Fast Delocalization in a Model of Quantum Kicked Rotator

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We present numerical results on a model in which quantization of chaos (instead of suppression of the diffusion) leads to a diffusive or even faster excitation with infinite conductance in the corresponding solid-state model. We suggest a possible mechanism for this effect.

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Since 1979 [1], the problem of quantum effects for the diffusive excitation of systems which are classically chaotic has attracted the attention of many researchers [2–5]. The reason for this interest seemed to be the importance of the quantum effects on the complexity of the corresponding classical dynamics and, in particular, the localization effect observed in the quantum version of the classical standard map [1–5]. To our knowledge, all proposed modifications to the initial kicked-rotator model [1] have led to excitation localization for the one-dimensional case with a perturbation periodic in time. It was later shown that quantum localization of chaos in such a mathematical model also leads to a better understanding of the microwave ionization of a highly excited hydrogen atom [4,6].

The other interesting feature of this phenomenon is that it can be considered as a dynamical counterpart of Anderson localization in solid-state physics [3]. In order to make a connection between the two problems, one needs to interpret the quantum number of the unperturbed rotator as the site number for some corresponding solid-state model. To our knowledge, all proposed modifications to the initial kicked-rotator model [1] have led to excitation localization.

We present here a model displaying chaotic classical behavior with unbounded quantum excitation. Contrary to the previously studied cases, we observe not only a diffusive excitation but also a free propagation in the momentum space in the region of the parameters for which the classical motion is completely chaotic. Therefore, in this case the conductivity of the corresponding solid-state model is infinite and therefore this effect may be interpreted as the existence of a ballistic excitation, which in some sense is analogous to a superconductive flow.

First, we briefly discuss the classical dynamics of our model. This is defined by the following area-preserving map of the plane:

$$\bar{p} = p + K\sin(x), \quad \bar{x} = x - L\sin(\bar{p}), \tag{1}$$

where K and L are positive parameters and the bars denote the new values of the variables after one iteration.

The physical meaning of p and x are the momentum and position of the classical particle. Also, the parameter K characterizes the strength of the periodic perturbation and the second equation in (1) comes from the dispersion law of the free propagation between two kicks. For $K=L$, this model was investigated in [7], where it was used as a model for the motion of a particle in a plane perpendicular to a magnetic field under the influence of some electromagnetic wave.

An example of the motion in the symmetric case ($K=L=1.2$) is given in Fig. 1(a) where the main part of the phase plane is covered by invariant circles and unbounded motion only takes place in a narrow stochastic layer around the separatrix. By decreasing the common value of the parameters, the chaotic region becomes exponentially small (for $K=L=0.4$ this region is negligible). For a larger value of the parameters, the stochastic region becomes larger and for $K=L\simeq 5$, there is no visible island of stability: Diffusive excitation in p and x takes place.

Figure 1(b) shows an example of the motion in phase space for the nonsymmetric case ($K=1.2$, $L=1$). The characteristic feature of the motion in this case, where the values of the parameters are not too large, is the existence of vertical invariant curves which restrict the motion in the x direction but allow unbounded motion in the momentum space p. As the value of the parameters increases, these curves are destroyed, allowing diffusion in the x direction, and for values of the parameters around $L\simeq 2$ and $K\simeq 4$, islands of stability are no longer visible. In such a situation, the motion is characterized by two diffusion coefficients in the p and x directions and the larger coefficient corresponds to the larger value of the two parameters. Notice that for small p and $KL=$const,

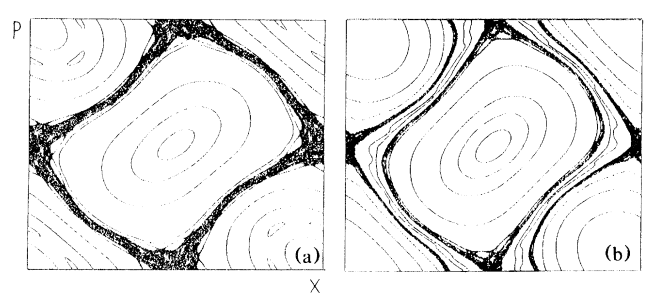

FIG. 1. Phase plane ($2\pi\times 2\pi$) of map (1) for parameter values of (a) $K=L=1.2$ and (b) $K=1.2$, $L=1$.

with $K \ll 1$, the motion is described by the standard map [8].

The quantized motion of model (1) is described by the following Hamiltonian:

$$H = L\cos(\hbar\hat{n}) + K\cos(x)\delta_1(t) , \qquad (2)$$

where we used units for which $\hbar$ is the dimensionless Planck constant, $\hat{n} = -id/dx$, $p = \hbar n$, and $\delta_1(t)$ is a periodic delta function of period 1.

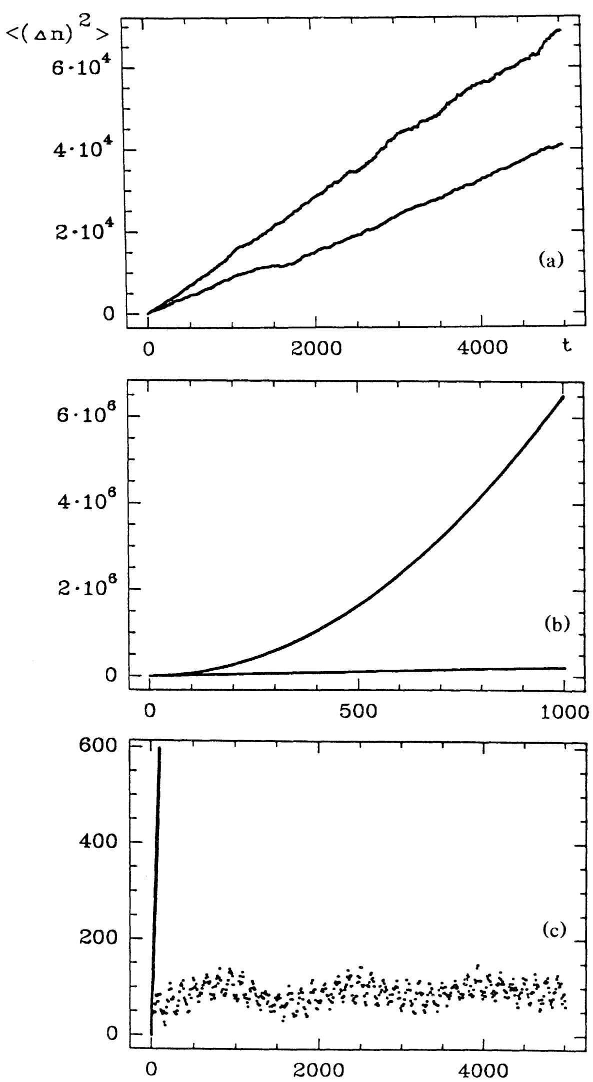

FIG. 2. Square width $\langle(\Delta n)^2\rangle$ for the momentum distribution $n = p/\hbar$, vs the number of iterations t in classical and quantum cases. (a) $K = L = 5$; the upper curve corresponds to the classical case, and the lower one to the quantum case with $\hbar = 2\pi/7.61803\ldots$. (b) $K = 4$, $L = 2$; the lower curve corresponds to the classical case, and the upper one to the quantum case with $\hbar = 2\pi/25.61803\ldots$. (c) $K = 2$, $L = 4$, the solid line corresponds to the classical case, and the dots to the quantum case with $\hbar = 2\pi/25.61803\ldots$. Initially $n = 0$. In all cases $\beta = 0$.

The corresponding evolution operator for one period is given by

$$\hat{U} = \exp\{-iL/\hbar\cos[\hbar(\hat{n}+\beta)]\}\exp[-K/\hbar\cos(x)] , \qquad (3)$$

where β is a quasimomentum for a wave propagating in the x direction. Because of the conservation of quasimomentum, the quasienergy eigenfunctions of the Hamiltonian (2) have the form $\exp(i\beta)\psi_\nu$, where functions ψ_ν are periodic in x. Therefore the operator $\hat{n}$ in $\hat{U}$ has only integer eigenvalues corresponding to unperturbed ($K = 0$) energy levels. The equation for quasienergy eigenfunction ψ_ν has the form $U\psi_\nu = \exp(-i\nu)\psi_\nu$, from where the quantum-dynamical model can be mapped [3,5] in a 1D solid-state model with incommensurate potential (for irrational values of $\hbar/2\pi$).

The quasiclassical limit of (3) is obtained when $\hbar \to 0$ for L,K fixed. Since for incommensurate solid-state models it is known [9] that delocalization can occur, the unbounded excitation for model (3) is not *a priori* excluded.

We carried out a numerical investigation of the model using the well-known procedure of [2,5]. The different types of behavior observed are shown in Fig. 2 for the classical parameter values for which no islands of stability were observed.

For the case where $K = L$, diffusive excitation takes place [Fig. 2(a)]. For $\hbar < 1$ the value of the quantum diffusion rate $D_q = (\hbar\Delta n)^2/\Delta t$ is of the same order as the classical value D_c. The probability distribution over the unperturbed levels, shown in Fig. 3, has a Gaussian form and so corroborates the diffusive nature of the excitation process. However, the fitting of the distribution by a Gaussian law also allows us to determine the diffusive and localized components, W_d and W_l, in the ψ function ($W_d + W_l = 1$). Numerically the value of W_d is determined as the coefficient corresponding to the Gaussian distribution in n which fits $W(n)$ in the tail and is normalized to unity (see also [1]). Since the values of both W_d and W_l are of the same order of magnitude (accord-

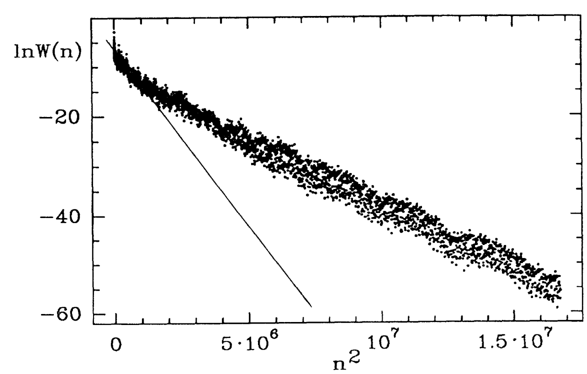

FIG. 3. Probability distribution $W(n)$ over unperturbed levels in the quantum case of Fig. 2(a) at $t = 5000$ (dots). The solid line is the classical Gaussian distribution corresponding to the diffusion rate of Fig. 2(a) at time $t = 5000$.

ing to Fig. 3 the value $W_d \simeq 0.2$), this suggests the possibility of a coexistence between the localized and the delocalized states.

The partial agreement between quantum and classical diffusion coefficients extends to values of $K=L$ larger than 1. For smaller values, the measure of the chaotic region in the classical model is very small and $D_c \simeq 0$. The value of the classical diffusion rate was computed by averaging over 1000 trajectories homogeneously distributed on a line with a constant value of the momentum p, corresponding to the quantum value. For the corresponding quantum case a finite diffusion rate was still observed and it has been found to be rather independent of the classical parameter values. For instance, if $K=L=0.4$, $D_q \simeq 0.02$ for $\hbar \simeq 0.06$. We think that the reason for such diffusion is connected with quantum jumps between different regions of stability separated by exponentially narrow chaotic layers.

For $K>L$ we always observed a quadratic growth in time of the square width $\langle(\Delta n)^2\rangle$ of the probability distribution over the unperturbed levels. A typical example is presented in Fig. 2(b). We would like to stress the fact that, at the same time, the classical motion is completely chaotic and has diffusive excitation. Therefore, contrary to previously studied models, quantization here leads to an excitation which is more effective than the corresponding classical one for a wide range of the parameters. It is important to underline that the origin of this effect is different from the well-known quantum resonance in the kicked rotator [2] since in our case there is no periodicity of $\hat{U}$ in the unperturbed level number n and the quadratic growth takes place for typical values of the parameters.

Our numerical investigations for values of $\hbar \simeq 1$ and the ratio $K/L \simeq 2$ have shown that the increase of the nonlinearity parameter K does not lead to an increase of the quadratic growth rate, $\gamma \lesssim \langle(\Delta n)^2\rangle/t^2$. For example, for $\hbar = 2\pi/7.61803$ and $K/L=2$, the value of γ remains approximately constant ($\gamma \simeq 3$) while the value of K varies over a wide range from 4 to 48.

We think that the reason for such a fast excitation can be understood qualitatively in the following way: As is seen from Fig. 1(b) in the case $K>L$, invariant curves appear only in the vertical direction. When such curves are present, the quadratic growth clearly takes place in both the classical and quantum cases. As the classical parameters K and L increase, such invariant curves are destroyed, but each one is then replaced by a cantorus which has zero measure in the classical case and leads to a transition to a diffusive regime (after destruction of all invariant curves). But, in the quantum case, the finite value of $\hbar$ leads to the existence of eigenfunctions which are concentrated near this invariant set. Such eigenstates, in a sense, are "scars," which are under extensive investigation at the present time (see [10]). But, in our case, these states are delocalized and since the average value of $\sin(x)$ is constant, they lead to a quadratic growth of the momentum. As the nonlinear parameter K increases (for $K/L = \text{const} > 1$), this quadratic growth becomes relatively weaker since the rate of growth γ remains approximately constant while the diffusion-rate growth is proportional to K^2.

This picture is consistent with the fact that for $K<L$, we also observe cases where the classical motion is completely chaotic but the quantum counterpart leads to a localization. In this case, since the cantori are horizontal, they may act as barriers for the diffusion in the quantum case [5,11,12], finally leading to the localization of the excitation; this is probably the case presented in Fig. 2(c). According to this interpretation, the cantorus may become penetrable as $\hbar$ decreases and then lead to the corresponding delocalization. We have indeed observed such an effect, for example, when $K=2$, $L=4$, $\hbar=2\pi/49.61803$. In some cases, the quantum delocalization is still present for $K<L$ and $\hbar \simeq 1$.

In conclusion we may say that, in the present work, we studied a model in which the quantization of chaos does not localize the excitation but instead leads to both a more efficient excitation and the existence of delocalized states. Such an excitation corresponds to an infinite diffusion rate and can therefore be interpreted as the existence of "superconductive," or ballistic, states for the corresponding solid-state model. These properties may be of some interest in studying incommensurate potentials in solid-state physics. We hope that some enlightening contribution from this field may give an explanation for the diffusive excitation in the symmetric case of map (1).

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Note added.—It is interesting to notice that the quantum model (2) can be considered as a kicked version of the Harper model [13]. After we submitted this Letter, an interesting paper [14] appeared in which diffusive and quadratic excitations have been observed in the Harper model. From our viewpoint the important difference between these two models is that the classical limit of the kicked Harper model (2) is chaotic (for $K,L>1$) while the motion in the Harper model, in this limit, is integrable.

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