# Initial Entropy Generation in Ultrarelativistic Nuclear Collisions

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Entropy generation during an ultrarelativistic nuclear collision is discussed. It is shown that the maximum entropy densities attained are limited by the time during which the collision takes place. This provides a strong upper limit on the maximum entropy density attained in a collision of given multiplicity density.

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The production of high-temperature matter in nuclear collisions is a topic of considerable interest. Studies of this matter have important implications for subjects ranging from the behavior of supernovae to the history of the very early Universe. Typically, the highest densities are achieved very early in these collisions, and conclusions depend very strongly on the assumptions about the initial stages of the collision. In this Letter, I discuss the physics of the very earliest stages of ultrarelativistic nuclear collisions.

I extend the work of Anishetty, Koehler, and McLerran,[1] by considering the actual process of entropy formation during the collision. This occurs over a scale of about 1 fm, which can be much longer than the equilibration time for the hot matter. In ultrarelativistic collisions, this entropy formation process provides the strongest limits on the entropy densities that are achieved.

In order to interpret experimental results, the common practice is to extrapolate the entropy density $s$ from the initial proper time $\tau_0$ to proper time $\tau$ using the Bjorken formula[2]

$$s(\tau)=\tau_0 s(\tau_0)/\tau . \tag{1}$$

This extrapolation is equivalent to assuming that entropy is conserved during the expansion of the hot matter, as the volume of the system is approximately $c\tau\sigma$, where $c$ is the speed of light and $\sigma$ is the cross section of the nucleus. The initial entropy density, in the central region with rapidity $y=0$, can then be estimated from the (observed) final particle rapidity density $dN/dy$:

$$s(\tau_0)=\frac{3.7dN/dy}{c\tau_0\sigma} . \tag{2}$$

This estimate is made by assuming that the particles are in the form of a massless (or ultrarelativistic) ideal gas, so that each particle carries 3.7 units of entropy.

In the picture of Anishetty, Koehler, and McLerran, the entropy is all generated at $\tau=0$. This occurs because they assume that the entropy is generated mainly by hard parton collisions that occur on a very short time scale. They thus find that $\tau_0$ is the time for the system to thermalize, which they estimate to be 0.5 fm.

The dominant mechanism for entropy generation in nuclear and hadronic collisions is the interaction of partons with rapidity differences of the order of 1, as discussed by Casher, Kogut, and Susskind.[3] As these partons have similar rapidities, the particles (and thus entropy) that they produce will have approximately the same rapidity as the mother partons. The entropy density in the central $(y=0)$ region will therefore be produced mainly through the interaction of partons with rapidities of order 1, the so-called wee partons. I assume in this Letter that all entropy is generated by these wee-parton interactions.

These partons are not compressed by a factor $1/\gamma$ as the nucleus is accelerated to high energy. Instead, as they always have the same rapidity (and hence momentum $p$), their wave functions extend over a region of size $\lambda \sim 1/p$ around the nucleus. The value of $\lambda$ is not too well known, but it is usually estimated to be about 1 fm (a typical hadronic size). I assume here that the wee-parton wave functions are constant over a box of width $\lambda$ surrounding the nucleus.

At time $t=0$, the nuclei begin to interact. The local entropy production rate is proportional to the wee-parton interaction rate, which is proportional to the product of the wee-parton wave functions. The rate of entropy growth is thus proportional to the volume overlap of the boxes (of size $\lambda$) surrounding the two nuclei, as the parton wave functions are constant inside this region. The entropy $S$ at time $t$ is then

$$S(t)=\begin{cases} k\sigma t^2, & 0<t<\lambda/2c, \\ k\sigma[-t^2+2(\lambda/c)t-(\lambda/c)^2/2], & \lambda/2c<t<\lambda/c, \end{cases} \tag{3}$$

where $\sigma$ is the interaction cross section and $k$ is an arbitrary constant.

This entropy is distributed over a volume of roughly $c\sigma t$, following the Bjorken scaling law (1). The entropy density as a function of time is thus

$$s(t)=\begin{cases} (k/c)t, & 0<t<\lambda/2c, \\ (k/c)[-t+2(\lambda/c)-(\lambda/c)^2/2t], & \lambda/2c<t<\lambda/c. \end{cases} \tag{4}$$

To obtain the entropy density as a function of proper time, I simply replace $t$ with $\tau$, as time and proper time are equivalent at $y=0$. This result is similar to that estimated by Kajantie, Raitio, and Ruuskanen.[4]

This analysis is somewhat incomplete, however, as I have assumed that entropy is produced instantaneously. That is not true, however—particles of energy $E$ are produced in a finite amount of time $\tau_f \sim 1/E$, and the entropy is then produced with a time scale of $\tau_c$ (the mean time between collisions). I then find that

$$s(\tau) = S(\tau - \tau_s)/\sigma c \tau \, , \tag{5}$$

where $\tau_s = \tau_f + \tau_c$ is the time for entropy formation and $S(\tau)$ is given by Eq. (3) with $t$ replaced by $\tau$. The value of $\tau_s$ can be estimated, following Ref. 1, to be about 0.5 fm, although this figure may be reduced for collisions of large nuclei due to the high density of produced particles.

Even in the most optimistic scenario where $\tau_s = 0$, the fact that the entropy production is dominated by soft interactions puts strong constraints on the maximum entropy density generated. Combining Eqs. (4) and (2) gives initially a linearly growing entropy density from $s(0)=0$ to $s(\lambda/2c) = 3.7(dN/dy)/\lambda\sigma \equiv s_\lambda$. The density then varies only slightly until $\tau = \lambda/c$, reaching a maximum value of $s(\lambda/c\sqrt{2}) = (4-2\sqrt{2})s_\lambda$, and then dropping back to $s(\lambda/c) = s_\lambda$. From then on, the density is given by the Bjorken scaling law (1), as entropy production is finished and entropy is therefore constant.

This scenario will, of course, be modified in the case that higher-energy interactions are more important than wee-parton interactions. However, this appears unlikely to be the case at currently achievable energies. More careful studies should be done, with attention paid especially to the effects of minijets,[5] as they could contribute significantly to the entropy generation on time scales shorter than $\lambda/c$, and nuclear shadowing,[6] which could limit the density of wee partons in ultrarelativistic collisions.

It is also possible to attempt to put in the actual parton wave functions and nuclear shape, in order to get a better determination of the actual maximum density reached. It is clear, however, that the maximum density can never be much higher than $s_\lambda$, although that density may be achieved for a time of order $\lambda/c$. In any case, the time scale over which particle production occurs should continue to be included in models of ultrarelativistic collisions, so that entropy and particle densities are not overestimated.

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