PHYSICAL REVIEW

LETTERS

VOLUME 66 25 FEBRUARY 1991 NUMBER 8

Direct Comparison of the Quantized Hall Resistance in Gallium Arsenide and Silicon

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(Received 16 November 1990)

Using an ultrasensitive, cryogenic, current-comparator bridge the quantized Hall resistance $R_H(2)$ in a GaAs/AlGaAs heterostructure has been compared directly with $R_H(4)$ in a silicon MOSFET. The measurements show that $R_H(2;\text{GaAs})/R_H(4;\text{Si}) = 2[1-0.22(3.5)\times 10^{-10}]$. Within the 1σ combined uncertainty of $\pm 3.5\times 10^{-10}$ the result suggests that the quantized Hall resistance is a universal quantity, independent of the host lattice and Landau-level index, and is probably equivalent to h/e^2, the relationship predicted theoretically.

PACS numbers: 06.20.Jr, 71.28.+d

Since the discovery in 1980 of the quantum Hall effect[1] many national laboratories have developed techniques of ever-increasing accuracy for comparing the quantized Hall resistance with room-temperature resistance standards. The most recent measurements[2] indicate that $R_H(i) = R_K/i$, where i is an integer and $R_K = 25\,812.807\ \Omega$ is the von Klitzing constant, is the same for different materials in which the two-dimensional electron gas is established within a 1 standard deviation (1σ) relative combined uncertainty of about 1 part in 10^8. There have been some reported measurements,[3] however, purporting to show that discrepancies greater than 1 part in 10^7 can occur. Since the adoption by practically all countries, from 1 January 1990, of the quantized Hall resistance as the primary standard of resistance, it is particularly important to resolve any possible discrepancies at the highest possible level of accuracy. It is equally important to confirm the theoretical prediction[4] that $R_K = h/e^2 = \alpha^{-1}\mu_0 c/2$, where h is the Planck constant, e is the fundamental charge, α^{-1} is the inverse fine-structure constant, c is the speed of light in vacuum, and μ_0 is the permeability of free space. Although the equality of R_K in different materials does not prove the exactness of this relationship, material independence is a significant factor in establishing the fundamental nature of R_K.

In this Letter we describe a sensitive 1:1- or 2:1-ratio cryogenic current-comparator (CCC) bridge[5] and its use for making the first direct comparison, in the same temperature enclosure and magnetic field, between the quantized Hall resistances in a gallium arsenide heterostructure and in a silicon metal-oxide-semiconductor field-effect transistor (MOSFET).

A diagram of the bridge circuit is shown in Fig. 1. The CCC consists of four 10000-turn windings surrounded by a conventional[6] superconducting shield. Magnetic flux in the CCC is sensed by an 8-turn niobium coil connected to the input coil of a commercial 20-MHz rf SQUID (superconducting quantum interference device). The whole structure, immersed in liquid helium at 4.2 K, is shielded from external fields by concentric cylinders of lead and low-temperature Mumetal. The current to flux transfer ratio of a single winding is 0.45 nA/φ_0, where φ_0 is the flux quantum. By connecting windings in series opposition the 1:1 ratio, between any pair of the three used for the measurements, was found to be unity to better than 5 parts in 10^{12}.

A primary constant current source supplies current I_{SD} to a GaAs heterostructure or a Si MOSFET, maintained at a temperature of about 0.45 K, in series with a

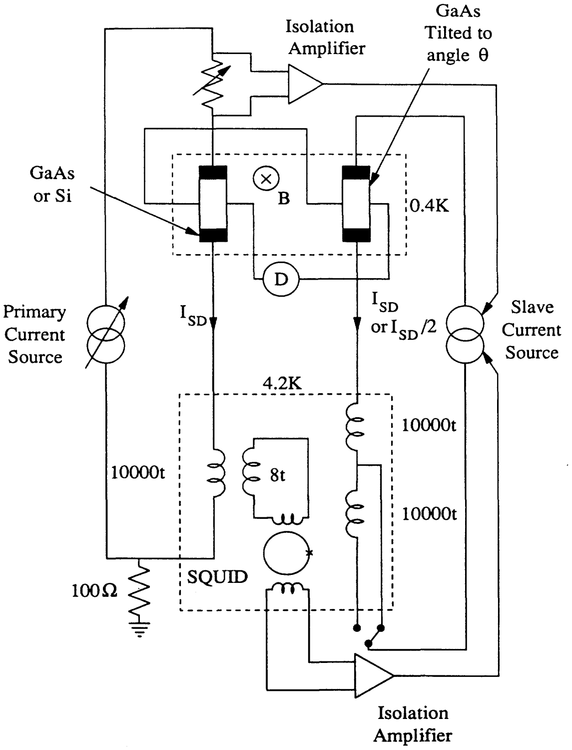
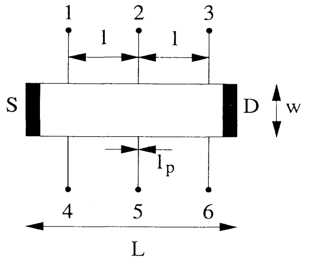


FIG. 1. Outline circuit diagram of CCC bridge. The CCC and SQUID and the devices are maintained at the temperatures indicated.

single winding of the comparator. A voltage derived from I_{SD} generates a slave current, I_{SD} (or $I_{SD}/2$), which feeds a second GaAs heterostructure, whose plane can be tilted to an angle θ with respect to the vertical magnetic induction B, in series with a single (or double) winding of the CCC. Any imbalance in the primary to slave current ratio is corrected to the CCC ratio by a negative feedback voltage, derived from the SQUID via an isolation amplifier and applied to the slave current source. The available dc loop gain of $\approx 10^7$ allowed the ratio of the currents to be maintained equal to the comparator ratio to better than 1 part in 10^{10} for all measurement currents.

The two halves of the bridge are connected together by a link joining equivalent Hall contacts on the high potential side of the two devices, while the low-potential contacts are connected to a null detector (an EM N11 nanovoltmeter).

The bridge is connected to the screen through a 100-Ω resistor (Fig. 1), which ensures that the CCC windings and the null detector can be maintained close to the screen potential without causing too much disturbance to the loop-control of the servo system. In addition to guarding against leakage currents between the Hall devices and the screen, the resistor also helps to reduce interference currents which may flow in capacitances between the comparator and screen. To eliminate direct leakage paths between connections to the Hall devices the isolation was maintained at $\approx 10^{14}$ Ω. With the magnetic field applied frequent checks were made for high resistances (>5 Ω) in the device contacts since it has been suggested[7] that these could lead to errors in R_K. In any case, the total resistance in the leads joining the high potential contacts should be less than about 20 Ω so that the current, measured to be less than 1 pA, flowing between them does not cause a significant systematic error.



Device	L (mm)	w (mm)	l (mm)	l_p (mm)	μ (T^{-1})	n_s ($\times 10^{15} m^{-2}$)
EPF 234	2.40	0.40	0.60	0.040	18.5	4.6
EPF 241	2.40	0.40	0.60	0.040	16.6	4.6
K402 #2	2.20	0.33	0.63	0.025	0.84	13.0

FIG. 2. Device characteristics. The dimensions are defined in the plan view. μ is the mobility and n_s is the two-dimensional electron concentration. For device K402 No. 2 the values of μ and n_s apply for $V_g = 48.2$ V.

The isolated output of the null detector was fed to a chart recorder and all measurements were taken from the recorder traces. A 10-GΩ resistor which could be connected in parallel with one of the GaAs devices provided a calibration signal.

In order to check the accuracy of the bridge the values of $R_H(2)$ of two GaAs/AlGaAs heterostructures, EPF 234 (device A) and EPF 241 (B), having similar characteristics and identical dimensions (Fig. 2), were compared at $T = 0.45$ K and $B = 9.6$ T via their 2-5 contacts. The values of the sheet resistivities ρ_{xx} between contacts 1-3 or 4-6, measured parallel to the current $I_{SD} = 40.5$ μA, were found to be zero within the measurement uncertainty of about 8 μΩ/□. The measurement proceeded by recording the null detector output for nineteen current sequences of the form $+I_{SD}, -I_{SD}, +I_{SD}$, to eliminate thermal emfs and their linear drifts with time, from which the ratio of the $R_H(2)$ values was calculated. The result of the comparison between devices A and B, as expressed by

$$R_H(i;A)/R_H(j;B) = (j/i)[1+\Delta_{ij}(\varepsilon_{ij})]\,, \tag{1}$$

where Δ_{ij} is the relative difference from the expected integer ratio j/i with a 1σ random error of the mean of

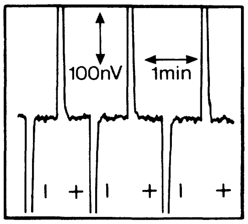


FIG. 3. Chart recording of the null-detector output for the direct comparison of R_H(4;Si) and R_H(2;GaAs) for several positive to negative current reversals. I_{SD}(Si) = 54 μA, I_{SD}(GaAs) = 27 μA, T = 0.43 K, and B = 13.4 T. The time constant of the null detector was 1.3 s.

$\pm\varepsilon_{ij}$, was $\Delta_{22} = -1.2(2.5)\times 10^{-10}$.

To investigate the possible effects of leakage currents driven by the Hall voltages through either the devices or external wiring, a 1:1 comparison was also made between the $R_H(4)$ values. Unfortunately, when operating at B = 4.8 T, the values of ρ_{xx} were no longer zero, leading to errors $\Delta R_H(4)$ in $R_H(4)$. It has been observed previously[8] for these devices, however, that $\Delta R_H(4) \simeq k\rho_{xx}$, where k is a constant depending on a particular pair of Hall contacts. For contacts 2-5 it was also observed that k reverses sign and has approximately the same value following a magnetic field reversal. To a good approximation, if ρ_{xx} is not too large, a more correct value of $R_H(4)$ can be obtained by taking the mean of measurements for both directions of B. For I_{SD} = 27 μA, Δ_{44} is $-12.3(1.6)\times 10^{-9}$ for $-B$, and $+14.9(1.9)\times 10^{-9}$ for $+B$. The unweighted mean is $\bar{\Delta}_{44} = 1.3(1.8)\times 10^{-9}$, which is not significantly different from zero. The corresponding mean values of ρ_{xx} were 315 and 488 $\mu\Omega/\square$ for devices EPF 234 and EPF 241, respectively.

Interchanging the role of the comparator windings did not lead to significant changes in Δ_{ii} suggesting there were no significant leakage currents associated with the connections to the windings.

Since Δ_{ii} is expected to be zero for two similar GaAs devices, the uncertainty of the Δ_{22} measurement, $\pm 2.5\times 10^{-10}$, is interpreted as the limit set on the systematic error of the bridge. Changing to a 2:1-ratio measurement should not have resulted in any significant addition to this estimate.

The GaAs heterostructure EPF 234 and a silicon MOSFET K402 No. 2 (see Fig. 2 for characteristics), having a particularly thick (800 nm) oxide layer and a very high breakdown current, were mounted in the sample holder at 0.43 K. Device EPF 234 was tilted by an angle $\theta = 44°$ so that its $i = 2$ plateau occurred when the magnetic induction normal to the plane of the device was 9.6 T, while K402 No. 2 was in the larger magnetic induction of 13.4 T. The gate voltage V_g of the Si device was adjusted to 48.2 V, corresponding to the center of the 0.8-V-wide $i = 4$ plateau. Measurements of ρ_{xx} and Δ_{24} were made over a current range from 13.5 to 67.5 μA and 6.75 to 33.75 μA for the Si and GaAs devices, respectively, for both directions of B. The values of ρ_{xx} were not significantly different from zero except at the two highest currents.

Figure 3 is a typical recorder trace of the output of the null detector for successive current reversals from which Δ_{24} and ε_{24} are derived [in Eq. (1), A and B are, respectively, the GaAs and silicon devices]. Values of Δ_{24} and $\pm\varepsilon_{24}$ in ppb (parts per billion or $\times 10^{-9}$) for different

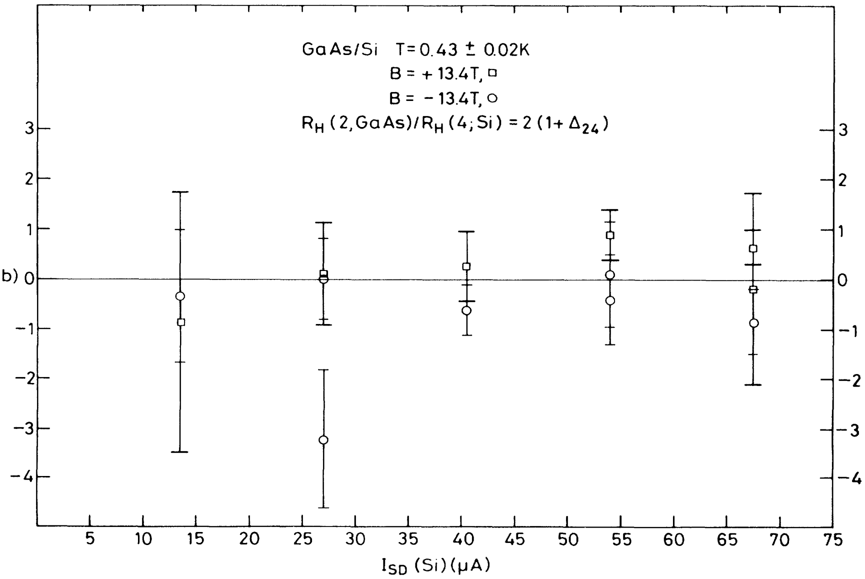


FIG. 4. Measurements of Δ_{24} expressed in ppb for the direct comparison of R_H(4;Si) and R_H(2;GaAs) as a function of I_{SD}(Si). The uncertainty bars represent the $\pm 1\sigma$ random error of the mean (ε_{24}) of Δ_{24}.

TABLE I. Mean values for $\pm B$ of surface resistivities $\bar{\rho}_{xx}(\mathrm{Si})$ and $\bar{\rho}_{xx}(\mathrm{GaAs})$, and $\bar{\Delta}_{24}(\varepsilon_{24})$ for device currents $I_{SD}(\mathrm{Si})$ and $I_{SD}(\mathrm{GaAs})$.

$I_{SD}(\mathrm{Si})$ (μA)	$\bar{\rho}_{xx}(\mathrm{Si})$ ($\mu\Omega/\square$)	$I_{SD}(\mathrm{GaAs})$ (μA)	$\bar{\rho}_{xx}(\mathrm{GaAs})$ ($\mu\Omega/\square$)	$\bar{\Delta}_{24}(\varepsilon_{24})$ (ppb)
13.5	0	6.75	0	−0.43(1.18)
27.0	3	13.5	0	−0.49(0.57)
40.5	8	20.25	0	−0.30(0.40)
54.0	14	27.0	21	0.48(0.38)
67.5	176	33.75	103	0.31(0.29)

currents $I_{SD}(\mathrm{Si})$ in the silicon device are plotted in Fig. 4. To eliminate the possibility of errors caused by contact misalignment together with nonzero values of ρ_{xx}, average values $\bar{\Delta}_{24}$ (Table I) were calculated for both directions of B. Although $\bar{\Delta}_{24}$ at the highest current is not significantly different from zero, because the ρ_{xx} values are relatively large it has not been included in the calculation of a weighted mean for $\bar{\Delta}_{24}$.

As a further check that the small, nonzero values of $\rho_{xx}(\mathrm{Si})$ cause insignificant errors, Δ_{24} was measured, with fixed currents, as a function of $\rho_{xx}(\mathrm{Si})$ by adjusting V_g by about 0.4 V towards the edge of the $i=4$ plateau, while the GaAs device acted as an unchanging reference standard. The values for $B=-13.4$ T, $I_{SD}(\mathrm{Si})=54$ μA, and $I_{SD}(\mathrm{GaAs})=27$ μA are plotted in Fig. 5. By fitting a straight line to the data points it can be shown that the Si device obeys the relationship $\Delta R_H(4;\mathrm{Si})=k\rho_{xx}(\mathrm{Si})$ with $k=0.048$, i.e., a relative error in $R_H(4;\mathrm{Si})$ greater than 1×10^{-10} occurs only for $\rho_{xx}(\mathrm{Si})>14$ $\mu\Omega/\square$. For $B=+13.4$ T a similar, but less well-defined relationship held over the whole range of ρ_{xx} with negative values of $\Delta[\Delta R_H(4;\mathrm{Si})]/\Delta\rho_{xx}(\mathrm{Si})$. For $\rho_{xx}(\mathrm{Si})$ less than 100 $\mu\Omega/\square$ the procedure of taking the mean of Δ_{24} for both directions of B to reduce errors is justified. These errors, caused by nonzero values of $\rho_{xx}(\mathrm{Si})$ are, in any case, only of marginal significance.

The final result, including the addition in quadrature of the bridge error estimated above, is

$$R_H(2;\mathrm{GaAs})/R_H(4;\mathrm{Si})=2[1-0.22(3.5)\times10^{-10}].$$

This shows that the quantized Hall resistances of GaAs and silicon are identical within the combined uncertainty of 3.5×10^{-10}.

The uncertainty associated with the value[9] recommeded for R_K to represent the ohm internationally is as high as 0.2 ppm, reflecting the spread in values obtained from several direct and indirect methods of measurement. The most accurate value, with a relative combined uncertainty of 6.9 ppb, is derived from a comparison of the experimental determination[10] of the electron magnetic moment anomaly a_e with a quantum electrodynamic calculation[11] of a_e from a series expansion in terms of the fine-structure constant α. This indirect method assumes that $R_K=h/e^2$ exactly.

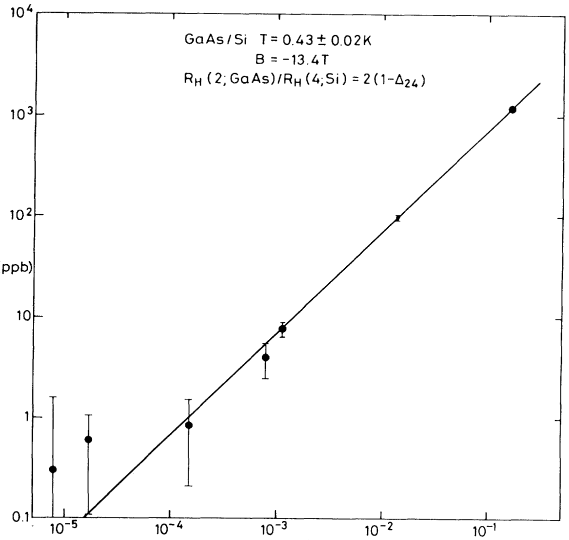


FIG. 5. Log-log plot of the variation of Δ_{24} in ppb as a function of $\rho_{xx}(\mathrm{Si})$. The line fitted to the data shows that $\Delta R_H(4;\mathrm{Si})=k\rho_{xx}(\mathrm{Si})$ with $k=0.048$.

The measurements reported in this Letter confirm that R_K is independent of the host material to within ±9.03 $\mu\Omega$, which is a substantially lower uncertainty than that claimed for any previous measurement. This result, therefore, gives added weight to the hypothesis that $R_K=h/e^2$, and a more accurate value of R_K, derived from the a_e experimental and theoretical values, could probably be recommended. However, before making this change it would be prudent to await the availability of at least one confirmatory alternative value of R_K having a comparable accuracy to that derived from a_e.

We thank Dr. W. Schwitz, Federal Office of Metrology, Berne, Switzerland, for the donation of devices EPF 234 and EPF 241 which were fabricated at the Federal Institute of Technology, Lausanne, Switzerland. K402 No. 2 was fabricated at the Science and Engineering Research Council Central Facility for Silicon Technology, University of Southampton, United Kingdom, for Professor L. Challis, University of Nottingham.

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