Early Stages of Nucleus-Nucleus Collisions: A Microscopic Calculation of the Initial Number of Degrees of Freedom

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A microscopic model for estimating n_0, the initial number of degrees of freedom used in calculating pre-equilibrium nucleon emission from heavy-ion reactions, is presented. The model follows the evolution of the geometrical and phase spaces during the process of the gradual fusioning of the target and the projectile. A good agreement is found between the calculated values of n_0 and the empirical values extracted from fits to nucleon spectra.

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Among the variety of models used to calculate the pre-equilibrium emission of nucleons from nucleus-nucleus collisions at intermediate energies, those based on the master-equation approach[1] all assume that the equilibration process starts from some simple configuration, characterized by an initial number of degrees of freedom, n_0. Although in view of the complexity of the equilibration of a multinucleon system n_0 should be a source term[2,3] rather than a single number, the use of a simple numerical value for n_0 has been surprisingly effective in describing the observed spectra with remarkable accuracy.[4] Thus in the Boltzmann-master-equation approach one has[2,5]

$$N(U)\Delta U \propto \frac{U^{n_0-1}-(U-\Delta U)^{n_0-1}}{(E^*)^{n_0-1}} \tag{1}$$

for the number of nucleons $N(U)$ emitted in the energy interval ΔU around the residual excitation energy U at the high-energy edge of the spectrum, with E^* the total excitation energy of the composite system. Similarly, in the exciton or the hybrid models[1]

$$N(U) \propto \frac{1}{(E^*)^{n_0-1}}\frac{dU^{n_0-1}}{dU}=(n_0-1)E^*\left(\frac{U}{E^*}\right)^{n_0-2}, \tag{2}$$

which is the limit of Eq. (1) for $\Delta U \to 0$.

The value of n_0 for a given collision is extracted from appropriate data as a phenomenological parameter characterizing the reaction. A simple way to obtain it is to apply Eq. (2) in its logarithmic form[6]

$$\ln[N(U)] \propto (n_0-2)\ln U \tag{3}$$

to the high-energy part of nucleon spectra from central collisions. A complete master-equation treatment of these spectra yields, ideally, the same value; in practice, the difference between the two is unessential.[4] In this paper we propose a microscopic model for calculating n_0. This model is based on simple geometrical and phase-space considerations with a few straightforward assumptions and it does not introduce any adjustable parameters.

The model.— In momentum (phase) space the colliding system is represented by two spheres of radii P_F, whose distance is P_P+P_T. P_F is the Fermi momentum, P_P and P_T are, respectively, the per nucleon momenta at the touching point of the projectile and the target (Fig. 1). Clearly,

$$P_F=(2m_0E_F)^{1/2},\quad P_P=(2A_Pm_0E_P)^{1/2}/A_P\,,\qquad P_T=(2A_Tm_0E_T)^{1/2}/A_T\,, \tag{4}$$

with m_0 the nucleon mass, E_F the Fermi energy, and the energies

$$E_P=\frac{A_T}{A_P+A_T}E_0,\quad E_T=\frac{A_P}{A_P+A_T}E_0\,. \tag{5}$$

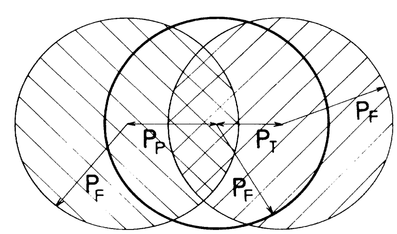

FIG. 1. The colliding system in momentum space. The heavy-line circle represents the final (equilibrated) composite system.

Here, E_0 is the center-of-mass energy of the collision, diminished by the corresponding Coulomb barrier V_{CB}, $E_0 = E_{c.m.} - V_{CB}$. To obtain the Fermi momentum we have used $E_F = 25$ MeV and for the Coulomb barrier we set $r_0 = 1.5$ fm.

The newly created composite system in its final state (i.e., after reaching the equilibrium) is represented by a third sphere of radius P_F, whose center coincides with the center of mass of the colliding system. The part of this sphere fed by both colliding nuclei is illustrated by the doubly shaded area in Fig. 1. This part has all the levels filled and hence it does not contribute particle-hole pairs to the excitation of the composite system. In all the remaining phase space within the spheres, particles and holes will be created by the collision. Shown schematically, holes from the projectile (h_P) will all stem from the region inside the composite system, where this sphere does not overlap with the projectile spheres [shaded area in Fig. 2(a)]; correspondingly, holes from the target (h_T) will stem from the region of no overlap between the target and the composite sphere [shaded area in Fig. 2(b)]. Note that the polar regions, fed neither by the projectile nor by the target, are counted twice. On the other hand, nucleons outside the composite sphere do not produce holes. Consequently, all these nucleons should be considered as particles [$p_P + p_T$, shaded areas in Fig. 2(c)]. From simple geometry it follows that the number of holes h_0 equals that of particles p_0. Finally, $n_0 = p_0 + h_0 = (p_P + p_T) + (h_P + h_T)$. Thus the number of particles and holes is calculated as the geometric overlap of parts of three spheres of equal radii in momentum space.

This number should be modified when the geometrical overlap of the two colliding nuclei is taken into account.

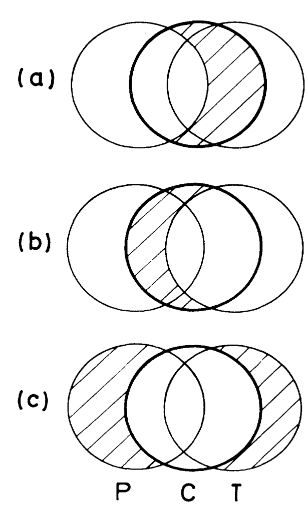

FIG. 2. Overlapping volumes for calculating the number of holes stemming from (a) the projectile, (b) the target, and (c) the number of particles. The depicted spheres are the same in all three parts, (a)-(c), and also identical to those in Fig. 1.

In the geometrical space the colliding system is represented by the projectile and target spheres of radii R_P and R_T, respectively. The overlap volume of the two colliding nuclei (projectile and target) is calculated at each moment of the collision. The time when two colliding nuclei touch is set as zero time; we stop the calculations at the time when the whole lighter partner has entered into the heavier partner. The whole process of gradual fusioning of the target and projectile is followed in steps of $\Delta t = 2.1 \times 10^{-23}$ s, assuming that the relative velocity of the collision partners is not affected significantly by friction. The time interval corresponds to the average time between two subsequent nucleon-nucleon collisions in nuclear matter.[2] We assume that the energy of the composite system gradually increases as the nucleons from the lighter partner, which we always assume to be the projectile, enter the target. Taking into account the conservation of the total number of particles, energy, and momentum we obtain the following relation for the dependence of the available excitation energy on the collision time t:

$$E_0(t) = E_0 \frac{A_T + A_P}{A_T + \Delta A(t)} \frac{\Delta A(t)}{A_P}, \tag{6}$$

where $\Delta A(t)$ is the number of nucleons from the lighter partner enveloped into the heavier partner at a given collision time t. At $t=0$, $\Delta A(0) = 0$ hence $E_0(0) = 0$, while at a certain $t = t_{max}$ when $\Delta A(t) = A_P$, $E_0(t_{max}) = E_0$, with E_0 given in Eq. (5). Obviously, since E_0 depends on t, all kinematical variables in the phase space (except P_F) will also depend on t. Therefore, we extend Eqs. (4) and (5) with the following set of equations:

$$P_P(t) = P_P[E_0(t)/E_0]^{1/2}, \quad P_T(t) = P_T[E_0(t)/E_0]^{1/2}, \tag{7}$$

$$E_P(t) = E_P E_0(t)/E_0, \quad E_T(t) = E_T E_0(t)/E_0, \tag{8}$$

where P_P, P_T, E_P, and E_T are defined in Eqs. (4) and (5). Figures 1 and 2 refer to the case when $t = t_{max}$ and $E_0(t_{max}) = E_0$; at $t = 0$ all the three spheres in the phase space coincide since $E_0(0) = 0$.

The calculation of $n_0(t)$ according to the model outlined above is straightforward. Figure 3 shows the dependence of the so-calculated values $n_0(t)$ on the collision time t for $^{32}S + ^{197}Au$ at 503.7 MeV. Obviously, we take as n_0 the maximal value of $n_0(t)$.

Comparison with empirical values.— Historically, the first estimate of n_0 was to set $n_0 = A_P$. The underlying idea was an extension from light-ion data, consistent with a complete break up of the projectile.[2] With the progress of the analysis, however, some energy dependence emerged. The value of n_0/A_P was found to increase with E_0/A_P, the available energy per nucleon. This dependence was observed for a wide range of reactions at energies up to about 20 MeV/nucleon.[7] More

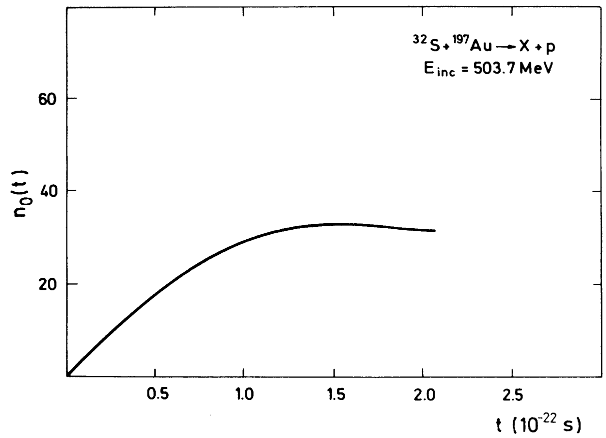

FIG. 3. Evolution of the calculated value of $n_0(t)$ with the collision time t.

recently, a linear dependence of E^*/n_0 on the energy brought in by the collision, $(E_{\rm lab}-V_{\rm CB})/A_P$, has been observed above 10 MeV/nucleon.[8]

Using the model described above, we calculated the values of n_0 for a number of nucleus-nucleus collisions and compared them to the values extracted from a fit to the data using a Boltzmann-master-equation analysis, where we have selected data from central collisions only.[8] This comparison is shown in Table I. The agreement with the fit values is fairly good. Varying the value of the Fermi energy energy does not destroy the agreement: Varying E_F by $\pm 15\%$ changes n_0 by only about 10%.

Finally, Fig. 4 displays the dependence of $E^*/n_0 = (E_{\rm c.m.}+Q_{\rm fus})/n_0$ on $(E_{\rm c.m.}-V_{\rm CB})/A_P$. On the top (open symbols) are the calculated values of E^*/n_0 for reactions induced by ^{16}O, ^{32}S, and ^{58}Ni projectiles on various targets; the corresponding empirical (extracted) values[8] are shown by the same solid symbols (bottom). The calculated values reproduce the empirical values very well. The agreement is corroborated also by the solid line in Fig. 4, which shows the best-fit line through the values calculated from the model. This line is given by

$$E^*/n_0 = 4.6 + 0.54(E_{\rm c.m.}-V_{\rm CB})/A_P\,. \qquad (9)$$

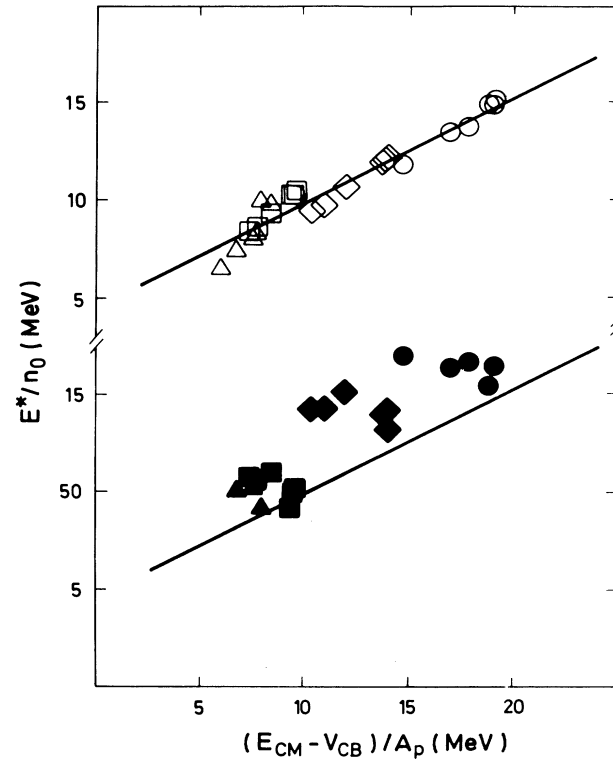

FIG. 4. Dependence of E^*/n_0 on the per nucleon energy above the Coulomb barrier for the systems from Table I. For each projectile (and incident energy) the extracted fitted values of n_0 are shown as solid symbols (bottom) and the corresponding calculated values as empty symbols (top). Symbols: circles for ^{16}O, squares and diamonds for ^{32}S, and triangles for ^{58}Ni projectiles. Lines: Eq. (9).

The empirical trend observed in Ref. 8 could be represented by the following equation:

$$E^*/n_0 = 6.8 + 0.54(E_{\rm c.m.}-V_{\rm CB})/A_P\,. \qquad (10)$$

[The difference between this equation and Eq. (2) of Ref. 8 stems from the use of c.m. energies instead of the laboratory ones.] The comparison between Eqs. (9) and (10) shows that the empirical dependence of E^*/n_0 on $(E_{\rm c.m.}-V_{\rm CB})/A_P$ from Ref. 8 and the one obtained in

TABLE I. Fit (extracted) (Ref. 8) and calculated values of n_0 for ^{16}O, ^{32}S, and ^{58}Ni projectiles and selected targets.

Projectile / Target	^{16}O (403 MeV) Fit	^{16}O (403 MeV) Calc.	^{32}S (504 MeV) Fit	^{32}S (504 MeV) Calc.	^{32}S (679 MeV) Fit	^{32}S (679 MeV) Calc.	^{58}Ni (876 MeV) Fit	^{58}Ni (876 MeV) Calc.
^{27}Al	16	23.0	23	29.5	23	34.7	26	28.4
^{46}Ti	19	23.0	28	34.5	28	40.9	35	47.4
^{60}Ni	19	23.0	29	34.2	29	40.8		59.3
^{120}Sn	21	23.1	35	33.8	35	40.7	46	58.0
^{124}Sn		23.2	35	33.9	35	40.8	46	58.2
^{197}Au	22	22.7	37	32.7	37	40.1	61	56.3

this work are essentially identical, except for a small systematic difference in the absolute values values of E^*/n_0.

To summarize, a microscopic model to calculate n_0, the value of the initial number of degrees of freedom that share the available excitation energy in a nucleus-nucleus collision (the initial exciton number), has been developed for the first time. The empirical values of n_0 were so far extracted from master-equation and plot analyses of nucleon spectra. By taking for n_0 its maximal value during the process of gradual fusioning of the target and projectile, the values of n_0 calculated from the model reproduce the empirical fit values rather well. They also reproduce the two main trends observed for n_0, namely, the increase of n_0/A_P with the available energy $(E_{\text{c.m.}} - V_{\text{CB}})/A_P$ and the linear dependence of E^*/n_0 (the share of the total excitation energy per initial degree of freedom) on this quantity.

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