Global Monopoles Do Not "Collapse"

An isolated static global monopole [1] has a spherically symmetric energy density. Goldhaber [2] has recently discussed a family of monopoles with conical symmetry and argued that global monopoles have an angular instability. He introduced a new coordinate $y=\ln[\tan(\theta/2)]$ to replace the polar angle and obtained an expression for the total energy

$$E=\int(\rho_1+\rho_2)dr\,dy\,d\varphi\,, \tag{1}$$

where

$$\rho_1=\frac{|\phi|^2}{2}\left[\sin^2\bar\theta+\left(\frac{\partial\bar\theta}{\partial y}\right)^2+\frac{r^2}{\cosh^2 y}\left(\frac{\partial\bar\theta}{\partial r}\right)^2\right], \tag{2}$$

$$\rho_2=\frac{1}{2}\left(\frac{\partial|\phi|}{\partial y}\right)^2+\frac{r^2\lambda}{4\cosh^2 y}(|\phi|^2-\eta^2)^2. \tag{3}$$

ϕ is an isovector Higgs field, with a vacuum expectation value η, and $\bar\theta$ is the polar component of ϕ in a spherical coordinate system. Goldhaber noted that if $|\phi|=\eta$ is a constant and $\partial\bar\theta/\partial r$ vanishes, then Eq. (2) is just the energy functional for the sine-Gordon equation. This suggests that there is a zero mode with corresponds to the translation (in y) of the sine-Gordon soliton: $\bar\theta(y;y_0)=2\times\arctan(e^{y-y_0})$. The soliton with $y_0=0$ describes the spherically symmetric monopole configuration, and a translation of this soliton to large $-y$ is the first step of Goldhaber's instability.

Except for the spherical ($y_0\equiv 0$) case, this sine-Gordon soliton is only a solution to the equations of motion if the monopole core is artificially held fixed as is implicit in Eqs. (2) and (3). In reality, the core is free to move, so there is no "soliton zero mode." Moving the soliton to $y_0<0$ transports some of the gradient energy from the southern hemisphere to the northern hemisphere and creates an effective tension pulling the monopole center north. If there were "translation invariance" in y, the soliton would continue to move toward larger values of $-y_0$ and the tension would increase arbitrarily high, which is clearly unphysical. This is a consequence of the fact that the translation of the soliton does not obey the equations of the motion at the monopole core.

One could attempt to create a moving "soliton" by starting with a spherically symmetric monopole and setting the initial $\dot\phi$ to that of a soliton moving at finite speed. This will be identical to Goldhaber's soliton at large distances from the monopole core, but a tension inhomogeneity will develop near the monopole core which will pull the core north until the monopole core is approximately spherically symmetric. Note that the static energy [3] of such a configuration will always be larger than that of the spherically symmetric monopole because $\bar\theta$ will become r dependent.

The second step of Goldhaber's angular instability involves slowly sending $|\phi|$ to zero in y after the soliton is moved to large $-y$. For the sake of discussion, let us imagine that Goldhaber's soliton can be moved to large $-y$, and examine the energy density of the "translated soliton" solution to see if it is energetically favorable to reduce $|\phi|$ to zero. When $|\phi|=\eta$, the energy density is just

$$\rho(|\phi|=\eta)=\rho_1(|\phi|=\eta)=\eta^2/\cosh^2(y-y_0)\,. \tag{4}$$

If we neglect the derivative term in Eq. (3), we can find a lower bound on ρ for points where $\phi=0$,

$$\rho(\phi=0)=\rho_2(\phi=0)\geq\lambda\eta^4r^2/4\cosh^2 y\,. \tag{5}$$

Comparison of Eqs. (4) and (5) reveals that for

$$\eta r>(2/\sqrt{\lambda})\cot(\theta_0/2)\,, \tag{6}$$

the energy is always smaller for $|\phi|=\eta$ than for $|\phi|=0$. [$\theta_0\equiv 2\arctan(e^{y_0})$.] Because the total energy of the monopole is dominated by the contribution from larger r, this implies that it is not energetically favorable for the monopole to "unwind." It is therefore impossible to construct a finite-sized "north-pointing teardrop monopole" contrary to Goldhaber's claim. In Goldhaber's paper [2], the r^2 factor is missing from the potential term of Eq. (3), and it appears that his erroneous conclusion might be a consequence of that.

In the realistic case of global monopoles that may have formed in the early Universe there is an instability of sorts. This is just the annihilation of monopoles and antimonopoles. Numerical simulations [4] of global monopole evolution have shown that this annihilation process limits the monopole density to about two monopole-antimonopole pairs for every horizon volume. We have seen that Goldhaber's claim that an isolated monopole can collapse is incorrect.

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[1] M. Barriola and A. Vilenkin, Phys. Rev. Lett. **63**, 341 (1989).
[2] A. S. Goldhaber, Phys. Rev. Lett. **63**, 2158 (1989).
[3] One must always calculate the energy of a global monopole in a sphere centered on the monopole core. Otherwise, one might conclude that simply translating a monopole can reduce its energy by moving some of the higher-energy regions of the monopole field outside the boundary.
[4] D. P. Bennett and S. H. Rhie, Phys. Rev. Lett. **65**, 1709 (1990).