Theory of Quantum Conduction of Supercurrent through a Constriction

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The dc Josephson current through a constriction in a two-dimensional superconductor-semiconductor-superconductor junction is calculated. It is shown that when the Fermi wavelength is comparable with the width of the constriction, the critical current shows a steplike variation as a function of the width of the constriction; this is reminiscent of the quantization of the normal-state conductance of point contacts in a two-dimensional electron gas.

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The hybrid Josephson field-effect transistor or superconducting transistor is one of the most promising candidates for three-terminal superconducting devices [1]. Its operation depends on the proximity coupling of two superconducting electrodes by the transport of Cooper pairs through a normal inversion layer or accumulation layer in a semiconductor. Several groups have realized prototypes and confirmed the switching of supercurrent by the field effect [2–4]. Most prototypes realized up to now are dirty systems such that the motion of Cooper pairs is diffusive due to impurity scatterings [5]. In the near future, however, with the progress in the techniques of microfabrication, it is likely that high-mobility superconducting transistors will be possible. These will provide the first examples of superconducting mesoscopic systems, where the phase coherence of not only the Cooper pairs but also of any pair of electrons will be maintained throughout the system. This additional phase coherence of ordinary electron pairs may introduce new features in the conduction of the supercurrent, and it is clearly of interest to explore the properties of such mesoscopic superconducting systems, where the motion of electrons as well as Cooper pairs is ballistic. In particular, when the Fermi wave length in the semiconductor is comparable to the system size, new quantum effects may be observed as a consequence of the quantum-mechanical nature of an individual electron. A superconducting transistor with a constriction in a normal inversion layer is not only a possible candidate for testing these ideas, but may also have significant implications in future technology. By an analogy with the quantum point contact (QPC), where such a new effect, i.e., the quantization of conductance in units of $2e^2/h$, has recently been observed [6–8], we will call such a superconducting transistor the superconducting quantum point contact (SQPC). It is of interest to note that the SQPC's may be realized by putting additional gates on the two-dimensional inversion layer, in the same way as in the QPC's.

In this Letter, we study the static Josephson effect of SQPC's in the ballistic regime using a simple two-dimensional model. Emphasis will be put upon how the discreteness of the energy levels of quasiparticles at the constriction affects the Josephson current at low temperatures. The problem of conduction of supercurrent through small numbers of channels has largely been restricted to resonant tunneling through localized impurity levels in dirty superconducting–doped-semiconductor–superconductor junctions [9]. It has rarely been discussed in the case of clean systems [10]. We demonstrate that a steplike change in the critical current may be observed in SQPC's under appropriate conditions as a function of the width of the constriction or of the carrier density, each of which can be controlled by a corresponding gate bias. This feature is the counterpart of the quantization of the normal-state conductance in QPC's [6–8]. However, there are several differences between these two phenomena; in SQPC's the steplike change is not always observed and, even when it is realized, the magnitude of the step is not universal.

The Josephson effect in superconductor-semiconductor-superconductor (S-Sm-S) junctions can be thought of as a combination of two processes (Fig. 1). The first one is the penetration of Cooper pairs through two S-Sm interfaces from a superconductor to a semiconductor and vice versa. This is simply the proximity effect. The other process is the transport of two electrons in the semiconductor from one S-Sm interface to the other while keeping their phase memory as a Cooper pair. Here we shall focus on the latter process, i.e., the communication of the superconducting phase coherence through small numbers of

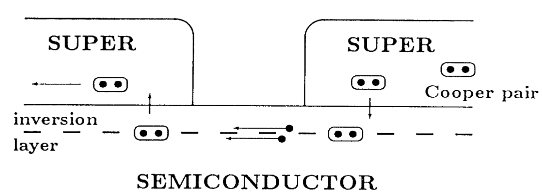

FIG. 1. A schematic picture of a superconducting transistor. The Josephson effect is a combination of the two processes: the penetration through the S-Sm interfaces and the conduction in the inversion layer. The gates by which the proximity coupling is controlled are not depicted in this figure.

open channels in the SQPC, and we are not concerned with the details of the proximity effect around the S-Sm interfaces. We adopt the following approximations to simplify the analysis, and to emphasize the physics involved. We treat the SQPC's as two-dimensional systems, in which the pair potential $\Delta(x,y)$ is assumed to be constant in the superconducting regions [Δ for $x<0$ and $\Delta\exp(i\varphi)$ for $x>L$], and vanishing in the normal region; see inset of Fig. 2. Moreover, we assume that the motion of quasiparticles of effective mass m is ballistic, and that reflections at the S-Sm interfaces are neglected, except for Andreev reflections [11]. The static Josephson current is calculated from the probability amplitudes of Andreev reflections, based on a recently developed formulation [12].

We shall begin with a qualitative discussion of the dc Josephson effect in the SQPC's within the adiabatic approximation. That is, the width of the constriction $W(x)$ is assumed to vary smoothly compared with the Fermi wavelength λ_F. The wave function of the quasiparticles belonging to the jth channel can be written as

$$\psi_j(x,y)=\left[\frac{2}{W(x)}\right]^{1/2}\sin\left[\frac{j\pi[y+\frac{1}{2}W(x)]}{W(x)}\right]\times\phi_j(x)\,, \tag{1}$$

where $\phi_j(x)$ is a (two-component) vector wave function. Solving the Bogoliubov–de Gennes equation [13], we obtain the probability amplitude for the Andreev reflection of an electronlike quasiparticle of energy E, injected from the left-hand side through the jth channel, as

$$a_j(\varphi,E)=\frac{\Delta\{\exp(i\Phi_j)-\exp(i\varphi)\}}{(E+\Omega)\exp(i\varphi)-(E-\Omega)\exp(i\Phi_j)}\,, \tag{2}$$

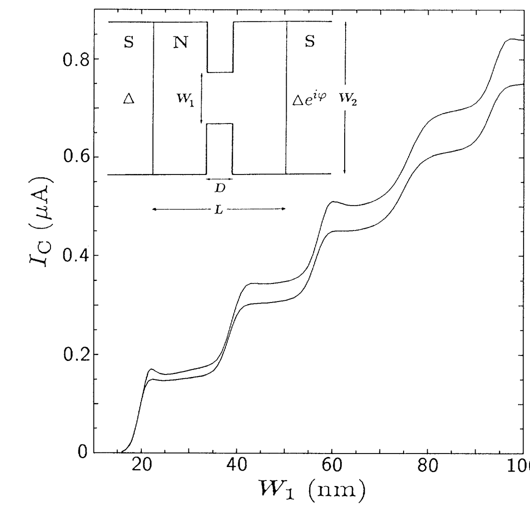

FIG. 2. Critical current as a function of the width of the constriction: The upper curve is for $T=0.5$ K and the lower curve is for $T=1.0$ K. Inset: Schematic of the model used in the numerical calculations.

where

$$\Phi_j(E)=\int_0^L dx\{k_j^+(x)-k_j^-(x)\}\,,$$
$$k_j^\pm(x)=\left[\frac{2m}{\hbar^2}(E_F\pm E)-\left(\frac{j\pi}{W(x)}\right)^2\right]^{1/2}, \tag{3}$$
$$\Omega=(E^2-\Delta^2)^{1/2}.$$

The dc Josephson current I can be calculated from the amplitude [12] by

$$I=\frac{e\Delta}{\hbar}\sum_j\frac{1}{\beta}\sum_{\omega_n}\frac{1}{\Omega_n}\{a_j(\varphi,i\omega_n)-a_j(-\varphi,i\omega_n)\}=-\frac{2e\Delta^2}{\hbar\beta}\sum_j\sum_{\omega_n}\frac{\sin\varphi}{(2\omega_n^2+\Delta^2)\cosh\tilde{\Phi}_j+2\omega_n\Omega_n\sinh\tilde{\Phi}_j+\Delta^2\cos\varphi}\,, \tag{4}$$

where $\tilde{\Phi}_j=-i\Phi_j(E\to i\omega_n)$, ω_n is the Matsubara frequency, $\pi(2n+1)/\beta$, and $\Omega_n=(\omega_n^2+\Delta^2)^{1/2}$. Equation (4) shows that the contributions of tunnel current through the closed channels are negligibly small and that the effect of the discreteness of the energy levels may not be seen in the Josephson current so clearly as in the normal-state conductance. In the case $L\gg\xi=\hbar v_F/\pi\Delta$, an approximate expression of the critical current I_c at zero temperature is obtained as

$$I_c\approx\frac{2e}{\pi L}\sum_{j=1}^{M}v_j\,, \tag{5}$$

where

$$v_j=\left[\frac{1}{L}\int_0^L dx\frac{m}{\hbar k_j(x)}\right]^{-1},$$
$$k_j(x)=\left[\frac{2mE_F}{\hbar^2}-\left(\frac{j\pi}{W(x)}\right)^2\right]^{1/2}, \tag{6}$$

and M is the largest integer to make v_j a real number [14,15]. The above expression, Eq. (5), can be understood as a product of $2e\Delta/\hbar$ and $\xi_j/L=\hbar v_j/\pi\Delta L$, and the latter factor indicates to what extent the phase memory of Cooper pairs can be preserved in the jth channel. This result is a natural extension of the well-known expression for the three-dimensional superconductor–normal-metal–superconductor junctions [16,17] to the case of SQPC's [18]. One can see from Eq. (5) that the critical current is entirely dependent on junction parameters, v_j and L, while the quantized magnitude of the normal-state conductance of QPC's is just a universal quantity, $2e^2/h$. Furthermore, it depends on the geometry of the system as to whether I_c shows steplike behavior as the number of open channels M is changed. For example, when the variation of $W(x)$ is sufficiently smooth, a change in the number of open channels may not be seen clearly from the critical current. We shall demonstrate this in the following calculations.

Next we examine the opposite situation in which the width changes abruptly between the wide and narrow regions (see inset of Fig. 2). The normal region of total length L consists of a narrow region of width W_1 and length D $(L>2D)$, and two wide regions of width W_2. The sample geometry is defined by hard-wall boundary conditions. The critical current is calculated numerically as a function of the width W_1 or the carrier density N which is assumed to be common to the superconducting and normal regions.

A wave function corresponding to an electronlike quasiparticle injected from the left superconductor is written as

$$\psi(x,y)=e^{ip_m^+x}\sin\{q_m(y+W_2/2)\}\begin{pmatrix}u\\ v\end{pmatrix}+\sum_{n=1}^{\infty}\left\{a_{mn}e^{ip_n^-x}\begin{pmatrix}v\\ u\end{pmatrix}+b_{mn}e^{-ip_n^+x}\begin{pmatrix}u\\ v\end{pmatrix}\right\}\sin\{q_n(y+W_2/2)\}\,, \tag{7}$$

if $x\le 0$, and

$$\psi(x,y)=\sum_{n=1}^{\infty}\left\{c_{mn}e^{ip_n^+x}\begin{pmatrix}ue^{i\varphi/2}\\ ve^{-i\varphi/2}\end{pmatrix}+d_{mn}e^{-ip_n^-x}\begin{pmatrix}ve^{i\varphi/2}\\ ue^{-i\varphi/2}\end{pmatrix}\right\}\sin\{q_n(y+W_2/2)\}\,, \tag{8}$$

if $x\ge L$, where

$$p_n^{\pm}=\left[\frac{2m}{\hbar^2}(E_F\pm\Omega)-q_n^2\right]^{1/2},\quad q_n=\frac{n\pi}{W_2},\quad u=\left[\frac{1}{2}\left(1+\frac{\Omega}{E}\right)\right]^{1/2},\quad v=\left[\frac{1}{2}\left(1-\frac{\Omega}{E}\right)\right]^{1/2}. \tag{9}$$

The coefficients a_{mn}, b_{mn}, c_{mn}, and d_{mn} are determined by the matching conditions at the superconductor-normal and wide-narrow interfaces. The dc Josephson current is related to $a_{mm}(\varphi,E)$ by

$$I=\frac{e\Delta}{2\hbar}\sum_m\frac{1}{\beta}\sum_{\omega_n}\frac{1}{\Omega_n}\left(1+\frac{p_m^-}{p_m^+}\right)\{a_{mm}(\varphi,i\omega_n)-a_{mm}(-\varphi,i\omega_n)\}\,. \tag{10}$$

The details of the calculations will be published elsewhere.

The results of the critical current are shown in Fig. 2 as a function of W_1 for a fixed carrier density ($N=5\times10^{11}$ cm^{-2}). We set $L=100$ nm, $D=20$ nm, $W_2=120$ nm, $\Delta(T=0\text{ K})=1$ meV, and $m=0.024m_e$, where m_e is the electronic mass [19]. With this carrier density, the increase of W_1 by an amount $\lambda_F/2=18$ nm amounts to the opening of another channel. A steplike change can be seen clearly, especially for small values of W_1. The structures are less obvious for large values of W_1. One can also see that each step has a finite slope, which means that the critical current is not quantized in a rigorous sense, as we mentioned in association with the adiabatic approach.

The critical current is also calculated as a function of the carrier density N. As expected, it changes in rather different ways according to the ratio of W_1/W_2 ($W_2=120$ nm) as shown in Fig. 3. The curve is almost steplike for $W_1/W_2=1/3$, while it appears more as a straight line with some structure for $W_1/W_2=2/3$.

The results shown in Figs. 2 and 3 show that the discreteness of the energy levels at the constriction leads to measurable steplike variations of order 0.2 μA in the critical current, which may be observed in future high-mobility superconducting quantum devices. In real systems, however, there always exist Schottky barriers at the S-Sm interfaces, which reduce the density of the Cooper pairs in the semiconductor [4]. Thus the critical current decreases by a factor of P^2, where P is the transmission probability through the barrier. This effect may be included by using a renormalized $P\Delta$ in our theory. Therefore, in order to observe the steplike feature of the critical current, it is necessary to find a good combination of a superconductor and a clean semiconductor.

In summary, we have studied the dc Josephson effect in SQPC's in the ballistic regime using simple two-dimensional models. We have found that in some cases the

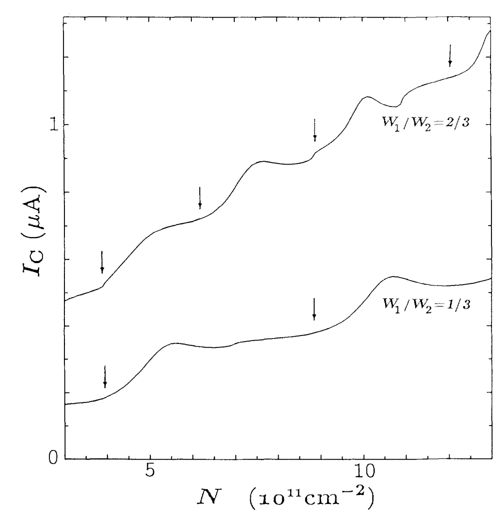

FIG. 3. Critical current as a function of the carrier density. The arrows indicate the densities at which another channel opens.

critical current shows a characteristic feature due to the discreteness of energy levels around the constriction. This will be a common feature of phase-coherent ballistic conduction of supercurrent through a constriction in superconducting quantum devices.

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