Nonlocal Dispersion of Edge Magnetoplasma Excitations in a Two-Dimensional Electron System

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Edge magnetoplasma excitations in a two-dimensional electron system have been studied in radio-frequency experiments which exhibit a strong nonlocal behavior of the plasmon dispersion. The nonlocality is shown to be caused by the diagonal conductivity σ_{xx}, which, via a length $l \propto \sigma_{xx}$, governs the spatial distribution of the plasma charge oscillations in the direction perpendicular to the plasmon wave vector, i.e., to the edge of the two-dimensional layer.

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The collective excitation spectrum of an electronic system at large wave vectors q is governed by well-known nonlocal effects [1]. Nonlocal effects are of the order $(qv_F/\omega_p)^2$, where v_F is the Fermi velocity and ω_p is the "local" plasmon frequency, and are usually rather small. Here we report on a very different, large nonlocal effect on the plasmon dispersion which is a unique property of a finite-size two-dimensional electron system (2DES) when the edges become important. We have observed this nonlocal effect in magnetic-field experiments on two-dimensional electron systems of macroscopic size (1×1 cm^2) for the low-frequency ($2\pi f \ll \omega_c$) branch of the plasma excitations, i.e., the edge magnetoplasmons (EMP). The nonlocal effect arises from a significant influence of the diagonal conductivity σ_{xx} on the EMP excitation which has not previously been observed. It can be characterized by a length $l \propto \sigma_{xx}$, which is related to the spatial extent of the plasma charge oscillations in the direction perpendicular to the plasmon wave vector, i.e., to the edge of a 2DES. It represents a kind of "transverse" nonlocality in contrast to the "longitudinal" nonlocality discussed above and also occurs for very small values of q.

EMP excitations have recently attracted much attention due to their unique properties, in particular, a surprisingly small damping which also occurs for the condition $2\pi f\tau \ll 1$, where τ is the $B=0$ momentum relaxation time [2–10]. The EMP frequency $f_{\rm EMP}$ has been calculated by different authors [10–13] and is shown to be essentially proportional to the Hall conductivity σ_{xy} and the wave vector $q=2\pi n/P$, where $n=1,2,\ldots,$ is the mode index and P is the perimeter of the finite-size 2DES. The theories in Refs. [10,11] include nonlocal effects and, in spite of their different underlying physical models, give practically the same form for $f_{\rm EMP}$ which is, if we neglect single-particle effects,

$$f_{\rm EMP} = (2\sigma_{xy} n/\tilde{\varepsilon} P)\ln(P/nl) . \qquad (1)$$

Here $\tilde{\varepsilon}$ is the effective dielectric function of the surrounding medium. The interesting quantity in the logarithmic term is l, which arises from a transverse nonlocality and has the following physical origin. Edge magnetoplasmons represent collective excitations where the individual electrons perform skipping cyclotron orbits along the boundary of the 2DES. Thus with increasing B the charge-density oscillations are concentrated in a very small region close to the edge. The interesting point for a 2DES is that the oscillating charge cannot be concentrated in a local δ-function "line" (which would lead to logarithmic divergences in the potential and electrostatic energy) [11]. This is a unique property of a 2DES and in contrast to surface plasmons at the surface of a 3DES (neglecting the spatial dispersion). Thus the oscillating charges of edge plasmons are inherently distributed over a certain region perpendicular to the edge and respond in a nonlocal manner. The calculation of this response, i.e., of l, is a serious problem, especially in the quantum Hall effect (QHE), since it requires a detailed microscopic description of the electronic system in the region near the edge. In the model of Volkov and Mikhailov [11], which is a classical electrodynamic description and assumes a sharp cutoff of the static 2D density profile at the edge, l is given by

$$l = \sigma_{xx}/\tilde{\varepsilon} f , \qquad (2)$$

where σ_{xx} is the—macroscopic—diagonal conductivity of the 2DES. In the model of Wassermeier *et al.* [10], l is set to the magnetic length $l_B=\sqrt{\hbar c/eB}$ at integer and fractional filling factors $\nu = hn_s c/eB$ (n_s is the electron density) and it is suggested that it be replaced by a localization length L outside the QHE regime. Previous experimental studies [4–10] have mostly investigated the EMP excitations at fixed B values near the centers of the Hall plateaus at low temperatures. It has been shown that EMP resonances occur whenever σ_{xx} is much smaller than σ_{xy}, and have frequencies $f_{\rm EMP}$ that, for a given sample, are essentially proportional to σ_{xy}. However, no influence of the nonlocality, i.e., the logarithmic term with a nonlocal quantity l, has ever been observed in the QHE regime. Some observations at high frequencies [14,15], where the QHE is destroyed [16], can in principle be interpreted using a logarithmic factor. It is so far not clear whether the nonlocality plays a role in the EMP dispersion and, especially, what is the microscopic origin of this transverse nonlocal effect. This is indeed a very interesting question, since the low-frequency EMP can be interpreted as a dynamic manifestation of the QHE, and the nonlocal length scale is closely related to the region of the edge currents [17] which are now a very popular model for describing transport in the integer and the fractional QHE.

We have investigated a 2DES in a GaAs-AlGaAs heterostructure with an electron density $n_s = 3\times10^{11}$ cm^{-2} and mobility $\mu = 2\times10^5$ cm^2/V s. Square samples with $P = 4$ cm have been measured in a nonresonant radio-frequency (rf) measurement cell [4,6,7]. The sample was positioned, as sketched in Fig. 1, in the center of a metal cylinder cell between four electrodes which act as antenna probes. Differently shaped electrodes were used to purposely vary the effective dielectric function. "Tip"-like antennas consisting of a 0.5-mm wire pointing onto the edge of the 2DES induce a small value of $\tilde{\varepsilon}$. "Flat" electrodes, which have a height of about 1 mm and a length of about 10 mm parallel to the edge of the 2DES, lead to a larger value of $\tilde{\varepsilon}$. We were able to vary the distance between the electrodes and the sample edges from $d = 3$ to 0.5 mm to increase $\tilde{\varepsilon}$. At large distances it was possible to place additional GaAs slices between the sample and electrodes to increase $\tilde{\varepsilon}$ without changing d. One electrode was connected to the generator, and one of the other three to the receiver. The signals received were the superposition of the EMP electric fields onto the constant-amplitude field supplied by the generator. We have measured the normalized amplitude, $U_N = U(B = \text{const};f)/U(B=0;f)$, and the phase shift, $\Theta = \Theta(B = \text{const};f) - \Theta(B=0;f)$, of such signals at different B values in sweeps of the generator frequency f. The experimental spectra shown in Fig. 1 clearly demonstrate that the EMP resonance of a 2DES exhibits the behavior of a single-frequency linear oscillator. From such curves we then determine the frequency f_{EMP} and the damping γ_{EMP} of the fundamental EMP mode ($n=1$) using the linear-oscillator approximation. This method gives, as has been demonstrated before [6], a 2% accuracy for f_{EMP} and 5% accuracy for γ_{EMP} values.

The obtained f_{EMP} and γ_{EMP} values versus ν are shown in Fig. 2. The upper $f_{\mathrm{EMP}}(\nu)$ curve resembles a slightly disturbed dc Hall conductivity curve $\sigma_{xy}(\nu)$. Such $f_{\mathrm{EMP}}(\nu)$ dependences have been previously studied in detail [4,6,8,9]. The curve in Fig. 2(a) has been measured with tiplike electrodes and a large distance d between the sample edges and the antenna probes. In Fig. 2 we also show the results obtained from the same sample using flat electrodes at the same distance with additional GaAs slices between the antennas and the sample edges, i.e., for an arrangement with a larger $\tilde{\varepsilon}$. The curves in Fig. 2(b) demonstrate a very unusual behavior, which so far has not been observed. In such an experimental geometry f_{EMP} decreases and strongly oscillates with ν while γ_{EMP} has correspondingly deep minima. This behavior is clearly pronounced in the original experimental spectra even without further evaluation as shown in the inset of Fig. 2.

We will now discuss the temperature dependence of the EMP excitations. The general trend is that with increasing temperature the strongly oscillating behavior of the frequency and damping becomes less pronounced. In particular, we find for $T > 30$ K a nearly linear dependence of $f_{\mathrm{EMP}}(\nu)$ and $\gamma_{\mathrm{EMP}}(\nu)$ on ν and a complete absence of any oscillations for $\nu > 2$. The most interesting

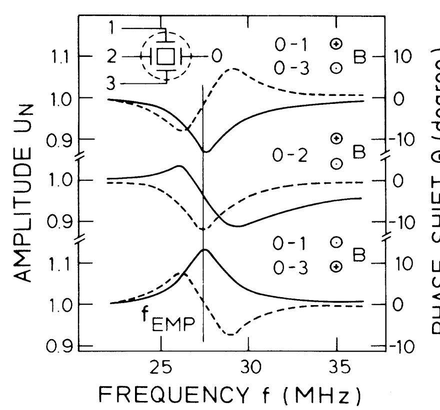

FIG. 1. Frequency dependences of the amplitude U_N (solid line) and the phase shift Θ (dashed line) of the rf signals measured with the electrodes 1, 2, and 3 when an ac voltage is applied to the electrode 0. The measurements have been performed at $T = 1.6$ K for opposite B directions and with additional GaAs slices between the "flat" electrodes and the sample edge at a distance $d = 2$ mm.

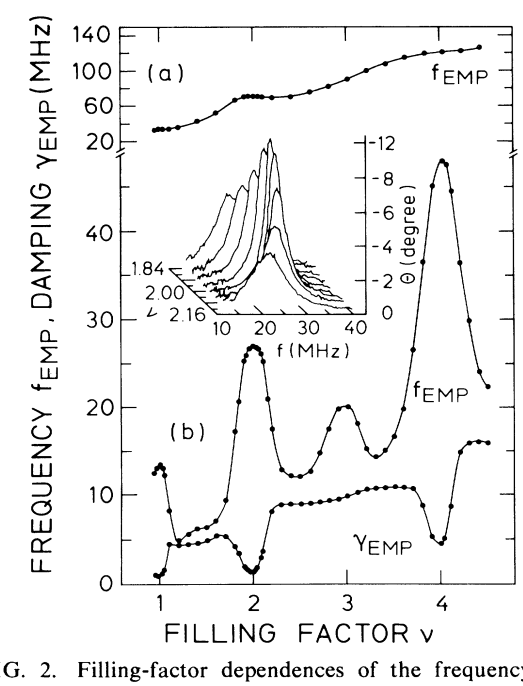

FIG. 2. Filling-factor dependences of the frequency f_{EMP} and the damping γ_{EMP} at $T = 1.6$ K for different experimental geometries. (a) Tip electrodes directed to the sample edges; $d = 2$ mm. (b) Flat electrodes with additional GaAs slices between the electrodes and the sample edges; $d = 2$ mm. Lines are guides to the eye. Inset: The f dependences of the phase shift Θ in the vicinity of $\nu = 2$ measured with electrodes 0 and 2.

effect shown in Fig. 3 occurs in the temperature regime where in dc transport measurements we observe an activated conductivity, $\sigma_{xx} \propto \exp(-W/2kT)$, with an activation energy W: Here, at integer filling factors $\nu=2$ and 4, the resonance frequency $f_{\rm EMP}$ exhibits a linear dependence on $1/T$. Its slope, measured at $\nu=2$ and 4, depends on $\tilde{\varepsilon}$ as can be shown by varying the geometry of the experimental arrangement, but is, within the limits of the experimental accuracy, identical for $\nu=2$ and 4 if the arrangement of the electrodes is not changed. Increasing $\tilde{\varepsilon}$ by decreasing the distance d between the electrodes and the 2DES leads to a decreased slope value, as shown for $\nu=2$ by Fig. 3(a) with $d=2$ mm and Fig. 3(c) with $d=1.7$ mm. Decreasing $\tilde{\varepsilon}$ by removing some of the GaAs slices at constant d leads to an increased value of the slope as shown for $\nu=2$ by Figs. 3(a) and 3(b).

These novel results in Figs. 2 and 3 clearly demonstrate that the EMP excitations are not only governed by σ_{xy} but are also strongly influenced by σ_{xx}. First, this follows from the oscillation behavior of $f_{\rm EMP}(\nu)$ at low T which then becomes less pronounced with increasing T. This is very similar to the behavior of $\sigma_{xx}(\nu)$. The temperature dependence also confirms the EMP dependence on σ_{xx}

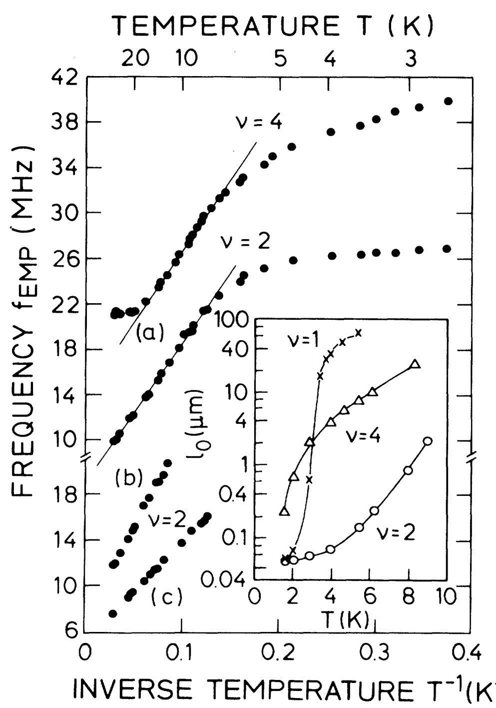

FIG. 3. The temperature dependences of the frequency $f_{\rm EMP}$ at $\nu=2$ and 4 measured with flat electrodes. (a) With GaAs slices and a distance $d=2$ mm. (b) For $d=2$ mm, with some GaAs slices removed. (c) With GaAs slices and $d=1.7$ mm. Straight lines show the slopes in the linear regime of $f_{\rm EMP}(1/T)$. Inset: The temperature dependence of the position l_0 for the charge "center of gravity" in the EMP wave at different ν.

and, moreover, shows that $f_{\rm EMP}$ depends on σ_{xx} via a logarithmic factor $\sigma_{xy}\ln(A/\sigma_{xx})$, where the constant A has the same dimensionality as σ_{xx}. This follows directly from the linear dependences of $f_{\rm EMP}$ and $\ln\sigma_{xx}$ on $1/T$ observed in the same temperature range where there is an activated behavior in the dc transport of the same GaAs-AlGaAs sample. It is also nicely demonstrated by the following observation. In the dc experiments the slope of $\ln\sigma_{xx}$ vs $1/T$ is proportional to the value of W, which is approximately the cyclotron energy $\hbar\omega_c$. Thus the slope is larger by a factor of 2 for $\nu=2$ as compared to $\nu=4$. In contrast to this, the slope of $f_{\rm EMP}$ vs $1/T$ is independent of ν since the proportionality of $\sigma_{xy} \propto \nu$ is cancelled by $\ln\sigma_{xx} \propto W \propto 1/\nu$.

Thus all the experimental results confirm qualitatively the EMP dispersion given in the model of Volkov and Mikhailov [11] [Eqs. (1) and (2)]. We can now check this theory quantitatively. To do this we need to know the values of σ_{xy} and σ_{xx} at $f=f_{\rm EMP}$ and the value of $\tilde{\varepsilon}$. The frequency dependence of σ_{xy} in the QHE regime has been shown [16,18] to be negligible up to $\sim$10 GHz, so we can use σ_{xy} obtained from dc measurements. The frequency dependence of σ_{xx} is so far not known for the studied f range at low T. But from the strong nonmonotonic behavior of $\gamma_{\rm EMP}(\nu)$ at low T in our experiments we conclude that there is a small frequency dependence. Increasing T leads to a linear dependence on $\gamma_{\rm EMP}(\nu)$ that saturates at higher T and thus shows that for $T>30$ K σ_{xx} has no frequency dependence. This means that at this T we can use the values of σ_{xx} obtained from dc measurements. $\tilde{\varepsilon}$ is thus the only unknown parameter and we will demonstrate the consistency of the model of Volkov and Mikhailov [11] by showing that different experimental observations give the same values of $\tilde{\varepsilon}$. At $T=34.5$ K, using (1) and (2), we calculate for $\nu=2$ a value of $\tilde{\varepsilon}=19.5$ using the dc values of σ_{xy} and σ_{xx} and the experimental value of $f_{\rm EMP}(\nu)$. With the ansatz $\sigma_{xx}=\sigma_0\exp(-W/2kT)$ we obtain from (1) and (2) for the fundamental mode $n=1$

$$f_{\rm EMP}=(2\sigma_{xy}/\tilde{\varepsilon}P)[\ln(P\tilde{\varepsilon}f_{\rm EMP}/\sigma_0)+W/2kT]\,. \qquad (3)$$

This gives us another independent method of determining $\tilde{\varepsilon}$, i.e., from the slopes of the linear $f_{\rm EMP}(1/T)$ regimes in Fig. 3. Using the W values from the dc measurements we find for $\nu=2$ and 4 the same value of $\tilde{\varepsilon}=19.2$. This demonstrates the consistency of the evaluation and thus confirms quantitatively the theory of Volkov and Mikhailov [11] in the temperature range from about 8 to 35 K in our samples. We note that in this temperature regime, the experimental values of $f_{\rm EMP}$ vary by a factor of less than 2. This means that the variation of the logarithmic term in (3) is more than a factor of 10 smaller than the variation of the term $W/2kT$, which thus totally dominates the slope of the $1/T$ dependence in the linear regime.

Finally, we would like to discuss the relevant length in our experiments explicitly. In the theory of Volkov and Mikhailov [11] the length l, given by (2), characterizes the spatial distribution of the EMP field inside a 2DES. A microscopically more physical quantity is the position l_0 of the charge "center of gravity" which from Ref. [11] is $l_0=(l/\pi)\ln(P/\pi l)$. In the inset of Fig. 3 we plot the temperature dependence of l_0 calculated from the experimental f_{EMP} values for different ν. The most interesting finding is the very strong localization of the edge plasma excitations with decreasing T. The plasmon "width" at $T=34.5$ K is rather large and has values of 280, 240, and 170 μm at $\nu=1$, 2, and 4, respectively. At low T in the QHE regime this width becomes extremely small (≈ 50 nm at $\nu=1$ and 2) and comparable with the "thickness" of the 2DES. It allows us to consider the EMP in the QHE regime as 1D plasmons with the width and the direction of the motion governed by B. Another interesting observation in the inset of Fig. 3 is that l_0 approaches a saturation value with decreasing T. Here it is not clear whether we have reached the limits of the classical model [11] or if this limited nonlocality is due to the microscopic properties in the edge regime, e.g., a variation of the electron density near the edge of a 2DES.

In summary, the edge magnetoplasmon dispersion exhibits a so far not observed strong B and T dependence that is shown to arise from a "transverse" nonlocal effect which is a unique property of a 2DES and is governed by the diagonal conductivity σ_{xx}.

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