Crossover to Effectively Two-Dimensional Vortices for High-T_c Superconductors

Petter Minnhagen and Peter Olsson
Department of Theoretical Physics, Umeå University, 901 87 Umeå, Sweden
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By means of Monte Carlo simulations and Coulomb-gas scaling it is shown that the vortex fluctuations in the three-dimensional anisotropic XY model become effectively two dimensional just above the critical temperature. This suggests that the resistance scaling function in the case of high-T_c superconductors should be more identical to the one for conventional superconducting films. Some experimental evidence for this is pointed out.

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Experiments on high-T_c superconductors strongly suggest that thermal vortex fluctuations associated with the CuO_2 planes are to a large extent responsible for the IV characteristics [1-7]. The support for this comes from measurements of the resistance parallel to the CuO_2 planes just above the superconducting transition temperature for $Bi_2Sr_2CaCu_2O_8$ crystals [1], $Tl_2Ba_2CaCu_2O_8$ crystals [2], and $YBa_2Cu_3O_7$ crystals [3,4], as well as from measurements of the nonlinear IV characteristics just below the transition for $Bi_2Sr_2CaCu_2O_8$ crystals [1,6,7] and for $YBa_2Cu_3O_7$ crystals [4].

Especially curious is the evidence found in Ref. [5] that the resistance scaling function for $Bi_2Sr_2CaCu_2O_8$ crystals appears to be nearly equal to the corresponding scaling function for conventional type-II superconducting films [8]. This suggests that the vortex fluctuations above the superconducting transition are effectively described by the same 2D (two-dimensional) Ginzburg-Landau Coulomb-gas model as are conventional 2D superconducting films [8,9]. Or, in other words, the superconducting planes for the $Bi_2Sr_2CaCu_2O_8$ crystals are in this sense effectively decoupled above T_c.

Analysis of the nonlinear IV characteristics for $Bi_2Sr_2CaCu_2O_8$ crystals just below T_c suggests, on the other hand, that the scaling function for the exponent of the nonlinear IV characteristics is distinctly different from the corresponding scaling function for conventional 2D superconductors [10]. This is less surprising since the interplane coupling is expected to cause a difference between the characteristics of thermal vortex fluctuations for layered superconductors, like high-T_c superconductors, and those for 2D superconductors. In the former case the "bare" vortex interaction associated with a plane increases linearly with distance, where for 2D superconductors it increases logarithmically [11,12]. Furthermore, on the level of a 3D (three-dimensional) anisotropic XY model, it has been shown that this linear increase also dominates the effective vortex interaction below T_c [13]. Thus from this point of view a distinct difference between the characteristics of vortex fluctuations should be expected between layered superconductors and 2D superconductors below T_c. However, above T_c the linear part of the effective vortex interaction vanishes [13]. Consequently, this particular distinction between 3D and 2D vortex fluctuations vanishes above T_c.

In the present Letter we show, on the level of a 3D anisotropic XY model, that the distinction between 2D and 3D vortex fluctuations apparently altogether ceases above T_c. In fact, the description of the vortex fluctuations for the 3D anisotropic XY model above T_c collapses onto the very same Coulomb-gas description as the 2D XY model.

The 3D anisotropic XY model on a cubic lattice (with lattice constant a) is given by the Hamiltonian

$$H_{XY}^{3D}=-\sum_{\langle ij\rangle_\parallel} J_\parallel\cos(\theta_i-\theta_j)-\sum_{\langle ij\rangle_\perp} J_\perp\cos(\theta_i-\theta_j)\,, \qquad (1)$$

where the sums are over nearest-neighbor pairs on a cubic lattice and $\langle ij\rangle_\parallel$ denotes nearest-neighbor pairs belonging to the same superconducting plane (the superconducting planes correspond to the fundamental horizontal planes of the cubic lattice), $\langle ij\rangle_\perp$ denotes pairs belonging to two adjacent planes, and $-\pi<\theta_i\le\pi$ is the phase associated with the lattice point i. $J_\parallel$ is the phase coupling within the superconducting planes and $J_\perp$ is the interplane coupling. High-T_c superconductors correspond to $J_\parallel\gg J_\perp$ [11]. We have performed standard Monte Carlo simulations on this model using periodic boundary conditions and lattice sizes up to $64\times64\times64$ [14,15].

In order to determine the critical temperature T_c we have used a finite-size scaling of the helicity modulus parallel to superconducting planes $\gamma_\parallel$. Precisely at T_c we have $\gamma_\parallel\propto1/N$ [16]. The helicity modulus $\gamma_\parallel$ may be expressed as

$$\gamma_\parallel(N,T)=\frac{J_\parallel}{2N^3a^2}\left\langle\sum_{\langle ij\rangle_\parallel}\cos(\theta_i-\theta_j)\right\rangle -\frac{J_\parallel^2}{TN^3a^2}\left\langle\left[\sum_{\langle ij\rangle_\parallel}\sin(\theta_i-\theta_j)\mathbf{e}_{ij}\cdot\hat{\mathbf{e}}\right]^2\right\rangle, \qquad (2)$$

where $\mathbf{e}_{ij}$ is the vector from site j to site i, $\hat{\mathbf{e}}$ is a unit vector with fixed direction parallel to the superconducting plane, the sums are over nearest-neighbor pairs in the same superconducting plane, N^3 is the total number of lattice points, and $\langle\cdots\rangle$ denotes thermal averages. Figure 1 shows a determination of T_c by means of the finite-size scaling of the helicity modulus [17]. For $J_\perp/J_\parallel=0.1$ we obtain $T_c/J_\parallel\approx1.33$ (as shown in Fig. 1) and for $J_\perp/J_\parallel=0.02$ we obtain $T_c/J_\parallel\approx1.14$.

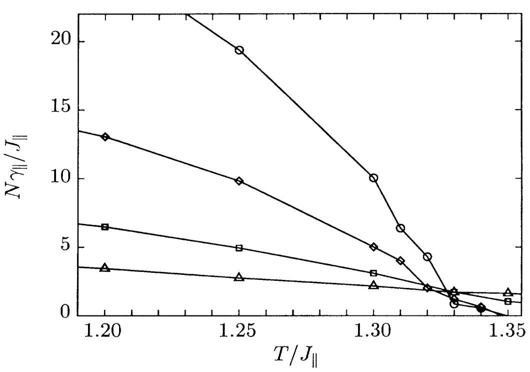

FIG. 1. Determination of T_c. Monte Carlo data for $\gamma_\parallel$ with $J_\perp/J_\parallel=0.1$ plotted as $N\gamma_\parallel/J_\parallel$ vs $T/J_\parallel$. The results are for four different lattice sizes $N\times N\times N$ with $N=8$, 16, 32, and 64 corresponding to triangles, squares, diamonds, and circles, respectively. The lines through the data points are guides to the eye. Finite-size scaling implies that $N\gamma_\parallel$ should be N independent precisely at T_c. As seen in the figure this condition locates T_c at $T_c\approx 1.33J_\parallel$.

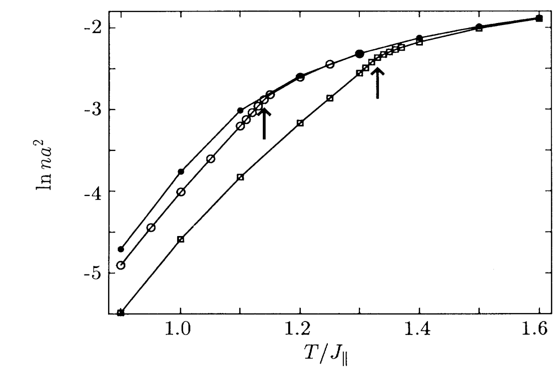

FIG. 2. Vortex density associated with a fundamental horizontal plane for the 3D anisotropic XY model on a cubic lattice. The dimensionless quantity na^2, where n is the vortex density and a is the lattice constant, is plotted as $\ln(na^2)$ vs $T/J_\parallel$. The results are from Monte Carlo simulations on a cubic lattice of size $N=64$. Solid circles, open circles, and open squares correspond to $J_\perp/J_\parallel=0$, 0.02, and 0.1, respectively. The two arrows mark the critical temperatures for $J_\perp/J_\parallel=0.02$ and 0.1 obtained as illustrated in Fig. 1. The lines are guides to the eye. The figure shows that above their respective critical temperatures the vortex densities for $J_\perp/J_\parallel=0.02$ and 0.1 become nearly equal to the vortex density for the 2D XY model. The conclusion is that the horizontal planes for the 3D anisotropic XY model to a large extent become decoupled above T_c.

The vortex concept for the 3D XY model is defined by means of the phase differences between nearest neighbors, $\theta_{ij}=\theta_i-\theta_j$, restricted to the interval $-\pi<\theta_{ij}\le\pi$. A vortex at a square means that the sum of these θ_{ij} around the square is nonzero. To be more precise we may identify a square by the four lattice points sitting at its corners, denoted by 1,2,3,4, in the counterclockwise direction. The sum of the phase differences is $\sum\theta_{ij}=\theta_{21}+\theta_{32}+\theta_{43}+\theta_{14}$ and the possible values for this sum are 0 and $\pm 2\pi$, corresponding to no vortex and a vortex with vorticity ± 1, respectively.

We have determined the vortex density in the superconducting planes n as a function of temperature T. Figure 2 shows the result for three coupling-constants ratios $J_\perp/J_\parallel=0$, 0.02, and 0.1 plotted as $\ln(na^2)$ against $T/J_\parallel$ (solid circles, open circles, and open squares, respectively). The two arrows in Fig. 2 denote T_c for $J_\perp/J_\parallel=0.02$ and 0.1, respectively. T_c for the 2D XY model (i.e., $J_\perp=0$) is $T_c/J_\parallel\approx 0.893$ [18]. The value $na^2=\frac{1}{3}$ [or $\ln(na^2)\approx -1.1$] is the limiting value for $T\to\infty$, corresponding to independent phases. The striking thing to note is that the densities of thermally excited vortices for $J_\perp/J_\parallel=0.02$ and 0.1, above their respective critical temperatures, are nearly equal to the thermal vortex density of the 2D XY model. *The inescapable conclusion is that the superconducting planes to a large extent become decoupled above T_c for the 3D anisotropic XY model.*

The surprising degree of two dimensionality of the vortex fluctuations above T_c can be made even more striking by invoking the Coulomb-gas scaling concept [8]. This concept is based on the following description of vortex-fluctuations for a 2D superfluid: A bare superfluid density ρ_0^{2D} is identified as the superfluid density in the absence of vortex fluctuations. The vortices are then introduced into this bare superfluid density. This gives rise to a description of vortex fluctuations in terms of a 2D Coulomb gas [8]. The properties of this Coulomb gas are controlled by an effective Coulomb-gas temperature variable, T^{CG}, given by $T^{CG}=T/2\pi\rho_0^{2D}(\hbar/m^*)^2$, where m^* is the mass of the superfluid particles [8]. It follows that T^{CG} for the 2D XY model is given by $T^{CG}=T/2\pi\gamma_0$, where γ_0 is the helicity modulus in the absence of vortices [8]. A dimensionless quantity defined within this Coulomb-gas model is only a function of T^{CG} [8]. The Coulomb-gas scaling concept is just the statement that such dimensionless quantities are, as functions of T^{CG}, "universal" for models which are described by the very same 2D Coulomb gas [8].

In order to test the universality of 2D vortex fluctuations for the 3D anisotropic XY model we calculate γ_0 by means of Monte Carlo simulations. This quantity is just $\gamma_\parallel$ for one particular plane calculated within the configurational subspace for which all vortices are excluded from this particular plane (but included for all other planes) [19]. Figure 3 shows γ_0 obtained for $J_\perp/J_\parallel=0$, 0.02, and 0.1 (solid circles, open circles, and open squares, respectively). Note that γ_0 is renormalized by the interplane coupling; for $J_\perp/J_\parallel=0.02$ only slightly, but for $J_\perp/J_\parallel=0.1$ somewhat more. In other words, the bare 2D superfluid density gets renormalized by the interplane coupling. *The amazing thing is that the vortex fluctuations apparently become two dimensional above T_c and are described by the very same Coulomb gas as the 2D XY model.* This is demonstrated in Fig. 4 where we have

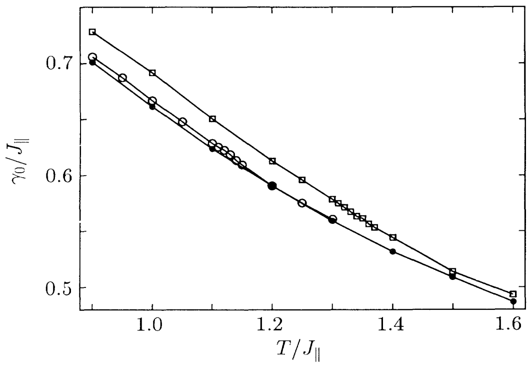

FIG. 3. The bare 2D superfluid density for the 3D anisotropic XY model. The bare 2D superfluid density is proportional to γ_0 which is the helicity modulus $\gamma_\parallel$ for a horizontal plane within the configurational subspace which excludes vortices on this particular plane. The figure shows γ_0 for $J_\perp/J_\parallel=0$, 0.02, and 0.1 corresponding to solid circles, open circles, and open squares, respectively. As seen in the figure γ_0 gets renormalized by the perpendicular coupling $J_\perp$, i.e., $J_\perp$ suppresses fluctuations and makes γ_0 larger.

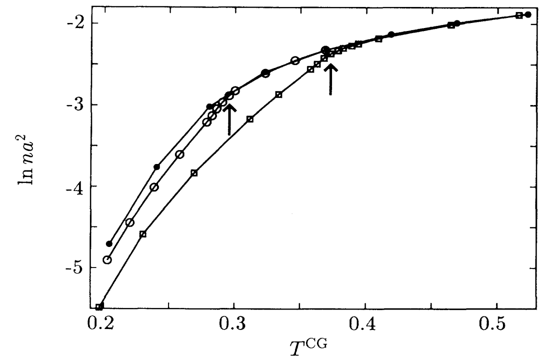

FIG. 4. 2D Coulomb-gas universality for the 3D anisotropic XY model. The dimensionless quantity na^2 as a function of the Coulomb-gas temperature variable; $T^{CG}=T/2\pi\gamma_0$ is a Coulomb-gas scaling function. The figure gives $\ln(na^2)$ vs T^{CG} for $J_\perp/J_\parallel=0$, 0.02, and 0.1 corresponding to solid circles, open circles, and open squares, respectively. The two arrows mark the critical temperatures for $J_\perp/J_\parallel=0.02$ and 0.1. The lines are guides to the eye. $\ln(na^2)$ as a function of T^{CG} for $J_\perp/J_\parallel=0.02$ and 0.1, above their respective critical temperatures, collapse onto the corresponding function for the 2D XY model ($J_\perp=0$). The conclusion is that the 3D anisotropic XY model, at least to a very good approximation, exhibits 2D Coulomb-gas universality above T_c.

plotted $\ln(na^2)$ as a function of T^{CG}. Above T_c (denoted by arrows in Fig. 4) the curves for $J_\perp/J_\parallel=0.02$ and 0.1 (open circles and squares, respectively) collapse onto the curve corresponding to the 2D XY model (solid circles).

We note that the 2D behavior of the vortex fluctuations above T_c does not imply that the transition has a Kosterlitz-Thouless character. The phase transition for the 3D anisotropic XY model is presumably of second order [20]. That the transition is not of the Kosterlitz-Thouless type is somewhat obvious directly from Fig. 4. A Kosterlitz-Thouless transition is possible only for $T_c^{CG} \leq \frac{1}{4}$ [8]. However, as seen in Fig. 4, the transitions for $J_\perp/J_\parallel=0.02$ and 0.1 take place at $T^{CG} > \frac{1}{4}$.

The main conclusion of the present Letter is thus that the vortex fluctuations for the 3D anisotropic XY model above T_c, at least to a very good approximation, are given by a 2D universal Coulomb-gas description.

The 3D anisotropic XY model can be viewed as a model of layered superconductors for which the magnitude variations of the order parameter have been suppressed. These magnitude variations can be included on the level of a Ginzburg-Landau description. For a 2D superconductor this leads to a description of vortex fluctuations in terms of the 2D Ginzburg-Landau Coulomb gas [8]. It has been shown for conventional type-II superconducting films that Coulomb-gas scaling is obeyed for the resistance ratio R/R_N just above the superconducting transition [8]. (R/R_N is proportional to $n_F\xi^2$, where n_F is the density of free vortices and ξ is the Ginzburg-Landau coherence length [8].) A description on the same level for layered superconductors, like high-T_c superconductors, involves, in addition, an interplane Josephson coupling in a similar way as the 3D anisotropic XY model [11,21]. This suggests, by analogy to our present results for the 3D anisotropic XY model, that vortex fluctuations for high-T_c superconductors above T_c are given by the 2D Ginzburg-Landau Coulomb-gas model. This was in fact precisely the curious evidence extracted from experiments on $Bi_2Sr_2CaCu_2O_8$ crystals in Ref. [5] mentioned above.

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