Subharmonic Pumping of a Josephson-Parametric Amplifier and the Pitchfork Instability

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We report the observation of substantial amplification due to multiwave (more than four) mixing in a Josephson-parametric amplifier. Amplification was achieved with subharmonic pumping at $\frac{2}{3}$, $\frac{2}{5}$, and $\frac{2}{7}$ of the signal frequency near 19 GHz. The processes responsible for the amplification were, correspondingly, five-, seven-, and nine-wave mixing. For the case of five-wave mixing we have observed amplification in excess of 35 dB and noise referred to input of a few degrees kelvin before the amplifier undergoes a pitchfork instability. The noise rise at the pitchfork bifurcation was also studied and compared with theory.

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A number of nonlinear optical media having second order, χ^2, and the third order, χ^3, nonlinear susceptibilities have been successfully used to achieve parametric gain using three-wave and four-wave mixing. These three-photon and four-photon processes have, in fact, been employed to obtain squeezed light by a number of groups [1-5]. However, materials with higher-order nonlinearities (χ^4, etc.) that would give a substantial parametric gain at optical frequencies are not available yet.

At microwave frequencies Josephson-parametric amplifiers have also been successfully operated in both three-wave and four-wave mixing modes when amplifiers were pumped with an external pump [6,7] or with a pump generated internally [8,9]. This earlier work (reviewed in Ref. [10]) was plagued by large excess noise. Lately a number of groups succeeded in producing devices that overcame the large-excess-noise problem [11-14]. In our investigations of one such device [15-18], all the noise was accounted for by equilibrium fluctuations of the amplifier's losses. Recently, vacuum squeezing at microwave frequencies was observed with the Josephson-parametric amplifier operated in a nondegenerate three-wave mixing mode [17]. This mode of operation was also used to study the noise properties of a system undergoing a Hopf bifurcation [18,19]. However, in the case of the Josephson-parametric amplifier one is not limited to the three-photon and four-photon processes. The nonlinear Josephson inductance employed in parametric amplification,

$$L_J = L_0 \frac{\sin^{-1}(i/I_c)}{i/I_c},$$

where i is the current, I_c is the critical current, and $L_0 = \hbar/2eI_c$, has higher-order nonlinear terms that are comparable in magnitude to the second- and third-order terms responsible for three-wave and four-wave mixing, respectively. In this paper we report the observation of gain from a Josephson-parametric amplifier in which these higher-order nonlinearities are employed in multiwave mixing processes involving five or more photons at microwave frequencies. This multiwave mixing was achieved by pumping the amplifier with a subharmonic of twice the signal carrier frequency. For example, in the case of the five-wave mixing the frequency of the pump was chosen to be $\frac{2}{3}$ of the fundamental frequency (or the frequency of the signal to be amplified). In this case three pump photons combined to produce two signal photons, making a total of five photons taking part in the process. Parametric processes involving more than two signal photons are possible but they are weaker because they are higher order in the weak signal fields. Therefore they would be difficult to observe.

Pumping a parametric amplifier with a subharmonic that has an even numerator and odd denominator insures that none of the harmonics lie at the signal frequency. This has enabled us to study the Josephson-parametric amplifier as it undergoes the pitchfork bifurcation without corruption from pump harmonics. For example, for the case of the five-wave mixing, with the pump at the frequency $\frac{2}{3}\nu_0$, the amplifier breaks into oscillation at the frequency ν_0 with no parasitic feedthrough at that frequency. We compare our results with the theoretical calculations [20] for a generic system undergoing a pitchfork bifurcation. For example, our data display a family of closely spaced peaks that emerge in the region of the bifurcation, which is a fingerprint of such a system. Recently, a similar behavior was observed in a system incorporating a driven magnetostrictive ribbon [21], with the data displaying the features predicted by the same theoretical calculations [20].

The Josephson-parametric amplifier used in this experiment is of the same design as the one that was used to demonstrate thermal [15] and vacuum noise [17] squeezing. The detailed description of this amplifier can be found in Refs. [15-19]. The apparatus was similar to the one used in previous investigations [17,18] that employed three-photon mixing. In those works the pump was obtained by doubling the output of the HP8673B signal generator at a frequency around 19 GHz. The pump at a frequency around 38 GHz was then piped into the input of the Josephson-parametric amplifier via a Q-band waveguide. For the current experiment pump frequencies occurred at various subharmonics of the fundamental frequency. As a result a semirigid beryllium-copper coaxial waveguide, heat sunk to the 1-K pot and the mixing chamber of an Oxford-1000 dilution refrigerator, was used to deliver the pump to the input of the Josephson-parametric amplifier. The other change was substituting an HP8673B signal generator for an HP8350B sweep oscillator as the local oscillator source because of the need for greater frequency stability.

Vacuum fluctuations or thermal noise input for the amplifier was provided by a variable-temperature microwave termination [22] that could be cycled between 30 mK and 2 K. The motorized cryogenic waveguide switch [23] allowed the Josephson-parametric amplifier (JPA) to be switched out of the circuit for calibration measurements. The detector can thus look directly at the variable-temperature termination. By cycling the temperature of the termination, accurate determination of the detector-system noise temperature could be made, which for this run was measured to be about 300 K. Performing the same calibration run with JPA switched in the circuit and comparing the results to a run with the JPA out readily gave the loss of the amplifier for the particular run. For the following data the amplifier's loss parameter η was measured to be between 0.480 and 0.698, the actual power loss for the signal as it passes through the amplifier being $1-\eta$.

A probe signal at a frequency of about 19 GHz was delivered to the input of the JPA via a K-band waveguide. Two 20-dB couplers insured that the room-temperature noise did not find its way into the input port of the JPA, the last one coupling the probe to signal going from the noise source to the input port of the JPA. The output signal from the JPA passed through three cryogenic isolators, and was amplified further by two high-electron-mobility (HEMT) field-effect-transistor amplifiers operated at 4.2 K. The isolators provided a total of 68-dB isolation from the room-temperature noise. The signal was then mixed down with the local oscillator at the frequency of 18.5 GHz generated by an HP8673B signal generator, amplified further by two Tron-Tech W1G2H amplifiers, and finally delivered to the input of the HP8566B spectrum analyzer. The frequency span of the analyzer was usually set to 20 or 50 MHz, the video bandwidth was set to 10 or 30 Hz, and the frequency resolution was set to 100 kHz, which resulted in a time span of a few minutes for a single trace. All the instruments that were used to collect data (HP6622 power supply, three HP8673B signal generators, and HP8655B spectrum analyzer) were controlled via a GPIB bus by an AT&T 6300 personal computer. During a single run the pump power, signal probe power, and the detuning from the fundamental frequency were stepped over the range of interest. The detuning was usually set between 1 and 20 MHz. The signal probe power was set to -70, -60, or -50 dBm. The pump power was stepped through a 10-dB range. 0.1-dB steps were used over a 2-dB interval in the region close to the threshold for oscillation; away from the region 0.5-dB steps were used. A total of 38 traces were collected for the single run for constant probe detuning and power. Two of the traces were taken with the pump off and a strong -40-dBm probe either on or off to determine the noise floor and the signal probe strength. The time required to collect the part of the data for constant probe power and detuning was between 1 and 2 h, and the complete run took from 12 to 18 h.

Most of the data for the five-photon process were taken with the pump at a frequency of $\nu_p = 12.580$ GHz. This frequency is $\frac{2}{3}$ of the fundamental frequency of $\nu_0 = 18.87$ GHz about which the amplified signal and idler signals appeared as the pump power was increased during a single sweep series. Figure 1 shows the results of one such run. It is composed from 36 individual traces of the received signal as a function of frequency for constant pump power. These data were collected with a signal-analyzer frequency span of 20 MHz about an i.f. that corresponded to an rf of 18.87 GHz. For large pump power the JPA breaks into oscillation giving rise to the sharp peak at 10 MHz which corresponds exactly to 18.87 GHz or $\frac{3}{2}$ of the pump frequency. The instability giving rise to this oscillation is a pitchfork bifurcation. The linewidth of the oscillation is instrument limited, and this oscillation is phase locked to the pump.

The ridge to the right of this peak at a frequency of 11 MHz is the amplified probe signal, detuned by 1 MHz from the fundamental frequency $\nu_0 = \frac{3}{2}\nu_p$ for these data. As the pump power is increased, the probe's gain increases until the JPA breaks into oscillation and undergoes a pitchfork bifurcation. Then the probe signal be-

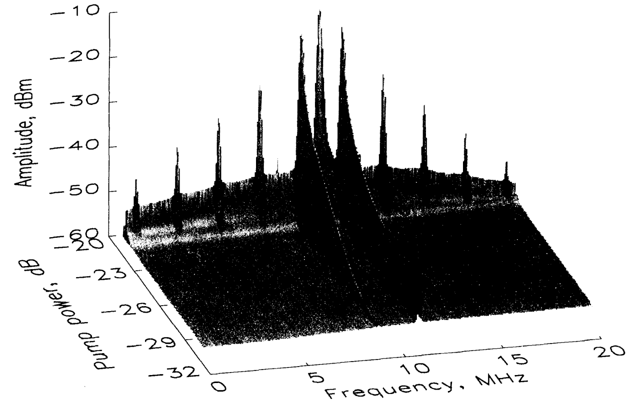

FIG. 1. The received signal as a function of frequency and relative pump power. The data was taken with the pump frequency $\nu_p = 12.58$ GHz. The signal analyzer's center frequency was set to 18.870 GHz which is exactly equal to $\frac{3}{2}\nu_p$. Detuning from the fundamental frequency $\nu_0 = 18.870$ GHz for this run was set to 1 MHz and the probe amplitude was -50 dBm.

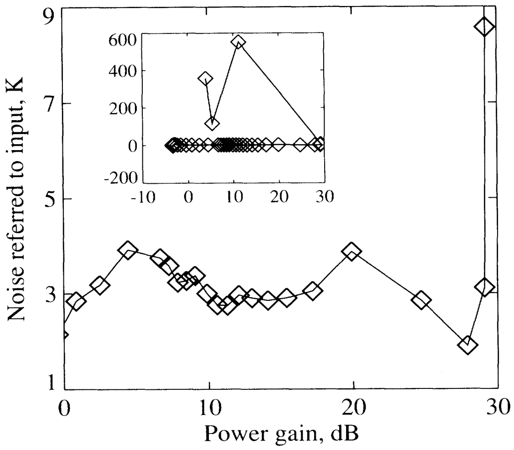

FIG. 2. Noise referred to input as a function of the probe gain in the region below the critical value of the pump power for the data displayed in Fig. 1. The line connects consecutive data points as the pump power is increased. In this regime the probe gain increases without any gain-related excess noise. Inset: The result of analysis for a complete data run, including the bifurcation region. As the pump power increases and the amplifier undergoes a pitchfork bifurcation, the noise referred to input rises dramatically.

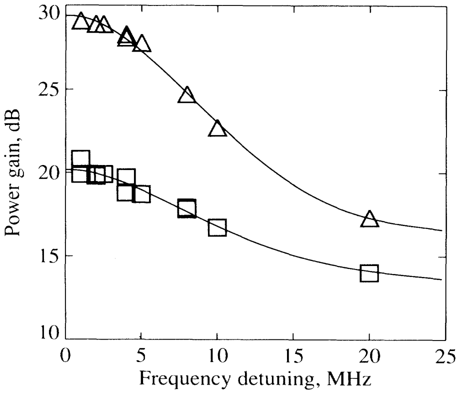

FIG. 3. The classical gain as a function of detuning from the fundamental frequency ν_0. The data that were used for calculation belong to a series of runs having different detuning frequencies, but all were taken for the same signal probe amplitude of -50 dBm. Triangles represent the data taken with the pump power just below the critical value for the instability, when the gain is at the maximum. Squares represent the data taken with a pump power 0.3 dB less. Solid lines through the two data sets are guides to the eye.

gins to drop with increasing pump power. The ridge to the left of the central pitchfork peak is the idler signal, spaced from the fundamental ν_0 by a detuning frequency of 1 MHz. As the pump power increases, the idler peak appears and grows until in the region of high amplification it becomes nearly a mirror image of the signal peak with respect to the central JPA's oscillation peak frequency ν_0. The other peaks of Fig. 1 are the result of higher-order mixing effects [20]. They appear in the region of the pump power just below the critical value for the onset of the bifurcation, and persist into the bifurcation regime in agreement with theoretical calculations [20].

Figure 2 shows the noise temperature referred to input of the JPA as a function of the gain for the probe signal as the pump power was stepped through the region of the pitchfork bifurcation. The analysis was performed on the data displayed in Fig. 1. Two additional traces were taken at the end of each run: one with the pump turned off and the probe at -40 dBm, and another with both pump and probe turned off. Measuring the amplitude of the probe signal of the former trace and that of the probe signal of each trace of the data run enables determination of the power gain. We also measured the noise under the signal probe peak and the noise floor at the same frequency for the trace with the pump and the probe turned off. The noise temperature at the output of the JPA was calculated using the known detector noise temperature and then divided by the classical gain times the JPA's loss to determine the noise temperature referred to input. As the plot of Fig. 2 shows, the amplifier is very well behaved until the gain is nearly 30 dB, without showing signs of any gain-dependent excess noise. As the pump power is increased further, the amplifier enters the regime of the pitchfork instability, where both increasing noise and decreasing classical probe gain result in a dramatic increase in the noise temperature referred to the input of the JPA, as shown in the inset of Fig. 2. This behavior is qualitatively similar to the one expected for a generic system undergoing a pitchfork bifurcation [24,25]. The average value of the noise referred to the input of the JPA is about 3 K, which is a factor of 2 greater than the expected value of ≈ 1.5 K. We attribute this discrepancy to an additional (probably four-wave) mixing process about the pump frequency that turns on at a lower pump power than the one used to collect the data. Such a mixing process would open an extra idler port and result in an increase in the noise seen at the signal frequency.

Figure 3 shows the parametric gain of the amplifier as a function of the detuning from the fundamental frequency ν_0 for two values of pump power below the critical value for the onset of the bifurcation Fig. 2. This figure was obtained by measuring the classical gain for the signal-analyzer traces taken at the same pump power level and signal probe level, but different detuning frequencies from ν_0. The data represented by triangles were taken with the pump power just below the onset of the bifurcation. The 3-dB gain bandwidth for this setting of high maximum amplification is only about 10 MHz, since the gain is symmetrical about zero frequency detuning. As the pump power is decreased by 0.3 dB and the maximum gain drops from ≈ 30 to ≈ 20 dB, the 3-dB gain bandwidth increases to about 20 MHz, as shown by the data represented by squares. When the maximum gain is decreased to 10 dB, the 3-dB gain bandwidth of the JPA becomes several hundred MHz.

We have also used $\nu_p = \frac{2}{5}\nu_0$ and $\nu_p = \frac{2}{7}\nu_0$, which resulted in seven-wave and nine-wave mixing. For these higher-order processes amplification of ≈ 15 dB resulted when the amplifier underwent a pitchfork bifurcation and nearly 30 dB was seen when the instability was of the Hopf type.

In conclusion, we have observed substantial gain resulting from subharmonic pumping of the Josephson-parametric amplifier. Amplification was due to the multiwave (more than four-wave) mixing. In the case of the five-wave mixing, when the pump frequency was set at $\nu_p = \frac{2}{3}\nu_0$, the maximum gain exceeded 30 dB. Several features were observed that agree with the theoretical calculations [20] for a generic system undergoing a pitchfork bifurcation.

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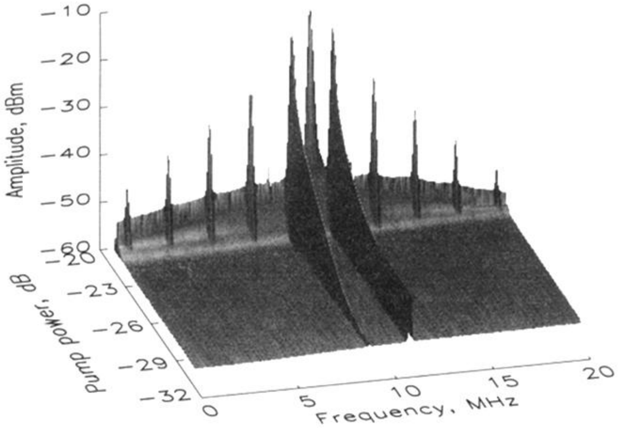

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