PHYSICAL REVIEW
LETTERS

VOLUME 67 | 26 AUGUST 1991 | NUMBER 9

Large-Scale Structure from Wiggly Cosmic Strings

Tanmay Vachaspati and Alexander Vilenkin
Tufts Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155
(Received 7 March 1991)

Recent simulations of the evolution of cosmic strings indicate the presence of small-scale structure on the strings. We show that wakes produced by such "wiggly" cosmic strings can result in the efficient formation of large-scale structure and large streaming velocities in the Universe without significantly affecting the microwave-background isotropy. We also argue that the motion of strings will lead to the generation of a primordial magnetic field. The most promising version of this scenario appears to be the one in which the Universe is dominated by light neutrinos.

PACS numbers: 98.80.Cq, 98.60.Eg

Superheavy strings produced at a phase transition in the early Universe can generate cosmological density fluctuations which can subsequently evolve into galaxies and large-scale structure [1,2]. Oscillating loops of string can serve as isolated seeds of accretion, while rapidly moving long strings can create planar overdensities in their wakes.

Most of the early work on this scenario was based on the "old picture" of string evolution. It was thought that the strings were smooth on the horizon scale, with about one long, relativistically moving string per horizon volume, and that about one horizon-size loop was chopped off the network per horizon volume per expansion time. The initial velocities of the loops were assumed to be small, and the loops were treated as a collection of stationary accretion centers. Accretion in the wakes of long strings was usually ignored.

The string evolution picture suggested by recent numerical simulations is quite different from the old one [3,4]. Long strings move slowly and have significant structure in the form of kinks and wiggles on scales much smaller than the horizon. This structure contributes nearly one-half to the total string energy in the radiation era and slightly less than one-third in the matter era. Moreover, stable loops are formed with sizes much smaller than the horizon and with large initial velocities. As a result, loops can become less important for structure formation than the wakes of long strings, and the vast amount of literature on the string scenario must therefore be rethought.

The formation and evolution of long-string wakes and their possible role in galaxy formation have been discussed in Refs. [5–10]. The main purpose of this Letter is to study how the cosmological effects of long strings are affected by their small-scale structure and by their slow motion. We shall see that some of the old conclusions should be significantly modified.

To an observer who cannot resolve the kinks and wiggles on a long string, the string will appear as smooth, but the effective mass per unit length μ and tension T will be different from their unperturbed values. It can be shown that the effective equation of state for a wiggly string is [11]

$$\mu T=\mu_0^2 , \qquad (1)$$

where μ_0 is the unperturbed string tension. The energy-momentum tensor of a wiggly string segment oriented along the z axis can be approximated as

$$T_\mu^\nu=\delta(x)\delta(y)\mathrm{diag}(\mu,0,0,T) . \qquad (2)$$

The gravitational field of the string can be found by solving the linearized Einstein equations with T_μ^ν from Eq. (2). This gives [12]

$$\begin{aligned} h_{00}&=h_{33}=4G(\mu-T)\ln(r/r_0) ,\\ h_{11}&=h_{22}=4G(\mu+T)\ln(r/r_0) , \end{aligned} \qquad (3)$$

where $h_{\mu\nu}=g_{\mu\nu}-\eta_{\mu\nu}$ is the metric perturbation, $r=(x^2+y^2)^{1/2}$, and r_0 is a constant of integration.

For an unperturbed string, $T=\mu=\mu_0$ and

$$h_{00}=h_{33}=0,\quad h_{11}=h_{22}=8G\mu_0\ln(r/r_0)\,. \tag{4}$$

A coordinate transformation brings this metric to a locally flat form,

$$ds^2=dt^2-dz^2-dr^2-(1-8G\mu_0)r^2d\phi^2\,. \tag{5}$$

It describes a conical space, which is just a Euclidean space with a wedge of angular size $\Delta_0=8\pi G\mu_0$ removed and the two faces of the wedge identified. A particle at rest with respect to a straight string experiences no gravitational force, but if the string moves with velocity v_s, then nearby matter gets a boost

$$u_i=4\pi G\mu_0 v_s\gamma_s \tag{6}$$

in the direction of the surface swept out by the string. Here, $\gamma_s=(1-v_s^2)^{-1/2}$. This effect is responsible for the formation of wakes [5] and for a discontinuous change of the microwave background temperature across a moving string [13]. Assuming that the string is perpendicular to the line of sight, the magnitude of the latter effect is

$$\delta T/T=8\pi G\mu_0 v_s\gamma_s\,. \tag{7}$$

Returning now to the wiggly string metric (3), we first consider the effect of the wiggles on light propagation. Assuming for simplicity that the direction of propagation is perpendicular to the string, we can write the relevant components of the metric in the form

$$ds^2=(1+h_{00})[dt^2-(dx^2+dy^2)]\,, \tag{8}$$

where we should identify the half lines $y=\pm 4\pi G\mu x$, $x\geq 0$. The conformal factor $(1+h_{00})$ does not affect light propagation and can be dropped. Then the resulting metric describes Minkowski space with a deficit angle $8\pi G\mu$, and we conclude that background temperature discontinuities produced by wiggly strings are given by the same Eq. (7) with μ_0 replaced by μ.

To study the formation of a wake behind a moving wiggly string, we will first look at the problem in the rest frame of the string where the particles are flowing past the string with a velocity v_s in the x direction. The linearized geodesic equations in the metric (8) can be written as

$$2\ddot{x}=-(1-\dot{x}^2-\dot{y}^2)\partial_x h_{00}\,, \tag{9}$$

$$2\ddot{y}=-(1-\dot{x}^2-\dot{y}^2)\partial_y h_{00}\,, \tag{10}$$

where the overdots denote derivatives with respect to t. We need only work to first order in $G\mu$, in which case (10) can be integrated over the unperturbed trajectory $x=v_st$, $y=y_0$. Then we can transform to the frame in which the string has a velocity v_s. The result for the velocity impulse in the y direction after the string has passed by is

$$u_i=-\frac{2\pi G(\mu-T)}{v_s\gamma_s}-4\pi G\mu v_s\gamma_s\,. \tag{11}$$

The second term is the usual velocity impulse due to the conical deficit angle. But, for small velocities, it is the new first term that dominates the deflection of particles. The origin of this term can be easily understood. From Eq. (3), the gravitational force on a nonrelativistic particle of mass m is $F=2mG(\mu-T)/r$. A particle with an impact parameter r is exposed to this force for a time $\Delta t\sim r/v_s$ and the resulting velocity is $u_i\sim(F/m)\Delta t\sim G(\mu-T)/v_s$.

String simulations show [3,4] that the coherence length of strings, beyond which the directions along the string are uncorrelated, is $\xi(t)\approx t$. The interstring separation $L(t)$ is of the same order of magnitude. In the matter era, $L(t)\approx 0.7t$. The rms string velocity on the scale of the smallest wiggles is [3] $\langle\langle v^2\rangle\rangle^{1/2}\approx 0.6$, but the coherent velocity obtained by averaging over a scale ξ is $v_s\sim 0.15$. The average mass per unit length and string tension are (in the matter era) $\mu\approx 1.4\mu_0$ and $T\approx 0.7\mu_0$. With these values, the first term in Eq. (11) is about 10 times larger than the second.

Allen and Shellard [4] emphasize that the slow motion of strings is occasionally interrupted by periods of rapid motion triggered by string intercommutings. Long strings occasionally self-intersect, producing a horizon-size loop which then rapidly collapses into myriads of tiny stable loops. If two different strings intercommute, the highly curved regions near the points of intercommuting develop a high velocity, $v_s\sim 1$, and also shed off a large number of tiny loops as they move. This rapid motion is necessary to maintain the scaling evolution of the network in which the distance between strings remains a sizable fraction of the horizon.

We now turn to the formation of large-scale structure in the Universe due to cosmic-string wakes. We start by assuming that the Universe is dominated by cold dissipationless dark matter with $\Omega_0=1$, as this is the simplest scenario. The more promising scenario in which the Universe is dominated by hot dark matter is discussed below.

Consider a wake formed behind a moving string segment of length $\sim\xi(t_i)$ at time t_i in the matter era ($t_i>t_{\rm eq}$). The distance traveled by the string in one Hubble time is $\sim v_st_i$, and thus the initial length and width of the wake are $l_i\sim t_i$ and $w_i\sim v_st_i$. The two opposite streams of matter in the wake overlap, and the mass density is enhanced by a factor of 2 within a wedge with an opening angle $2u_i/v_s$, where u_i is from Eq. (11). The average thickness of this wedge is $d_i\sim u_it_i$. The initial surface density of the wake is

$$\sigma_i\approx 2\rho(t_i)d_i\approx\frac{2}{3}\frac{\mu-T}{v_st_i}\,, \tag{12}$$

where $\rho(t)=(6\pi Gt^2)^{-1}$ is the average density of the Universe, and its total mass is $M_i\approx\sigma_il_iw_i\approx(\mu-T)t_i$. Note that M_i is independent of the string velocity v_s. If the string moves faster, the wake is wider, but the surface density is decreased proportionately. We note also that the velocity perturbation (11) is produced at distances up to $\sim w_i$ from the plane of the wake. For $r > w_i$, the gravitational field of the string is like that of a stationary rod and $u_i \sim G(\mu - T)t_i/r$.

As the Universe expands, the length and width of the wake grow like the scale factor, $a(t) \sim t^{2/3}$, while the total mass of the wake grows by gravitational instability like $M \propto a(t)$. As a result, the wake thickness (defined as the turnaround distance) and surface density evolve like $d \propto a^2(t)$ and $\sigma \propto a^{-1}(t)$. At the present time $(t = t_0)$ the wake has dimensions

$$t_i z_i \times v_s t_i z_i \times u_i t_i z_i^2 \tag{13}$$

and surface density

$$\sigma_0 \approx (\mu - T) z_i^{1/2}/v_s t_0 \,, \tag{14}$$

where z_i is the redshift at t_i. The fraction of the total mass of the Universe accreted onto wakes which were formed at time $\sim t_i$ can be estimated as

$$f \approx 2 w_i d_i z_i / L^2(t_i) \approx 8\pi G(\mu - T) z_i \,. \tag{15}$$

Wakes produced by slow- and fast-moving strings differ in their dimensions and surface density. For fast-moving strings they have the form of sheets with dimensions $t_i z_i \times t_i z_i \times u_i t_i z_i^2$, while for slow strings they may have a filamentary appearance. For example, for slow-string wakes produced around $t_{\rm eq}$, the wake thickness in Eq. (13) is somewhat larger than its width. This indicates that the wake has the shape of a cylinder, rather than of a sheet. Its length is still $\sim z_i t_i$, while the diameter is $\sim (u_i v_s)^{1/2} z_i^{3/2} t_i$, which is the geometric mean of the width and thickness in Eq. (13). As we explained, the masses in both types of wakes are comparable, but the surface density in the slow-string wakes is much higher. If a string segment moves coherently for more than one Hubble time, the resulting wake will have a variable surface density, with denser parts being the one formed at earlier times.

Cold-dark-matter wakes can also be formed during the radiation era $(t < t_{\rm eq})$, but in this case the gravitational instability sets in only at $t \sim t_{\rm eq}$. It can be shown that the surface density of the resulting wakes is proportional [6] to $(t_i/t_{\rm eq})^{1/2}$, while the fraction of dark matter accreted onto wakes formed within a Hubble time of t_i is roughly independent of t_i. The main problem with the strings-plus-cold-dark-matter scenario is that tiny wakes formed at early times and cometlike wakes produced by rapidly moving small loops introduce excessive power on small scales. Albrecht and Stebbins [14] argue that wakes formed at $t > t_{\rm eq}$ are drowned in this short-wavelength noise and give little advantage in explaining the large-scale structure compared to, say, the cold-dark-matter scenario with a Zeldovich spectrum of adiabatic perturbations. It is perhaps too early to draw a final conclusion on this issue, since it is still under active investigation.

In a universe dominated by light neutrinos, wake perturbations are damped by neutrino-free streaming on comoving scales smaller than $\lambda_\nu(t) \sim v_\nu(t)t$, where $v_\nu(t) \approx v_{\rm eq}(t_{\rm eq}/t)^{2/3}$ is the rms velocity of neutrinos and $v_{\rm eq} \approx 0.2$. On larger scales the evolution of perturbations is similar to that in cold dark matter. In this scenario most of the accreted matter resides in wakes formed at $t \sim t_{\rm eq}$, and Eq. (15) gives the following estimate for the total fraction of matter accreted into wakes:

$$f_{\rm tot} \sim 20 G\mu_0 z_{\rm eq} \sim 0.4 h^2 \mu_6 \,. \tag{16}$$

Here, h is the Hubble constant in units of 100 km/s Mpc, the Universe is assumed to have critical density, $\Omega = 1$, $\mu_6 = G\mu_0/10^{-6}$, and in the last step we have used the values of μ and T from the simulations. Thin wakes of small relativistic loops are strongly suppressed by the neutrino-free streaming [15], and it appears that loops play a negligible role in structure formation. Equation (16) then implies that most of the matter in the Universe remains unclustered at the present time [10]. This may explain why dynamical measurements in clusters give values of Ω substantially smaller than 1. The characteristic scale of the large-scale structure in this scenario is $t_{\rm eq} z_{\rm eq} \sim 10 h^{-2}$ Mpc. With $h = 0.5$ it is comparable to the scale suggested by observations [16] $(\sim 25 h^{-1}$ Mpc). The present surface density of the neutrino wakes produced after $t_{\rm eq}$ is smaller than that of the wakes produced at $t_{\rm eq}$ but only by a factor $\propto (t_{\rm eq}/t_i)^{1/3}$. This means that structures on scales larger than $10 h^{-2}$ Mpc can also be prominent in this scenario.

The large-scale velocities predicted at the present time are [9] $u_0 \approx 0.4 u_i z_i^{1/2}$. For sheetlike wakes from rapidly moving stings, it gives $u_0 \sim 300 \mu_6 h$ km/s, where we assumed that $t_i \sim t_{\rm eq}$ and $v_s \gamma_s \approx 1$. These velocity perturbations extend over regions of size $(10 h^{-2}\ {\rm Mpc})^3$ and may account for the observed large-scale streaming velocities [17]. Reasonable values of u_0 are obtained, e.g., for $h \sim 0.5$, $\mu_6 \sim 4$. We note that in some regions of space the motion of matter can be affected by two or more different strings. The streaming velocity in such regions will typically be enhanced, and a smaller value of μ may suffice to explain the observations. Alternatively, the same value of μ may explain streaming velocities on a larger scale.

We next consider the question of when nonlinear structures begin to form in a neutrino-dominated universe. For a cold-dark-matter wake formed at time t_i, all matter initially within a distance $u_i t_i z_i / z$ will be accreted onto the wake by the redshift z. A neutrino wake will go nonlinear at the redshift $z_{\rm nl}$ when the comoving scale of $\lambda_\nu(t_i)$ becomes less than the distance to which the matter has been swept by the wakes [8]: $u_i t_i (a_{\rm nl}/a_i)^2 \approx \lambda_\nu(t_i) \times a_{\rm nl}/a_i$, where the scale factor $a(t)$ is related to the redshift by $1 + z = a(t_0)/a(t)$. For filamentary wakes, this gives [18] $1 + z_{\rm nl} \approx 4.5 \mu_6 h^2$ independent of z_i. With $h = 0.5$ and $\mu_6 = 4$, we have $z_{\rm nl} \approx 3.5$. For sheetlike wakes, we find $1 + z_{\rm nl} \approx 2\mu_6 h^2$. Observations do indicate that $z=2-3$ is the epoch of intensive galaxy and quasar formation [19].

Baryonic wakes in a neutrino-dominated universe start collapsing after baryons decouple from radiation, $t > t_{\rm dec}$. However, since baryons constitute only a small fraction of the total density, the growth of these wakes is strongly suppressed. Baryonic wakes could nonetheless be cosmologically significant if the energy output from the primordial stars formed in the wakes might trigger some kind of explosive amplification and lead to preferential galaxy formation along these wakes [7]. They could also explain the existence of quasars at redshifts greater than 3. The scale of baryonic wakes, $t_{\rm dec} z_{\rm dec} \sim 50h^{-1}$ Mpc, is comparable to the largest-scale structure observed in the Universe.

The wiggliness of the string network implies that the wakes will not be uniform but will have a substructure on the scale of the wiggles. For neutrino wakes the substructure will be erased on scales smaller than the neutrino-free-streaming scale, while for baryonic wakes it may extend all the way to the damping scale due to gravitational radiation, $\Gamma G\mu_0 t$. Here, $\Gamma \sim 100$ is a numerical factor coming from the rate of gravitational radiation [20]. For example, wakes produced at $t_{\rm dec}$ will have a substructure on comoving scales larger than $0.6h^{-3}$ Mpc and $5h^{-1}\mu_6$ kpc for neutrino and baryonic wakes, respectively. We expect this substructure to help fragmentation of the wakes into smaller objects. Note that the wakes formed at $\sim t_{\rm eq}$ have very little substructure. Their fragmentation may be helped by small loops passing through the wakes at later times.

The flow of matter in baryonic wakes is expected to be turbulent. The string small-scale structure induces velocity gradients in the flow, and order-of-magnitude estimates show that the corresponding Reynolds numbers are very large. The characteristic scale of turbulence $\sim \Gamma G\mu_0 t_i$ at time t_i is comparable to the width of the wake. The typical velocity on this scale is $\sim 4\pi G\mu_0$ and the corresponding vorticity is $\omega \sim 4\pi/\Gamma t_i \sim 0.1 t_i^{-1}$. Vorticity in the baryonic flow can give rise to a primordial magnetic field via the Harrison mechanism [21]. With $t_i \sim t_{\rm dec}$, the resulting magnetic field can be estimated as [21] $B(t_{\rm dec}) \sim 10^{-4}\omega \sim 10^{-17}h$ G on a comoving scale $\sim 5h^{-1}\mu_6$ kpc. It is known [22] that a field strength of about 10^{-21} G in a protogalaxy would be enough to seed a galactic dynamo so as to produce the observed galactic field of 10^{-6} G. In addition, in our case, the presence of turbulence opens up the possibility of a turbulent dynamo that could also amplify the field. We also expect vorticity on larger scales due to the motion of long wiggly strings. A reliable estimate of turbulent vorticity and of the resulting magnetic field may require numerical simulations.

In conclusion, we summarize the promising features of the structure formation scenario based on wiggly cosmic strings in a neutrino-dominated universe. (i) Wakes formed by the strings provide an efficient mechanism for generating the large-scale structure and large-scale streaming velocities. Both filamentary and sheetlike structures will be formed. (ii) Voids between the wakes are essentially unperturbed, and most of the matter in the Universe remains unclustered (hence there is no contradiction with $\Omega = 1$). (iii) String wiggles induce inhomogeneities in the wakes which lead to wake fragmentation. (iv) Neutrino wakes collapse at $z \sim 2-3$, which is the epoch of intensive galaxy formation. Baryonic wakes can account for galaxies and quasars at higher redshifts. (v) No new exotic particles are required. (vi) Turbulent vorticity in the wakes leads to the generation of magnetic fields that may be seeds for the galactic magnetic fields.

We are grateful to George Field and Albert Stebbins for discussions. This work was supported in part by the National Science Foundation.

Note added.— After this work was completed, we received a preprint by Vollick [23], who also studied the wakes due to wiggly strings and their effect on structure formation.

[1] Ya. B. Zeldovich, Mon. Not. R. Astron. Soc. **192**, 663 (1980).

[2] A. Vilenkin, Phys. Rev. Lett. **46**, 1169 (1981); **46**, 1496(E) (1981).

[3] D. P. Bennet and F. R. Bouchet, Phys. Rev. Lett. **60**, 257 (1988). See also the papers by Bennett and Bouchet in *The Formation and Evolution of Cosmic Strings,* edited by G. W. Gibbons, S. W. Hawking, and T. Vachaspati (Cambridge Univ. Press, Cambridge, 1990).

[4] B. Allen and E. P. S. Shellard, Phys. Rev. Lett. **64**, 119 (1990); also in *The Formation and Evolution of Cosmic Strings* (Ref. [3]).

[5] J. Silk and A. Vilenkin, Phys. Rev. Lett. **53**, 1700 (1984).

[6] T. Vachaspati, Phys. Rev. Lett. **57**, 1655 (1986).

[7] M. J. Rees, Mon. Not. R. Astron. Soc. **222**, 21 (1986).

[8] A. Stebbins, S. Veeraraghavan, R. Brandenberger, J. Silk, and N. Turok, Astrophys. J. **322**, 1 (1987).

[9] T. Hara and S. Miyoshi, Prog. Theor. Phys. **81**, 1187 (1990).

[10] L. Perivolaropoulos, R. Brandenberger, and A. Stebbins, Phys. Rev. D **41**, 1764 (1990); Int. J. Mod. Phys. A **5**, 1633 (1990).

[11] B. Carter, Phys. Rev. D **41**, 3886 (1990); A. Vilenkin, Phys. Rev. D **41**, 3038 (1990).

[12] A. Vilenkin, Phys. Rev. D **23**, 852 (1981).

[13] N. Kaiser and A. Stebbins, Nature (London) **310**, 391 (1984); J. R. Gott, Astrophys. J. **288**, 422 (1985).

[14] A. Albrecht and A. Stebbins, Fermilab report (unpublished).

[15] E. Bertschinger, in *The Formation and Evolution of Cosmic Strings* (Ref. [3]).

[16] V. de Lapparent, M. Geller, and J. Huchra, Astrophys. J. Lett. **302**, L1 (1986).

[17] D. Burstein *et al.*, in *Galaxy Distances and Deviations from the Hubble Flow,* edited by B. Madore and R. Tully (Reidel, Dordrecht, 1983); C. Collins, R. Joseph, and N. Robertson, Nature (London) **320**, 506 (1986).

[18] Our estimate of $z_{\rm nl}$ was derived assuming planar geometry and applies only if $w > d$ at $z \sim z_{\rm nl}$. This gives the

condition $z_i < z_* \equiv (v_s/v_{eq}) z_{eq}$. For $z_i > z_*$, the expression for $1 + z_{nl}$ acquires an additional factor z_*/z_i. We ignore this modification, since it affects only the filamentary wakes in the narrow interval $z_{eq} > z_i > 0.75 z_{eq}$.

[19] P. J. E. Peebles and J. Silk, Nature (London) **346**, 233 (1990).

[20] T. Vachaspati and A. Vilenkin, Phys. Rev. D **31**, 3052 (1985); J. Quashnock and D. N. Spergel, Phys. Rev. D **42**, 2505 (1990).

[21] E. R. Harrison, Mon. Not. R. Astron. Soc. **147**, 279 (1970).

[22] *Magnetic Fields in Astrophysics,* edited by Ya. B. Zeldovich, A. A. Ruzmaikin, and D. D. Sokoloff (Gordon and Breach, New York, 1983).

[23] D. N. Vollick, University of British Columbia report (to be published).