Useful Theorem on Vanishing Threshold Contribution to $\sin^2\theta_W$ in a Class of Grand Unified Theories

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Superheavy particles near the unification threshold introduce modifications to precise grand-unified-theory (GUT) predictions. We establish a very useful and general theorem valid in a class of models possessing the custodial symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C \times P$ at the highest intermediate scale, according to which the one-loop GUT-threshold contribution to $\sin^2\theta_W$ by every class of superheavy particles (gauge bosons, Higgs scalars, and additional fermions) vanishes. The result also applies with supersymmetry, infinite towers, or higher-dimensional operators, and is independent of other intermediate symmetries at lower scales.

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After the discovery of the standard gauge theory $SU(3)_C \times SU(2)_L \times U(1)_Y$ $(=G_{\rm st})$, many attempts have been made towards achieving a truly unified theory with a single gauge coupling. Currently, a number of grand unified theories (GUTs) such as SO(10), SO(18), E_6, and others including supersymmetric SU(5) are being taken as prospective candidates awaiting experimental tests. Interesting predictions, including proton lifetime (τ_p), $\sin^2\theta_W$, fermion masses, strong and weak *CP* violations, and an attractive inflationary big-bang cosmology, are found to be possible in models with one or more intermediate gauge symmetries (IGSs). While the origin of some GUTs might be traced to the higher-dimensional unification with gravity, certain others with $N=1$ supergravity emerge as effective low-energy theories of superstrings. Although earlier predictions included contributions of particles sufficiently lighter than the unification mass (M_U), the effective-gauge-theory approach[1] has demonstrated that there could be very significant GUT-threshold effects due to superheavy masses $(\simeq M_U)$. In models such as SO(10), SO(18), E_6, and others, Higgs scalars necessary for spontaneous symmetry breaking occur as small components of much larger representations, giving rise to many superheavy scalars with masses around M_U and larger threshold modifications to the predicted values of τ_p and $\sin^2\theta_W$. More specifically, a factor-of-10 nondegeneracy[2] among the superheavy masses has been found[3] to decrease $\sin^2\theta_W$ by 0.02 in SO(10) with the IGS $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ $(\equiv G_{2213}, g_{2L} \neq g_{2R})$, compared to the earlier results[4] using the mechanism of decoupling P (=parity) and $SU(2)_R$ breakings.[5] Since the Coleman-Weinberg mechanism[2] restricts the mass of only one scalar component in the larger Higgs representation of a GUT, the assumption that all superheavy-component masses differ from M_U by a factor of 10 is reasonably stringent. We use the notation G_{224P} to represent the Pati-Salam gauge group, $SU(2)_L \times SU(2)_R \times SU(4)_C \times P$, including parity $(=P)$, the left-right discrete symmetry, with equal couplings for $SU(2)_L$ and $SU(2)_R$ $(g_{2L}=g_{2R})$. G_{224} represents the same gauge group but without P $(g_{2L} \neq g_{2R})$. When SO(10) breaks through the vacuum expectation value of the G_{224} singlet contained in the Higgs representation **54** **(210)**, G_{224P} (G_{224}) survives as an IGS. GUT-threshold effects in SO(10) including G_{224} and other IGSs have been investigated in Ref. 6 and found to modify $\sin^2\theta_W$ (τ_p) by 0.02–0.06 (2–4 orders of magnitude) for an assumed nondegeneracy factor of 30 in superheavy Higgs-scalar masses.

The purpose of the present Letter is to point out, for the first time, the existence of a class of models containing G_{224P} where the one-loop contribution to the GUT-threshold correction on $\sin^2\theta_W$ vanishes according to the following theorem and proof.

Theorem.— In all grand unified theories where the custodial symmetry G_{224P} occurs at the highest intermediate scale $(M_P < M_U)$, the one-loop GUT-threshold contribution to $\sin^2\theta_W$ by every class of superheavy particles vanishes.

Proof.— We use renormalization-group equations (RGEs) for the coupling constant $g_i(\mu)$ of the gauge group G_i occurring in the IGS or the standard symmetry expressed as $\mathcal{G} = G_1 \times G_2 \times \cdots$,

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M)} + \frac{a_i}{2\pi}\ln\frac{M}{\mu} + \frac{1}{8\pi^2}\sum_j b_{ij}\int_\mu^M \alpha_j(\mu)\frac{d\mu}{\mu}\,, \tag{1}$$

$i,j=1,2,3\ldots$, where $\alpha_i(\mu)=g_i^2(\mu)/4\pi$, a_i (b_{ij}) is the one- (two-) loop coefficient of the β function, and μ (M) is the lower (higher) scale. At first we establish the theorem in the simplest case of any GUT G [=SO(10), SO(12), SO(14), SO(16), SO(18), SU(16), E_6, etc.] with the single IGS

$$G \xrightarrow{M_U} G_{224P} \xrightarrow{M_P} G_{\rm st} \xrightarrow{M_W} SU(3)_C \times U(1)_{\rm em}\,. \tag{2}$$

The superheavy-particle effects on $g_i(\mu)$ for $\mu \simeq M_U$ in supersymmetric (SUSY) and non-SUSY GUTs are well known to have the form, at the one-loop level,[1,3]

$$\frac{1}{\alpha_i(\mu)}=\frac{1}{\alpha_G}-\frac{\lambda_i(\mu)}{12\pi},\quad i=2L,2R,4C\,, \tag{3}$$

where α_G corresponds to the GUT coupling. It must be stressed that our proof does not depend upon the exact nature of the expression for $\lambda_i(\mu)$. It goes through in all models so long as the threshold correction has the form (3). The contribution of at least the low-lying components in infinite towers has the form (3). Also the string-loop effects, demonstrated[7] to have the form (3) for the standard effective theory ($i=2L,Y,3C$) are expected to retain the same form for G_{224P}. Using (1) and (3), we obtain

$$\begin{aligned}
\ln\frac{M_U}{M_P}&=\frac{1}{a_{2L}'+a_{2R}'-2a_{4C}'}\Bigg\{2\pi(\alpha^{-1}-\tfrac{8}{3}\alpha_s^{-1})-(\tfrac{5}{3}a_Y+a_{2L}-\tfrac{8}{3}a_{3C})\ln\frac{M_P}{M_W}\\
&\qquad-\frac{1}{4\pi}(\tfrac{5}{3}\phi_Y^P+\phi_{2L}^P-\tfrac{8}{3}\phi_{3C}^P+\phi_{2L}^U+\phi_{2R}^U-2\phi_{4C}^U)+\tfrac{1}{6}(\lambda_{2L}^U+\lambda_{2R}^U-2\lambda_{4C}^U)\Bigg\}\,,\\
\sin^2\theta_W&=\frac{1}{a_{2L}'+a_{2R}'-2a_{4C}'}\Bigg\{a_{2L}'-a_{4C}'+\frac{\alpha}{\alpha_s}(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')\\
&\qquad+\frac{3\alpha}{16\pi}[(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')(\tfrac{5}{3}a_Y+a_{2L}-\tfrac{8}{3}a_{3C})\\
&\qquad\qquad-\tfrac{5}{3}(a_{2L}'+a_{2R}'-2a_{4C}')(a_Y-a_{2L})]\ln\frac{M_P}{M_W}\\
&\qquad+\frac{3\alpha}{64\pi^2}[(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')(\tfrac{5}{3}\phi_Y^P+\phi_{2L}^P-\tfrac{8}{3}\phi_{3C}^P+\phi_{2R}^U+\phi_{2L}^U-2\phi_{4C}^U)\\
&\qquad\qquad-(a_{2L}'+a_{2R}'-2a_{4C}')(\tfrac{5}{3}\phi_Y^P-\tfrac{5}{3}\phi_{2L}^P+\phi_{2R}^U+\tfrac{2}{3}\phi_{4C}^U-\tfrac{5}{3}\phi_{2L}^U)]\\
&\qquad+\frac{\alpha}{96\pi}[3(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')(2\lambda_{4C}^U-\lambda_{2L}^U-\lambda_{2R}^U)\\
&\qquad\qquad+(a_{2L}'+a_{2R}'-2a_{4C}')(2\lambda_{4C}^U+3\lambda_{2R}^U-5\lambda_{2L}^U)]\Bigg\}\,,
\end{aligned}\tag{4}$$

where a_i' (b_{ij}') is the one- (two-) loop coefficient corresponding to G_{224P}, $\lambda_i^U=\lambda_i(M_U)$,

$$\phi_i^P=\sum_j b_{ij}\int_{M_W}^{M_P}\alpha_j(\mu)\frac{d\mu}{\mu},\quad \phi_i^U=\sum_j b_{ij}'\int_{M_P}^{M_U}\alpha_j(\mu)\frac{d\mu}{\mu}\,,$$

and we have ignored negligible threshold corrections at lower scales. It is clear that the one-loop superheavy-particle contributions are

$$\begin{aligned}
\Delta_1\ln\frac{M_U}{M_P}&=\frac{1}{6}\frac{\lambda_{2L}^U+\lambda_{2R}^U-2\lambda_{4C}^U}{a_{2L}'+a_{2R}'-2a_{4C}'}\,,\\
\Delta_1\sin^2\theta_W&=\frac{\alpha}{96\pi}[3(a_{2R}'+\tfrac{2}{3}a_{4C}'-\tfrac{5}{3}a_{2L}')(2\lambda_{4C}^U-\lambda_{2L}^U-\lambda_{2R}^U)\\
&\qquad+(a_{2L}'+a_{2R}'-2a_{4C}')(2\lambda_{4C}^U+3\lambda_{2R}^U-5\lambda_{2L}^U)](a_{2L}'+a_{2R}'-2a_{4C}')^{-1}\,,
\end{aligned}\tag{5}$$

where we have used the subscript 1 indicating that the corrections are one-loop effects. Since left-right symmetry is preserved due to the presence of G_{224P} for $\mu\geq M_P$, $a_{2L}'=a_{2R}'$ and $\lambda_{2L}^U=\lambda_{2R}^U$. These constraints, when used in (5), yield

$$\begin{aligned}
\Delta_1\ln\frac{M_U}{M_P}&=\frac{\lambda_{2L}^U-\lambda_{4C}^U}{6(a_{2L}'-a_{4C}')}\,,\\
\Delta_1\sin^2\theta_W&=0\,.
\end{aligned}\tag{6}$$

Q.E.D.

In proving the theorem we have not used any specific particle content to compute a_i, a_i', or λ_i^U. Thus the theorem is independent of the nature and number of particles, light, heavy, or superheavy (degenerate or nondegenerate), whether they are components of supersymmetric GUTs or infinite towers resulting from compactification of extra dimensions. Using more IGSs below M_P, with or without decoupling P and $SU(2)_R$ breakings, as shown in Fig. 1, we have checked that the analytic expressions (5) and (6) are independent of these symmetries. This is understood in view of the well-known fact that the GUT-threshold correction to M_U

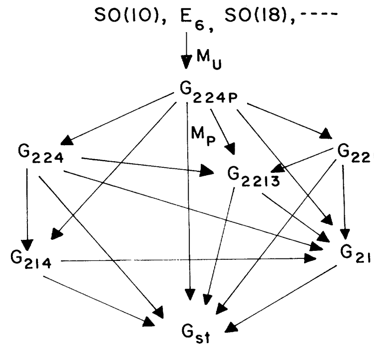

FIG. 1. Schematic representation of some models with vanishing GUT-threshold contribution to $\sin^2\theta_W$, where $G_{214} = SU(2)_L \times U(1)_R \times SU(4)_C$ and other symbols are as explained in the text.

and $\sin^2\theta_W$ involves only those a_i coefficients of (1) corresponding to the symmetry group immediately below M_U. Equations (5) and (6) have the same analytic structure irrespective of the GUT symmetry breaking down to G_{224P}, although the numerical values of a_i, a_i', and λ_i^U might differ depending upon the particle content. Also in every case of Fig. 1, if a GUT of higher rank (e.g., E_6) breaks down to another GUT of lower rank [e.g., SO(10)] containing G_{224P} at a scale $M_U' > M_U$, then $\Delta_1 \sin^2\theta_W = 0$. Thus the theorem is general and is valid in a large class of models.

The twin ideas of left-right symmetry and quark-lepton unification were combined by Pati and Salam[8] through the partial unification symmetry G_{224P}. While establishing this novel and profound property of a class of GUTs, we find that G_{224P} acts as a custodial symmetry and its presence is uniquely essential at the highest intermediate scale $M_P < M_U$ as no other gauge group replacing it achieves $\Delta_1 \sin^2\theta_W = 0$.

The theorem has the potential to hold in other cases where the threshold corrections might not be due to superheavy-particle loops[7] or perturbative origin.[9] For example, in the model (2) with $G = SO(10)$ investigated by Shafi and Wetterich,[9] the threshold corrections at $\mu = M_U$ are of the form (3) due to the five-dimensional operator $\mathrm{Tr}(F_{\mu\nu}\phi_{(54)}F^{\mu\nu})$ [$\phi_{(54)}$ = Higgs field 54 $\subset SO(10)$, $F_{\mu\nu}$ = gauge-field tensor] when $\langle\phi_{(54)}\rangle \neq 0$. In this case λ_i^U involves an unknown parameter and the compactification scale (M_C). The procedure outlined in the proof then leads to the vanishing-operator (o) threshold effect $\Delta_o \sin^2\theta_W = 0$. If the model originates from spontaneous compactification of extra dimensions, then the results achieved here yield $\Delta \sin^2\theta_W = 0$, where $\Delta = \Delta_1 + \Delta_i + \Delta_o$ and Δ_i is the effect of the infinite towers when M_C is not very different from M_U. We expect this mechanism to help in building realistic unified models envisaged in Ref. 10.

Investigations in different models are now in progress in ordinary and SUSY GUTs including those inspired by superstrings.[7] However, some immediate consequences in the SO(10) model without SUSY or infinite towers are noted below. We find $\Delta_1 \sin^2\theta_W = 0$ whatever the number of IGSs below the parity-breaking scale M_P ($< M_U$), or the size of the Higgs representations; consequently, most of the intermediate scales do not change significantly due to the GUT-threshold effects. Denoting τ_p^l (τ_p) as the threshold-corrected (-uncorrected) prediction in the model (2) with $G = SO(10)$, we find $M_P = 4\times 10^{13}$ GeV, $M_U = 1.2\times 10^{15}$ GeV, $\sin^2\theta_W = 0.23$, and $r = \tau_p^l/\tau_p = 3^{\pm 1}$ ($10^{\pm 2}$) under the reasonably stringent assumption that the superheavy Higgs scalars are degenerate (nondegenerate) differing by a factor of 10 from M_U, where the + (−) sign applies for masses lighter (heavier) than M_U. Similarly, in the model of Ref. 11, $r \simeq 1$ ($10^{\pm 2}$) for the same degeneracy (nondegeneracy) factor. Interestingly, in the model considered important experimentally,[5]

$$SO(10) \xrightarrow{54} G_{224P} \xrightarrow{210} G_{224} \xrightarrow{210} SU(2)_L \times U(1)_R \times U(1)_{B-L} \times SU(3)_C (\equiv G_{2113}) \xrightarrow{126} G_{\mathrm{st}},$$

$\Delta_1 \sin^2\theta_W = 0$ and other predictions (except τ_p^l) are predominantly those of Ref. 5 when GUT-threshold effects are included although there are three or four large representations and three IGSs. A similar stringent assumption on the degeneracy (nondegeneracy) factor in superheavy-scalar masses yields $r = 6^{\pm 1}$ ($10^{\pm 2.5}$) and the value $r = 10^{-2.5}$ brings the model prediction closer to the experimentally accessible range on the proton lifetime.

As the theorem applies in the presence of the custodial symmetry G_{224P} with unbroken parity at the highest intermediate scale, it does not contradict larger threshold corrections to $\sin^2\theta_W$ in those models where P is broken at the GUT scale.[6]

Finally, we conclude that we have identified a class of models where uncertainties in the $\sin^2\theta_W$ prediction due to the GUT-threshold effects are drastically reduced as the dominant one-loop contribution is zero. The theorem is expected to provide important guidelines for future model building.

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