paper [2] we have shown the following for Wilson quarks in the quenched QCD: (1) When the quark mass is defined properly by a PCAC (partial conservation of axial-vector current) relation, for a given β ($\beta=6/g^2$) and K, its value is independent of whether the system is in the high-temperature phase or in the low-temperature phase. Therefore one is able to define the chiral limit irrespective of the phase of a gluon system. (2) The temperature dependence of the pion screening mass in the chiral limit is consistent with the physical picture that the spontaneously broken chiral symmetry is recovered at a temperature identical with the deconfining transition.

Although it is expected that the pure gauge theory resembles full QCD in respect to the confinement and the chiral property, what is really important is to investigate these properties in full QCD. In the physical world of QCD, only the masses of the u and d quarks are much smaller than those of other quarks, the deconfining temperature, and the QCD scale parameter. Therefore full QCD with two degenerate light quarks is a good approximation to the physical world. It is thus highly important to investigate chiral properties of Wilson quarks, because the Wilson formulation of quarks is the only known way to describe two flavors of light quarks in terms of a local action. Hence in this paper we would like to investigate the chiral properties of Wilson quarks in full QCD with two flavors. Our primary interests here are whether the value of the quark mass m_q, defined as in the previous paper, is independent of the phase, and whether chiral SU(2) and/or U(1) symmetries are recovered in the high-temperature phase. Of course, the second question is meaningful only when the answer for the first one is affirmative because only in this case can the chiral limit be defined irrespective of the phase, as stressed above.

We consider QCD with two degenerate quarks. We take the standard one-plaquette action and the $r=1$ Wilson fermion action. We make simulations at $\beta=5.5$ on an $8\times8\times20\times N_t$ lattice ($N_t=4$ or 8) with $K=0.15$, 0.16, 0.1615, 0.163 for $N_t=4$ and with $K=0.15$, 0.155 for $N_t=8$. We call the direction whose linear extension is 20 the z direction. We use an antiperiodic boundary condition for quarks in the t direction and periodic boundary conditions for quarks in the spatial directions as well as for gluons in all directions. Investigation [3–5] of the β dependence of the Polyakov loops shows that the systems for all the hopping parameters at $\beta=5.5$ are in the high- (low-) temperature phase on the lattice with $N_t=4$ ($N_t=8$). Gauge configurations are generated by the hybrid Monte Carlo algorithm (HMCA) [6] with molecular-dynamics step size $\Delta\tau=0.025$ and the numbers of steps for one trajectory $n_{\text{MD}}=40$ (except for the case at $K=0.155$ on the $N_t=8$ lattice; $\Delta\tau=0.02$ and $n_{\text{MD}}=50$ for this case). The inversion of the quark matrix ($x=D^{-1}b$) is made by a minimal residual method with incomplete LU (lower triangle matrix–upper triangle matrix) decomposition [7,8]. The stopping condition is $r=||b-Dx||/\sqrt{3\times4V}=4.5\times10^{-7}$. The acceptance ratio is (70–80)% for all cases. After thermalization of 1000 trajectories or 500 trajectories, depending on the initial configuration, typically 20 gauge configurations are saved, separated by 50 trajectories (except for the case at $K=0.150$ on the $N_t=4$ lattice; 100 trajectories for this case). We check the equilibrium of the system by monitoring $\langle e^{-\Delta S}\rangle$.

Hadronic propagators in the z direction are calculated on the $8\times8\times40\times N_t$ lattice obtained by doubling the original $8\times8\times20\times N_t$ lattice. We calculate the propagators of flavor nonsinglet mesons, π ($\bar\psi\gamma_5\tau^a\psi$), ρ ($\bar\psi\gamma_i\tau^a\psi$), δ ($\bar\psi\tau^a\psi$), B ($\bar\psi\sigma_i\tau^a\psi$), A_1 ($\bar\psi\gamma_5\gamma_i\tau^a\psi$), and $\tilde\rho$ ($\bar\psi\gamma_0\gamma_i\tau^a\psi$) ($i=1,2$), as well as those of baryons, N (nucleon), Δ (delta), and their chiral partners.

We define the quark mass in the continuum theory, in the same way as in the previous work [8], by

$$\langle 0|\partial_\mu A_\mu(x)|\pi\rangle = 2m_q\langle 0|P_5(x)|\pi\rangle . \tag{1}$$

Here, $A_\mu(x)=\bar\psi(x)\gamma_5\gamma_\mu\psi(x)$ and $P_5(x)=\bar\psi(x)\gamma_5\psi(x)$ with the flavor index suppressed, and the pion is in the zero-momentum state. Thus our definition of the quark mass on the lattice is given by

$$-m_\pi\langle 0|A_4(n)|\pi\rangle = 2m_q\langle 0|P_5(n)|\pi\rangle . \tag{2}$$

Alternatively we may replace the derivative ∂_μ in Eq. (1) with the discrete derivative on the lattice. Noting the relations

$$\langle 0|A_4(t\pm 1)|\pi\rangle = e^{\mp m_\pi}\langle 0|A_4(t)|\pi\rangle , \tag{3}$$

we have

$$\langle 0|\partial_\mu A_\mu(x)|\pi\rangle = c_\delta\langle 0|\nabla_4 A_4(t)|\pi\rangle \tag{4}$$

in the continuum theory. Here c_δ is a correction factor. For example, when we put $\nabla_4 A_4(t)=[A_4(t+1)-A_4(t-1)]/2$, $c_\delta=2m_\pi/(e^{+m_\pi}-e^{-m_\pi})$, and when $\nabla_4 A_4(t)=A_4(t+1)-A_4(t)$, $c_\delta=m_\pi/(1-e^{-m_\pi})$. Hence we have an alternative form of the definition of the quark mass on the lattice given by

$$c_\delta\langle 0|\nabla_4 A_4(n)|\pi\rangle = 2m_q\langle 0|P_5(n)|\pi\rangle . \tag{5}$$

Note that Eq. (3) holds on the lattice as well as in the continuum theory and therefore this definition is completely equivalent to the definion of Eq. (2).

Thus we calculate the quark mass in finite-temperature QCD by

$$2m_q = m_\pi \lim_{z\,\mathrm{large}} R(z) , \tag{6}$$

with

$$R(z) = -\frac{\langle \sum_{x,y,t} A_z(x,y,z,t)\pi(0)\rangle}{\langle \sum_{x,y,t}\pi(x,y,z,t)\pi(0)\rangle} . \tag{7}$$

At zero temperature, this definition is reduced to Eq. (2). Similarly we may calculate the quark mass by a formula which corresponds to the definition given by Eq. (5). We have checked that each of them gives identical results for the quark mass within statistical errors.

The quark mass defined by relation (2) was first numerically calculated in Ref. [8] [see Eq. (5.3) and Fig. 11 therein] in quenched QCD. Independently and slightly later, Maiani and Martinelli [9] [see Eq. (26) and Fig. 1 therein] calculated, also in quenched QCD, the quark mass similarly defined, being based on careful analyses [10] of chiral Ward identities of Wilson fermions on a lattice. Both works confirmed the relation $m_\pi^2 \sim m_q$ at zero temperature.

The screening masses [11] of the hadrons at each K are determined by fitting the propagators typically for $z=9$–20 with one-mass forms. The errors for the masses are estimated by the jackknife method. All the screening masses in the high-temperature phase can be determined with small errors, in contrast to the case of the low-temperature phase where the propagators of the particles other than the s-wave particles (π, ρ, N, and Δ) are not good enough to determine their screening masses with small errors.

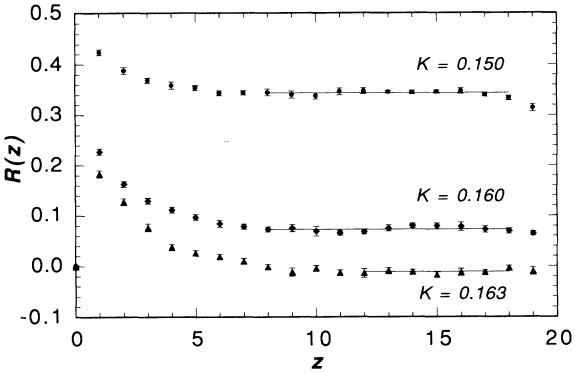

FIG. 1. Typical results for $R(z)$ which tends to $2m_q/m_\pi$ for large z, together with the fitted lines. The data are on the $N_t=4$ lattice.

In Fig. 1 we present typical results for $R(z)$ on the $N_t=4$ lattice. The $R(z)$ is flat for large z. We determine the quark mass by fitting the data in the flat region (typically for $z=10$–18) with $2m_q/m_\pi$ by the jackknife method. The region of z for the fit is chosen in such a way that Δm_π is zero within 1σ, when the $R(z)$ is fitted by the form $2m_q/m_\pi \exp(-\Delta m_\pi z)$.

We present in Fig. 2 the results for the quark masses and the pion screening masses squared. The m_q on the two lattices as well as m_π^2 on the $N_t=8$ lattice are fitted by linear functions of $1/K$. As the figure shows, the m_q's obtained for the $N_t=8$ lattice are almost (but not exactly) on the line fitted to those for the $N_t=4$ lattice. The K_c^{quark}'s where m_q vanishes are 0.1627(1) and 0.1621(8) on the $N_t=4$ and $N_t=8$ lattices, respectively. The two

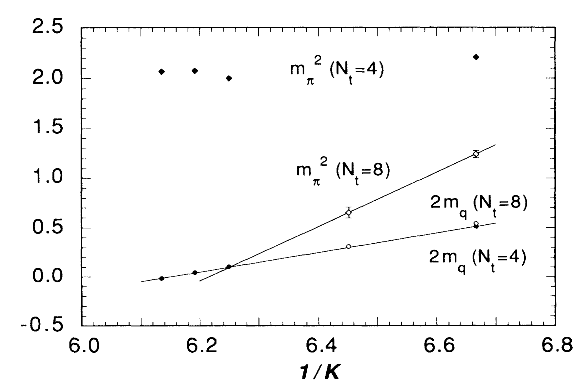

FIG. 2. $2m_q$ and m_π^2 vs $1/K$. The straight lines are the fit to the data for $2m_q$ on the $N_t=4$ lattice and the fit to the data for m_π^2 on the $N_t=8$ lattice.

K_c^{quark}'s are almost identical. Thus we conclude that the quark masses are almost independent of the phase. The coincidence of the quark mass on the two lattices is not exact. The differences slightly exceed the errors estimated. One possible explanation for this is that the errors for the quark masses are underestimated: It is not certain that the separation of configurations (50 or 100 trajectories) is large enough to regard configurations as statistically independent. (We find that the autocorrelation for the 1×1 Wilson loop is about 100 trajectories.) An alternative explanation is that the slight differences are effects due to the $O(a)$ chiral-symmetry breaking of Wilson quarks. Note that the K_c^π where m_π vanishes in the low-temperature phase is 0.1610(12) (this K_c^π is also consistent with those given by Ukawa [3] and Gupta *et al.* [4]) and approximately agrees with the K_c^{quark}'s. It should be emphasized that we can define the chiral limit uniquely within small differences irrespective of the phase for the case of Wilson quarks with two degenerate flavors.

Note that we are able to calculate the hadron propagators around K_c ($K=0.1615$ and 0.163) in the high-temperature phase as in quenched QCD: The number of iterations needed for the matrix inversion increases only moderately. For example, the number of iterations at $K=0.163$ with $N_t=4$ is approximately equal to that at $K=0.155$ with $N_t=8$. This is consistent with our observation that D has no zero modes around $K=K_c$: The smallest eigenvalues of the $\gamma_5 D$ around $K=0.163$ obtained by the Lanczos method are not small (of order 0.1) for all 20 configurations. However, this is in conflict with what is stated in Ref. [5].

As Fig. 2 shows, m_π^2 on the $N_t=8$ lattice is proportional to the quark mass. This is exactly what we expect when we regard the pion as a Goldstone boson associated with spontaneously broken chiral SU(2) symmetry. On the other hand, m_π in the high-temperature phase does not vanish at the critical hopping parameter. Thus the behavior of the pion screening mass is consistent with the picture that the spontaneously broken chiral symmetry is recovered in the high-temperature phase.

Now let us discuss how chiral multiplets are realized in the high-temperature phase of QCD. At zero temperature, the chiral SU(2) symmetry is spontaneously broken, while the chiral U(1) symmetry is explicitly broken by the U(1) anomaly. At high temperatures, besides the SU(2) symmetry, U(1) symmetry is expected to be restored, associated with the diminution of the U(1) anomaly. Although we have calculated the screening masses only for flavor nonsinglet hadrons, we can distinguish the restoration of the U(1) symmetry from that of the SU(2) symmetry: For the restoration of the U(1) symmetry the δ-π and $\tilde{\rho}$-B multiplets are realized, while for the restoration of the SU(2) symmetry, the ρ-A_1 multiplet is realized. In contrast to the meson multiplets, the restoration of either symmetry implies multiplets of baryons and their chiral partners. Note that we can see what kinds of

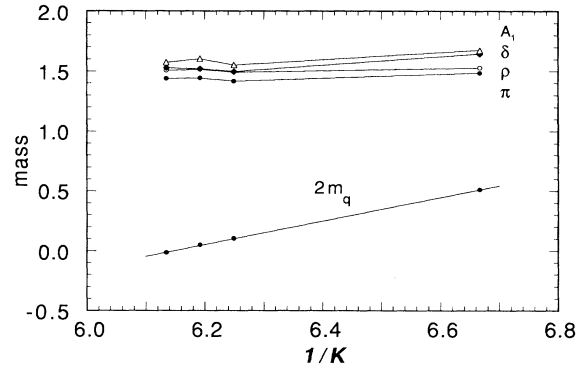

FIG. 3. Typical meson screening masses together with $2m_q$ on the $N_t=4$ lattice.

multiplets are realized around K_c without extrapolation in the high-temperature phase. Figure 3 shows the results for the meson screening masses on the $N_t=4$ lattices. We see from the figure that both the π-δ and ρ-A_1 multiplets are almost degenerate at $K=K_c$. We also find that the $\tilde{\rho}$-B multiplet as well as the baryon multiplets are almost degenerate. Therefore we conclude that both U(1) and SU(2) chiral symmetries are almost recovered in the high-temperature phase (at $\beta=5.5$ on the $N_t=4$ lattice with $K\sim 0.163$). Our results suggest that the $O(a)$ chiral-symmetry breaking is already small at $\beta=5.5$. Similar calculations for Wilson quarks have been also done by other groups [3,12]. See also the calculations [13] of the screening masses with staggered quarks.

Note that the screening masses of the mesons and the baryons are, respectively, roughly equal to $2\pi T$ and $3\pi T$ ($T=\frac{1}{4}$) which are the lowest Matsubara frequencies in the corresponding channels. This means that quarks are almost free [14]. a^{-1} at $\beta=5.5$ is roughly 1.6–1.8 GeV [15], which implies that the temperature on the $N_t=4$ lattice is about 2 or 3 times the deconfining temperature T_c. Further calculations at temperatures closer to T_c are necessary to see whether hadronic modes [11] really exist in the high-temperature phase of full QCD with Wilson quarks.

In a previous paper [2] we have shown in quenched QCD with Wilson quarks that the deconfining transition temperature is, within the precision of the calculation, identical to the restoration temperature of the flavor-nonsinglet chiral symmetry. We would like to investigate in the near future whether the same relation is satisfied for the SU(2) symmetry in full QCD and whether the U(1) symmetry is restored exactly at the deconfining temperature or is restored asymptotically.

The numerical calculations reported here have been performed with HITAC S820/80 at KEK. We would like to thank members of KEK for their warm hospitality and strong support. We also would like to thank S. Gottlieb, M. Okawa, D. Toussaint, and A. Ukawa for valuable discussions. Part of this work is supported by University of Tsukuba Project Research.

---

[1] K. G. Wilson, in *New Phenomena in Subnuclear Physics,* edited by A. Zichichi (Plenum, New York, 1977).
[2] Y. Iwasaki, Y. Tsuboi, and T. Yoshié, Phys. Lett. B **220**, 602 (1989).
[3] A. Ukawa, Nucl. Phys. B (Proc. Suppl.) **9**, 463 (1989).
[4] R. Gupta, A. Patel, C. F. Baillie, G. Guralnik, G. W. Kilcup, and S. R. Sharpe, Phys. Rev. D **40**, 2072 (1989).
[5] K. Bitar, A. D. Kennedy, and P. Rossi, Phys. Lett. B **234**, 333 (1990).
[6] S. Duane, A. D. Kennedy, B. J. Pendlenton, and D. Roweth, Phys. Lett. B **195**, 216 (1987).
[7] Y. Oyanagi, Comput. Phys. Commun. **42**, 333 (1986).
[8] S. Itoh, Y. Iwasaki, Y. Oyanagi, and T. Yoshié, Nucl. Phys. **B274**, 33 (1986).
[9] L. Maiani and G. Martinelli, Phys. Lett. B **178**, 265 (1986).
[10] M. Bochicchio, L. Maiani, G. Martinelli, G. Rossi, and M. Testa, Nucl. Phys. **B262**, 331 (1985).
[11] C. E. DeTar, Phys. Rev. D **32**, 276 (1985).
[12] K. Bitar *et al.*, in *Proceedings of the International Conference on Lattice Field Theory, Tallahassee, Florida, 8–12 October 1990* [Nucl. Phys. B (Proc. Suppl.) **20**, 272 (1990)].
[13] C. DeTar and J. Kogut, Phys. Rev. Lett. **59**, 399 (1987); Phys. Rev. D **36**, 2828 (1987); S. Gottlieb, W. Liu, D. Toussaint, R. L. Renken, and R. L. Sugar, Phys. Rev. Lett. **59**, 1881 (1987).
[14] A. Irbaeck, in *Proceedings of the International Conference on Lattice Field Theory* (Ref. [12]), p. 280.
[15] M. Fukugita, Y. Oyanagi, and A. Ukawa, Phys. Lett. B **203**, 145 (1988).