Break Structure of Forward Peaks of Elastic pp and $\bar{p}p$ Scattering at High Energies

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We examine the implication of the unitarity bounds for the differential cross section to the break structure of the forward peak of elastic pp and $\bar{p}p$ scattering observed at the squared momentum transfer $|t|=0.1$–0.15 $(\text{GeV}/c)^2$ at high energies. The break is suggested to occur at around $8.8/R_0^2<|t|<14.7/R_0^2$, R_0 being the interaction radius given in terms of σ_t^2/σ_{el}. This fits well with the experimental data of pp scattering in the energy region $\sqrt{s}=10$–60 GeV and $\bar{p}p$ in 10–546 GeV. Scaling with the variable $t\sigma_t^2/\sigma_{el}$ is discussed.

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It was pointed out by Carrigan [1] two decades ago that the forward peak of the differential cross section of elastic pp scattering shows a break structure in the energy region $P_L=19$–30 GeV/c. The logarithmic slope b changes in the region of the squared momentum transfer $|t|\sim 0.15$ $(\text{GeV}/c)^2$ by an amount ~ 2 $(\text{GeV}/c)^{-2}$. Since then this break structure has been observed at higher energies, from below the energies of the CERN ISR to the energies of the CERN $Sp\bar{p}S$, for pp and $\bar{p}p$ scattering [2–4]. Similar curved structures of the forward peaks have been observed in $\pi^{\pm}p$ and $K^{\pm}p$ scattering [2]. The origin of the break, however, seems not to have been fully explored. Calculations of some theoretical models such as the Chou-Yang model [5] show a similar feature (in general a curved structure rather than a break). In most cases the structure is due to the curvature property of the input amplitudes as well as the effects of multiple scattering.

The purpose of this Letter is to examine this problem from the perspective of the unitarity bound to the differential cross section and to point out interesting relations between the unitarity bound and the structure of the forward peak.

We ignore the effects of spin and consider two spinless particles of equal masses scattered by some finite-range interaction. The center-of-mass system (c.m.s.) scattering amplitude $f(k,t)$ can be given in terms of the partial-wave amplitude $f_l(k)$ as

$$f(k,t)=\sum_{l=0}^{L}(2l+1)f_l(k)P_l\left(1+\frac{t}{2k^2}\right), \tag{1}$$

where k and t are the c.m.s. momentum and the squared momentum transfer, respectively, and L is the cutoff angular momentum arising from the finite interaction range. We define the interaction radius R_0 by $kR_0=L+1$.

By using the Bessel-function representation of the Legendre polynomials, we have the following asymptotic form at small $-t$ for the stationary values of the differential cross section from the expression derived by Arushanov when the total cross section (σ_t) and the elastic cross section (σ_{el}) are imposed as constraints [6,7]:

$$\frac{d\sigma^{(\pm)}}{dt}=\frac{\sigma_t^2}{16\pi}\{A(t)\pm\alpha B(t)\}^2, \tag{2}$$

with

$$A(t)=2J_1(R_0\sqrt{|t|})/R_0\sqrt{|t|},$$
$$B(t)=[\{J_0(R_0\sqrt{|t|})\}^2+\{J_1(R_0\sqrt{|t|})\}^2-A^2(t)]^{1/2}, \tag{3}$$

where J_0 and J_1 are the Bessel functions. The quantity α is given by

$$\alpha=[(x-x_0)/x_0]^{1/2}, \tag{4}$$

where x is the elasticity σ_{el}/σ_t and x_0 is $\sigma_t/4\pi R_0^2$. In the following analysis we use the Bessel-function representation (2) of the bounds, which is well justified in the energy and momentum-transfer region under consideration.

We obtain two stationary values of the differential cross section, $d\sigma^{(+)}/dt$ and $d\sigma^{(-)}/dt$, since the constraint by the elastic cross section is quadratic in the scattering amplitude. The extremal properties of these two stationary values were studied in Ref. [7]. At small momentum transfers there are three momentum-transfer points, t_α, t_β, and t_γ, which characterize the extremal properties.

The value $d\sigma^{(+)}/dt$ gives a saddle-point value between $t=0$ and t_α, an upper bound between $t=t_\alpha$ and t_γ, and beyond t_γ a saddle point again, while $d\sigma^{(-)}/dt$ gives a lower bound between $t=0$ and t_β, then a saddle point up to $t=t_\gamma$, and an upper bound beyond t_γ. The values of t_α, t_β, and t_γ are determined by the following conditions:

$$\alpha A(t)=B(t) \text{ at } t=t_\alpha, \tag{5}$$

$$A(t)=\alpha B(t) \text{ at } t=t_\beta, \tag{6}$$

and

$$A(t)=0 \text{ at } t=t_\gamma. \tag{7}$$

In the following analysis we are concerned with the upper bound. Our suggestion is that two points, t_β and t_γ, will be closely associated with the break or the curvature structure. This stems from the following simple observation. In Eq. (2) the amplitude $A(t)$ is that of the uniform grey disk, while the amplitude $B(t)$ represents the possible scope of departure from the uniform disk. The amplitude $A(t)$ is rapidly decreasing, with antibreak curvature, while $B(t)$ is increasing as t is passing the point t_β. The actual amplitude may also be decomposed into parts corresponding to these two components. Then, the component of nonuniform disk may develop structure around the point t_β where $A(t)=\alpha B(t)$. The second point t_γ is where the upper bound itself shows the break structure as will be shown later and corresponds to the zero of $A(t)$. Here the nonuniform component exposes itself fully. Our conjecture is, therefore, that the change of the t dependence will start to develop around t_β and be complete near t_γ.

Let us examine the plausibility of this conjecture. The essential factor which determines these momentum-transfer points is the interaction radius R_0. We have to take R_0 consistent with the partial-wave-unitarity condition. The strongest restriction comes from the condition at $t=t_\alpha$ and we take R_0 such that $\mathrm{Im} f_L=0$ there, L being the maximum angular momentum [8]. This implies

$$P_L(1+t/2k^2)\approx J_0(R_0\sqrt{|t|})=0 \text{ at } t=t_\alpha . \quad (8)$$

The first zero $j_{0,1}$ of the Bessel function J_0 gives

$$R_0|t_\alpha|^{1/2}=j_{0,1}\approx 2.405 . \quad (9)$$

In this case Eq. (5) takes a simple form

$$|t_\alpha|=16\pi x/\sigma_t . \quad (10)$$

Equations (9) and (10) determine t_α and R_0. Furthermore, α assumes a value

$$\alpha=[R_0^2|t_\alpha|/4-1]^{1/2}\approx 0.668 . \quad (11)$$

The smallest root $|t_\beta|$ of Eq. (6) for α of (11) is given by

$$R_0^2|t_\beta|\approx 8.756 . \quad (12)$$

The point t_γ is given by the first zero of $A(t)$ as

$$R_0^2|t_\gamma|\approx 14.68 . \quad (13)$$

We show an example of $d\sigma^{(+)}/dt$ and also $d\sigma^{(-)}/dt$ divided by $\sigma_t^2/16\pi$ at small momentum transfers in Fig. 1, where the solid parts of the curves represent either the maximum or the minimum limits, while the dashed parts give the saddle points. Here we have taken $\sigma_t=61.9$ mb, $x=0.215$, and $R_0=1.82$ fm. The values of the cross section and the elasticity are those of $\bar{p}p$ scattering at $\sqrt{s}=546$ GeV for which differential cross-section data [4] are also shown in Fig. 1. The value $R_0=1.82$ fm is given by Eqs. (9) and (10).

From the experimental data of the total and elastic cross sections, we have evaluated the values of R_0 in the energy region $\sqrt{s}=10$–60 GeV for pp and 10–1800 GeV for $\bar{p}p$, and these are shown in Table I together with $R_b\equiv 2\sqrt{b}$ [9], where b is the forward slope. Here we have used the values of the empirical fit to the total and elastic cross sections given by the Particle Data Group [10] and the values of R_b are the results of our fit to the experimental values of slope data.

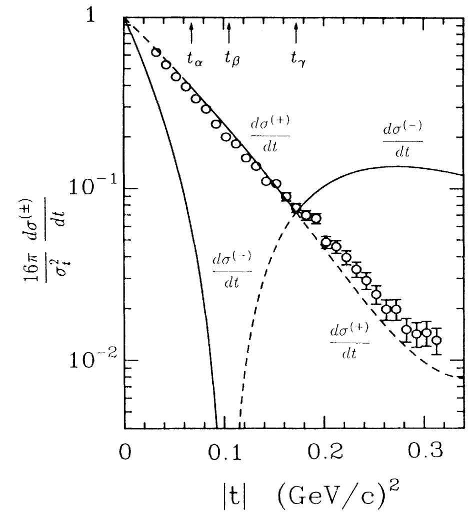

FIG. 1. Behavior of $d\sigma^{(\pm)}/dt$ divided by $\sigma_t^2/16\pi$ for the parameters $\sigma_t=61.9$ mb, $x=0.215$, and $R_0=1.82$ fm. We also show the experimental data of the differential cross section of $\bar{p}p$ scattering at $\sqrt{s}=546$ GeV [4] normalized in the same way.

The break structure in the region $|t|=0.1$–0.15 $(GeV/c)^2$.—As was noted above, we expect the break to occur between t_β and t_γ. Though it is difficult to determine uniquely the breaking point by experiment, a

TABLE I. The interaction radius R_0 in units of fm and the break points t_β and t_γ in units of $(\mathrm{GeV}/c)^2$ together with the radius $R_b\equiv 2\sqrt{b}$. The c.m.s. total energy $\sqrt{s}$ is in units of GeV.

	$\sqrt{s}$	R_0	$\lvert t_\beta\rvert$	$\lvert t_\gamma\rvert$	R_b
pp	10	1.51	0.149	0.250	1.28
	20	1.60	0.133	0.224	1.34
	30	1.64	0.127	0.213	1.38
	40	1.66	0.124	0.208	1.41
	50	1.68	0.122	0.204	1.43
	60	1.69	0.120	0.201	1.44
$\bar{p}p$	10	1.48	0.156	0.261	1.40
	20	1.60	0.133	0.223	1.42
	30	1.65	0.125	0.210	1.44
	40	1.68	0.121	0.203	1.45
	50	1.69	0.119	0.199	1.46
	60	1.71	0.117	0.196	1.47
	200	1.77	0.109	0.182	1.51
	546	1.82	0.103	0.173	1.55
	1800	1.89	0.096	0.160	1.59

change of the slopes has been observed in the region $|t|=0.1\text{–}0.15$ $(\mathrm{GeV}/c)^2$ with a tendency to move forward as energy increases. The experimental data of the differential cross section indicate that the break looks as though it appears around t_β in the region $|t_\beta| \le |t| \le |t_\gamma|$. These features of the experimental data are quite consistent with the results for t_β and t_γ shown in Table I, supporting our conjecture.

Scaling with $R_0^2 t$.—The bound structure is determined by the radius $R_0 = j_{0,1}(\sigma_t/16\pi x)^{1/2}$. This leads us to propose the scaling of the experimental data with the variable $R_0^2 t$. The success of the scaling with the dimensional parameter σ_t in the ISR region and the failure in the S$\bar{p}p$S region are well known. For example, σ_t/b is constant in the ISR region, but rising at the S$\bar{p}p$S energies [11]. If we take σ_t/xb instead, then the constant behavior is recovered. In a sense this is almost trivial, as the forward slope b is given by

$$b = \sigma_t/16\pi x \, , \tag{14}$$

in the purely imaginary exponentially decreasing amplitude. Incidentally t_α^{-1} is equal to b of Eq. (14), though it is not clear whether this is mere coincidence or not.

To see this scaling feature, we plot the experimental data of $\bar{p}p$ scattering [3,4,12] with respect to $\sigma_t|t|/x$ in Fig. 2. If we overlay $d\sigma^{(+)}/dt$, we find the experimental data cluster just below the upper bound $d\sigma^{(+)}/dt$ in the region $|t_\beta| \le |t| \le |t_\gamma|$ as they should. Beyond t_γ the cross sections do not cluster along a single curve. The scaling hypothesis seems to hold for the differential cross section up to $\sqrt{s}=1800$ GeV at small momentum transfers $|t| \le |t_\gamma|$. If we look at the experimental data more closely, however, the observed differential cross sections at 1800 GeV have no break structure: The present data decrease exponentially up to $|t| \approx 0.6$ $(\mathrm{GeV}/c)^2$ without showing any noticeable structure [12]. Therefore, the scaling is only approximate. The point t_γ is interesting. The experimental differential cross sections pass over $d\sigma^{(+)}/dt$ around this point at all energies in the range $\sqrt{s}=10\text{–}1800$ GeV. Measurements of the pp and $\bar{p}p$ data at energies higher than 1800 GeV are very interesting in order to see whether the exponentially decreasing cross section will become convex or not and whether the point t_γ will still be the crossing point.

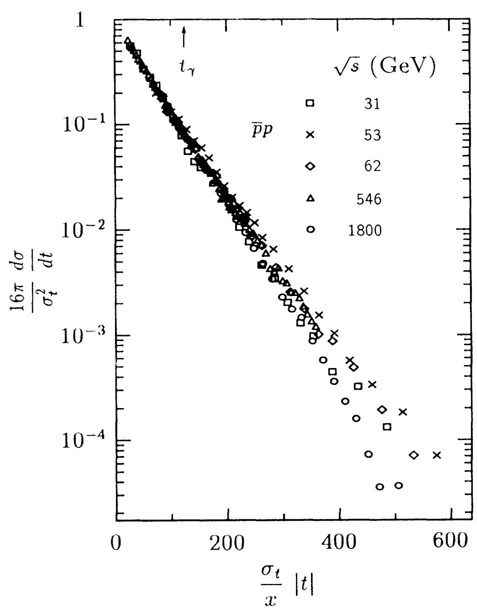

FIG. 2. Scaling behavior of the differential cross section $d\sigma/dt$ divided by $\sigma_t^2/16\pi$. The observed differential cross sections of $\bar{p}p$ scattering [3,4,12] are plotted against the scaling variable $\sigma_t|t|/x$.

The difference between R_0 and R_b can be considered as representing roughly the size of the surface region. If we take the approximate value (14) for b, then the ratio R_0/R_b is 1.20. The surface region is proportional to the interaction radius. Roughly this is a feature shared by the Chou-Yang model. The experimental values of the ratio are nearly constant at 1.18–1.19 for pp scattering and weakly increasing from 1.06 at 10 GeV to 1.19 at 1800 GeV for $\bar{p}p$ scattering.

In this paper we have discussed the implication of the unitarity bounds to the differential cross section in pp and $\bar{p}p$ scattering. The interaction radius R_0 is determined uniquely by (9) and (10) in terms of σ_t and σ_{el}. The break is suggested to be between t_β and t_γ which specify the unitarity bound structures, and we have found consistency of this expectation with the experimental data. The observed cross sections lie very closely to the one of the extremal solution, $d\sigma^{(+)}/dt$, indicating the near saturation of the unitarity limits. The scaling hypothesis with the variable $\sigma_t t/x$ has been examined and has been found to be consistent with the experiments up to 546 GeV. There may be a change in the structure of the forward peak around 1 TeV from concave to convex curvature. The differential cross section seems to cross over $d\sigma^{(+)}/dt$ at $t=t_\gamma$ at all energies up to 1.8 TeV. It will be interesting to see what changes occur in the pp and $\bar{p}p$ differential cross sections in the S$\bar{p}p$S-Fermilab Tevatron-CERN Large Hadron Collider-Superconducting Super Collider energy region.

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[8] It might be considered that no upper limit will exist for R_0, but the optimal values of the partial-wave amplitudes $\mathrm{Im} f_l^{(+)}$ giving $d\sigma^{(+)}/dt$ become negative at large l near L for R_0 larger than some critical value. This critical value is given by the condition (8).

[9] In general b is one-half the average value of the impact parameter squared as weighted by the scattering amplitude in the impact-parameter representation.

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