Interface Tension in an SU(N) Gauge Theory at High Temperature

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For an SU(N) gauge theory without dynamical fermions, at temperatures above the deconfining transition there are Z(N) degenerate vacua. The interface tension between distinct Z(N) vacua is computed in weak coupling by semiclassical techniques.

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The properties of QCD at nonzero temperature are relevant to nuclear collisions at ultrarelativistic energies and for the early Universe. To date, most attention has focused on bulk thermodynamics, such as the order of possible phase transitions. To characterize finite or inhomogeneous systems, however, requires an understanding of more than bulk properties. For example, if the theory has degenerate vacua, then in an infinite volume the ground state lies uniquely in one of them. In contrast, in a finite volume bubbles of different vacua can form. The evolution of these bubbles is a surface phenomenon, controlled by the interface tension between the different vacua.

In SU(N) gauge theories without dynamical fermions, there is a global Z(N) symmetry associated with the center of the gauge group.[1] Because of confinement, at zero temperature the vacuum is Z(N) invariant, but for temperatures T above that of the deconfining transition, the Z(N) symmetry is spontaneously broken,[2] and there are N degenerate vacua.

In this Letter we compute the interface tension between different Z(N) vacua in the deconfined phase of an SU(N) gauge theory. The interface tension is related to the probability for tunneling in an effective action, which is computed semiclassically in weak coupling. As dynamical fermions are left out, our results can only be compared with the results of numerical simulations.[3] Nevertheless, we feel that the computation of the interface tension in an SU(N) gauge theory is of general interest on two counts. First, while the semiclassical calculation of interface tension is familiar,[4] its application here is not. The effective action which applies to Z(N) interfaces has an unusual form, and includes terms at both the classical and quantum levels. Let g be the gauge coupling constant, which without loss of generality we can take to be positive. At small g, the action of a solution to the classical equations of motion, such as an instanton, is invariably proportional to $1/g^2$. Thus it is natural to expect that the interface tension α also behaves like $1/g^2$. Instead, because the Z(N) interface is the action for the solution to effective equations of motion, α is proportional just to $1/g$. Second, as this effective action is calculable from first principles, α is equal to a pure number times T^3/g, where the pure number depends only upon the N of SU(N), Eq. (7). Since we use semiclassical methods, our result is only valid at weak coupling, at tempertures high above the phase transition. Thus our result in Eq. (7) is valid to leading order in $1/g$ at small g; effects at higher loops orders generate corrections to α of order g^0, g^1, etc.

We work at a temperature T in the imaginary-time formalism, so the Euclidean time runs from 0 to $\beta=1/T$. The system is of length L_t in the x and y directions, and L in the z direction, with L_t and L much larger than β. At nonzero temperature the fundamental order parameter is the Wilson or Polyakov line, which is the path-ordered trace

$$\Omega(\vec{x})=\frac{1}{N}\mathrm{Tr}\mathcal{P}\exp\left(ig\int_0^\beta A_0(\vec{x},t)dt\right). \tag{1}$$

We can assume that the gauge fields obey strictly periodic boundary conditions. Since the time direction is of finite extent, states with constant A_0 are inequivalent. By a global color rotation, any constant A_0 can be rotated into a form in which it is a diagonal matrix:

$$(A_0)_{ij}=(\pi T/g)q^i\delta_{ij}. \tag{2}$$

A_0 is written in the fundamental representation, $i,j=1,\ldots,N$; adjoint indices are denoted by $a=1,\ldots,N^2-1$. We parametrize the color vector q^i as

$$q^i=2q/N+2\tilde{q}^i,\quad i=1,\ldots,N-1,$$
$$\sum_{i=1}^{N-1}\tilde{q}^i=0,\quad q^N=-\frac{N-1}{N}2q. \tag{3}$$

These coordinates obey the requirement that the trace of A_0 vanish.

With this parametrization of A_0 we can understand the Z(N) structure of the vacuum. The ordinary perturbative vacuum is $A_0=0$, $\Omega=1$. The other Z(N) degenerate vacua occur when $\tilde{q}^i=0$ and $q=j$, with j an integer: then $\Omega=\exp(2\pi ij/N)$. The N degenerate vacua occur for $j=0,1,\ldots,N-1$.

To construct a Z(N) interface, we parametrize A_0 as in (2) and (3), but assume that the q^i's are a function of z. To interpolate between $\Omega=1$ at $z=0$ and $\Omega=\exp(2\pi i/N)$ at $z=L$, we impose the boundary conditions $q(0)=0$ and $q(L)=1$, $\tilde{q}^i(0)=\tilde{q}^i(L)=0$.

Using the ansatz of (2) and (3), the classical action is

$$S_{\rm cl}=\int d^4x\,\tfrac{1}{4}(F^a_{\mu\nu})^2=\beta L_t^2\int_0^L dz\,\frac{1}{2}\left(\frac{dA_0^a}{dz}\right)^2=\beta L_t^2\int_0^L dz\,\frac{4\pi^2T^2}{g^2}\left[\frac{N-1}{N}\left(\frac{dq}{dz}\right)^2+\sum_{i=1}^{N-1}\left(\frac{d\tilde q^{\,i}}{dz}\right)^2\right]. \tag{4}$$

Classically, there is no potential for A_0, and all fields with $A_0\neq 0$ are degenerate. This degeneracy is lifted at the one-loop level. To include these effects, we compute the determinant for one-loop fluctuations about a background A_0 field. We treat the background A_0 field as constant in z, and justify this after the fact. The calculation of the determinant for constant A_0 is standard,[5] with a result independent of the choice of gauge fixing. For our parametrization of A_0,

$$S_{\rm qu}={\rm tr}\ln\{-(\partial_0+g[A_0,\,])^2-\partial^2\}=\beta L_t^2L\frac{2\pi^2T^4}{3}\left[2\sum_{i=1}^{N-1}V(q-\tilde q^{\,i})+\sum_{i,j=1}^{N-1}V(\tilde q^{\,i}-\tilde q^{\,j})\right], \tag{5}$$

where the potential $V(x)=[x]^2(1-[x])^2$, with $[x]=|x|_{\rm mod 1}$. The $Z(N)$ symmetry of the vacuum is now manifest, for when the Wilson line $\Omega=\exp(2\pi ij/N)$, the quantum action vanishes: $S_{\rm qu}=0$ at $\tilde q^{\,i}=0$ and $q=j$. These points are absolute minima of the potential, since with other choices of $\tilde q^{\,i}$ and q, $S_{\rm qu}>0$.

To calculate the interface tension, we construct an effective action as the sum of the classical and quantum terms, $S_{\rm cl}$ and $S_{\rm qu}$. Since the boundary condition includes $\tilde q^{\,i}(z)=0$ at $z=0$ and L, we set $\tilde q^{\,i}(z)=0$ for all z. Our final result is an effective action in terms of the single field $q(z)$:

$$S_{\rm eff}=S_{\rm cl}+S_{\rm qu}=\beta L_t^2\frac{4(N-1)\pi^2}{\sqrt{3N}}\frac{T^3}{g}S'_{\rm eff},$$
$$S'_{\rm eff}=\int dz'\left\{\left(\frac{dq}{dz'}\right)^2+[q]^2(1-[q])^2\right\}, \tag{6}$$
$$z'=(N/3)^{1/2}gTz.$$

For convenience, we have rescaled the z coordinate into the dimensionless variable z'. With this form of the effective action it is trivial to compute the action of the interface.[4] Taking $L\to\infty$, we want the solution to the equations of motion for $S'_{\rm eff}$ which satisfies the boundary conditions $q(-\infty)=0$ and $q(+\infty)=1$. This solution is $q(z')=\exp(z')/[1+\exp(z')]$, with action $S'_{\rm eff}=\frac{1}{3}$. Plugging this back in, we find that the interface tension α, defined by $S_{\rm eff}=\beta L_t^2\alpha$, is

$$\alpha=\frac{4(N-1)\pi^2}{3\sqrt{3N}}\frac{T^3}{g}, \tag{7}$$

which is our final result.

Our derivation of α has glossed over several assumptions which need to be justified. In a semiclassical calculation, the path which dominates the functional integral is that with minimal action. In color space our stationary point is very simple, just a straight line between $q=0$ and $q=1$, with $\tilde q^{\,i}(z)=0$ for all z. By (3), there are $N-2$ independent $\tilde q$'s, so they only enter for $N\geq 3$. To prove that this is the path with the least action, consider the effective action analogous to $S'_{\rm eff}$, Eq. (6), but now including the kinetic energy for the $\tilde q$ fields. Construct the energy for this action, which is a conserved quantity. This energy vanishes, since at the end points both the kinetic and potential terms do. Using this, the action of the relevant path is equal to $2\int ds[V_{\rm tot}(q,\tilde q)]^{1/2}$, where $V_{\rm tot}(q,\tilde q)$ is the total potential for q and $\tilde q$, and ds is the path length in function space, $ds^2=dq^2+\sum_i(d\tilde q^{\,i})^2$. Obviously, our straight-line solution has the smallest path length in s. Further, in SU(3) one can show that at fixed q the smallest value of $V_{\rm tot}(q,\tilde q)$ occurs for $\tilde q=0$. Since both the arclength ds and $(V_{\rm tot})^{1/2}$ are then bounded from below by the straight-line path, this has minimal action. Because of the complexity of $\tilde q$ space, we were not able to prove that the straight-line path has minimal action for $N\geq 4$, although we suspect this is so. Thus our result in (7) is only valid for SU(2) and SU(3).

The really surprising feature of our derivation is that we can start with $S_{\rm qu}$, which is computed for constant A_0, and use it to compute the properties of the $Z(N)$ interface, which is certainly not constant in z. This is allowed because the form of the stationary point is slowly varying in z: The width of the solution is for z' of order 1, which in terms of the original coordinate z is $1/gT$. Hence for small g, the width is $1/g$ times the natural length scale, $1/T$, and the interface is very broad. Consider now what would happen if one computed the correction to the one-loop determinant in (5) from momentum dependence in the background A_0 field. This would give terms such as $\int dz\,g^2(dA_0/dz)^2$, which represents wave-function renormalization for the background fields. Using the properties of our solution for A_0, we see that this only contributes to α through terms that are of order g, vs $1/g$ in (7). In essence, each derivative d/dz brings in a factor of the inverse width of the solution, which is proportional to g.

The result in (7) is valid only for small g, to leading order in $1/g$. Expanding the effective action $S'_{\rm eff}$ to quadratic order in fluctuations of $q(z)$, followed by integration over these fluctuations, will produce corrections of order 1 to α; at this order, it will also be necessary to deal with the collective coordinates of the stationary point and the like.[4] Corrections of higher order, such as

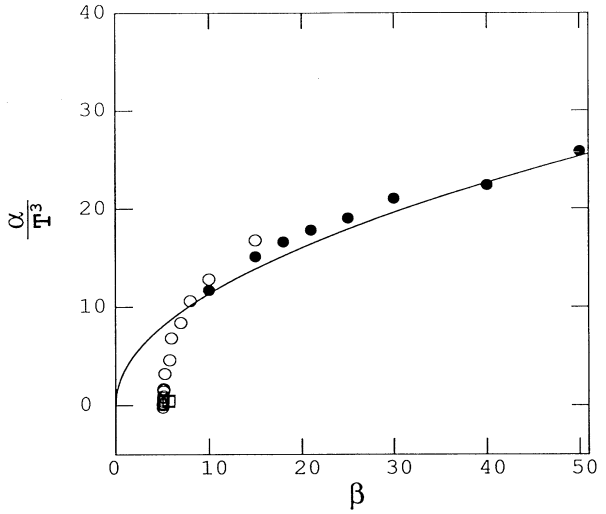

FIG. 1. Comparison with data from Ref. 3. Open circles represent the data from the Helsinki group for a $2\times8\times8\times40$ lattice. The solid circles are their preliminary unpublished data on a $2\times8\times8\times20$ lattice. The squares are data from the Boston University group on a $4\times16\times16\times32$ lattice.

those of order g in α, will be far more involved to compute, although calculable in principle. Certainly at order g it will be necessary to sort out fluctuations in the background field A_0 from those which produced the effective action, $S_{\rm qu}$ of (5), in the first place.

It is worth understanding the surprising appearance of $1/g$, instead of $1/g^2$, for the action of the stationary point. Suppose that one were not so clever, and started with just the classical action, $S_{\rm cl}$ of (4). The equations of motion are those of a free field, $d^2q/dz^2=0$, so a solution which satisfies the boundary conditions is just $q\simeq z/L$, with action $S_{\rm cl}\simeq 1/g^2L$. Notice that this action vanishes as L goes to infinity: This is not an interface, but a smooth variation from one value of A_0 to another. Now compute the one-loop corrections to this solution. To estimate their form we replace L times the potential in (5) by $\int_0^L dz$ times the potential. Since q is uniformly of order 1 over the integral, the result is of order L, $S_{\rm qu}\simeq L$. Thus at large L while the classical action vanishes, the quantum fluctuations diverge. The distance at which the effects of the quantum fluctuations become large is when $S_{\rm cl}\simeq 1/g^2L$ is of order $S_{\rm qu}\simeq L$, which is $L\simeq 1/gT$. This is precisely the distance scale which characterizes the width of the solution of the effective action, which incorporates both classical and quantum effects. Thus for boxes which are very small in the z direction, L on the order of $1/T$, there is no true interface, and we can use the classical action alone. Once L is large, however, on the order of $1/gT$, the effective action derived above must be employed.

Our results can be compared with numerical simulations on the lattice, Fig. 1. While the agreement is spectacular, this may be fortuitous. The numerical simulations were done with lattices which were only two lattice spacings in the temperature direction; for such lattices there may well be significant corrections to our result in (7). These and related questions are under study.

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