

Constraints on the Gravitational Properties of Antiprotons and Positrons from Cyclotron-Frequency Measurements

Richard J. Hughes and Michael H. Holzscheiter

Physics Division, P-15, Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545

(Received 15 October 1990)

Tests of the weak equivalence principle for antiprotons and positrons are inferred from the results of particle-antiparticle cyclotron-frequency comparisons. The potential improvements from future, higher-precision experiments are discussed.

PACS numbers: 04.80.+z, 14.20.Dh, 14.60.Cd, 32.80.Pj

A fundamental question in physics which has yet to be addressed experimentally is whether particles of antimatter, such as the antiproton or positron, obey the weak equivalence principle.^{1,2} An experiment that will test this principle directly for antiprotons is under development³ at present. Meanwhile, high-precision experimental results on the equality of the inertial masses of protons and antiprotons have recently become available from an experiment which compares the cyclotron frequencies of the particles in the same magnetic field.⁴ (Similar results for electrons and positrons, although at a lower precision, have been in existence for almost ten years.⁵) These results are usually regarded as very sensitive tests of *CPT* symmetry. However, in this Letter we will show that they can instead provide tests of the weak equivalence principle for a gravitational coupling to the energy of positrons and antiprotons, under the assumption of exact *CPT* symmetry.

This possibility arises because the frequencies in question constitute local "clocks" and as such are subject to a gravitational redshift, which may be formulated as a test of weak equivalence for their energy content.⁶ To see this, we will consider a modified version^{6,7} of Einstein's gedanken experiment⁸ in which the gravitational redshift can be derived from conservation of energy in the gravitational field.

We start with some local clock based on the photon frequency in the transition $A^* \rightarrow A + \gamma$ between a system A and its excited state A^* . In a uniform gravitational field, A (A^*) has gravitational acceleration g (g^*) and inertial mass m (m^*), respectively. The relative rates of two such clocks at different heights in the gravitational field may be compared by exchanging photons.⁸ We start with system A located at some height h_1 and system A^* above it at height h_2 , with $h_2 - h_1 = l$. Next, A and A^* are interchanged, with the output of an amount of energy,

$$E_{\text{out}} = (m^* g^* - mg)l. \quad (1)$$

System A^* , now located at the lower height h_1 , is allowed to decay to its ground state A by emission of a photon of frequency ω_1 with $\hbar\omega_1 = (m^* - m)c^2$, which

defines the local clock frequency. This photon is allowed to propagate up to height h_2 , where its frequency will have suffered a generalized redshift to some value ω_2 , according to the local clock at h_2 ,

$$\hbar\omega_2 = \hbar\omega_1(1 - g_R l/c^2), \quad (2)$$

where the parameter g_R has the dimensions of an acceleration. In order to excite the system A at the upper level h_2 and recover the initial configuration, the energy carried by the photon must be augmented by an amount

$$E_{\text{in}} = (m^* - m)g_R l. \quad (3)$$

Conservation of energy requires $E_{\text{in}} = E_{\text{out}}$, and hence

$$g_R = \frac{m^* g^* - mg}{m^* - m} = \frac{\Delta(\text{clock weight})}{\Delta(\text{clock mass})}. \quad (4)$$

Thus, the gravitational redshift is a test of weak equivalence for the energy content of the clock, and the conventional redshift, for which $g_R = g$, only arises if weak equivalence, $g^* = g$, is obeyed. (It is important to note that the gravitational redshift has nothing to do with the "weight of light."^{9,10})

If *CPT* symmetry is assumed to be exact, the particle and antiparticle cyclotron-frequency clocks will have identical rates at "infinity" (beyond the range of any equivalence-principle-violating interaction). However, from the above argument, if the proton (or electron, respectively) respects the weak equivalence principle, and we assume that any violation of equivalence for the antiproton (positron) occurs through an anomalous coupling strength of gravity to its energy, the antiproton's (positron's) cyclotron frequency will redshift by an amount different from the proton (electron) when they are lowered to the same height in a gravitational field from "infinity,"^{11,12} resulting in a measurable frequency difference. Since there is a redshift in any theory in which gravity couples to energy,⁹ this argument applies to a tensor field coupled to the energy-momentum tensor, as well as to a scalar field coupled to its trace, but not to a vector interaction, since this would be coupled to some conserved quantum number rather than to energy.^{9,10} However, vector interactions, as well as more general

scalar interactions, are already constrained strongly by “fifth-force” experiments.¹³ Conversely, a gravitational coupling to energy with a universal strength for matter but with an anomalous strength for antimatter is not directly constrained by fifth-force experiments.⁹

To make this general argument quantitative we consider a phenomenological model for a violation of weak equivalence for antiprotons or positrons.^{11,12} Near the surface of the Earth, protons, electrons, and electromagnetism experience conventional gravity through the interaction Lagrangian

$$\mathcal{L} = \frac{1}{2} h_{\mu\nu} T^{\mu\nu}, \quad (5)$$

where $T^{\mu\nu}$ is the energy-momentum tensor of the particles and electromagnetism, the (weak) tensor gravitational field is

$$h_{\mu\nu} = (2U/c^2) \text{diag}(1, 1, 1, 1), \quad (6)$$

and U is the Newtonian gravitational potential. [We use the convention in which the metric of flat space-time is $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$.] A violation of weak equivalence for antiprotons or positrons is accommodated through the interaction

$$\mathcal{L}' = \frac{1}{2} \alpha h_{\mu\nu} T^{\mu\nu} \quad (7)$$

of gravity with the energy-momentum tensor of the antiparticles, where α is an adjustable coupling parameter. Hence, if protons and electrons experience a gravitational acceleration g , their antiparticles experience an anomalous acceleration $g' = \alpha g$ (different α parameters should be introduced for the various antiparticles under consideration) in this model. Later we will consider a second possibility in which the antiparticles violate weak equivalence because of a new, finite-range tensor field $h'_{\mu\nu}$ that only couples to their energy-momentum tensor.

The term (5) or (7) has to be added to the action of a single particle of charge e and inertial mass m in an electromagnetic field to obtain the effect of gravity on the particle's cyclotron frequency, $\omega_c = eB/m$, where B is the magnetic-field strength. Since the energy-momentum tensor $T^{\mu\nu}$ separates into particle and electromagnetic field components, the gravitational effects are contained in the particle action, which for matter is

$$S \simeq \int dt [-mc^2(1+U/c^2) + \frac{1}{2} mv^2(1-3U/c^2)] \quad (8)$$

(where v is the speed of the particle, and we have assumed $v^2/c^2, |U/c^2| \ll 1$), and in the electromagnetic field action

$$S_{\text{em}} \simeq \frac{1}{8\pi c} \int d^4x [(1-2U/c^2)\mathbf{E}^2 - (1+2U/c^2)\mathbf{B}^2], \quad (9)$$

but not in the particle-electromagnetic-field interaction term. From Eq. (8) we see that in these flat space-time coordinates we must make the inertial-mass replacement¹⁴

$$m \rightarrow m(1-3U/c^2) \quad (10)$$

for the electron or the proton in the Lorentz-force law, and from Eq. (9) we see that the gravitational field acts like a “medium” with permeability and permittivity¹⁴ $\mu = \epsilon = 1 - 2U/c^2$. Therefore, in these flat space-time coordinates the effect of gravity on the magnetic field produced by a given current distribution is obtained by the substitution

$$B \rightarrow B(1-2U/c^2). \quad (11)$$

The net result of the replacements (10) and (11) on the cyclotron frequency of the electron or proton is

$$\omega_c \rightarrow \omega_c(1+U/c^2) \quad (12)$$

in these coordinates. [This technique, which is suitable for the study of relative clock rates to $O(U/c^2)$, was used by Thirring¹⁴ to derive the conventional gravitational redshift in a clock based on atomic transition frequencies. However, it can be shown that^{9,10} because of the interaction (5) any clock (made of “matter”) suffers the frequency shift (12) with respect to this time coordinate. Therefore, when a local time variable $\bar{t} \equiv (1+U/c^2)t$ measured with respect to a local matter clock is introduced, the electron or proton cyclotron frequency is independent of height in the gravitational potential. Moreover, when the unit of length L is replaced by a local unit $\bar{L} \equiv (1-U/c^2)L$ based on the Bohr radius of the hydrogen atom, for instance,^{10,14} the Lorentz-force law assumes its normal form for a particle of matter of mass m and charge e in a magnetic field B . These transformations will be discussed in greater detail in a future paper.¹⁵]

For the cyclotron frequency $\bar{\omega}_c$ of the antiproton or the positron in the same magnetic field, we have, instead of Eq. (10),

$$m \rightarrow m(1-3\alpha U/c^2), \quad (13)$$

and hence, for these antiparticles we have

$$\bar{\omega}_c \rightarrow \bar{\omega}_c(1+[3\alpha-2]U/c^2), \quad (14)$$

so that there is a frequency difference between the matter and antimatter clocks of

$$(\bar{\omega}_c - \omega_c)/\omega_c = 3(\alpha-1)U/c^2 \quad (15)$$

at the same height, in this model. The potential dependence¹⁶ of the frequency difference (15) when $\alpha \neq 1$ is an inescapable consequence of the violation of equivalence, rather than a specific feature of our model. For a massless field this means that we would no longer have the freedom to change the value of the potential by a constant that is implied by the Newtonian field equations. In this case, following Good,¹⁶ it is logical to choose the zero of U to be at “infinity,” so that the frequency

difference (15) vanishes “in the absence of gravity.” No such prescription is necessary when the corresponding field has a finite mass, and we note that a conservative limit on the rest mass of the graviton is¹⁷ $m_g \leq 1.1 \times 10^{-29}$ eV, corresponding to a range of at least 580 kpc, although a range of ~ 10 Mpc is not excluded. We therefore follow Kenyon¹⁶ and choose the potential U in Eq. (15) to be the potential of the local supercluster, with $|U/c^2| \approx 3 \times 10^{-5}$.

The high-precision antiproton inertial-mass measurement of Gabrielse *et al.*,⁴ which involves the measurement of the cyclotron frequencies of the proton and antiproton in the same magnetic field, has already reached a precision of 4×10^{-8} on the frequency ratio (15). This result therefore conservatively constrains any weak-equivalence-principle violation in the coupling of a single tensor gravitational field to antiprotons to obey

$$|\alpha - 1| < 5 \times 10^{-4}. \quad (16)$$

A similar interpretation can be applied to the high-precision ion-trap measurements of electron and positron cyclotron frequencies. For these particles the frequency-difference ratio (15) has been measured to be less than 10^{-7} ,⁵ and so the positron is constrained to respect the weak equivalence principle with a precision of

$$|\alpha - 1| < 10^{-3}. \quad (17)$$

Therefore, the gravitational acceleration of the antiproton (positron) could differ from that of matter by no more than $5 \times 10^{-4}g$ ($10^{-3}g$, respectively) if the gravitational force is mediated by a single tensor field.

It is also possible to consider a scenario in which the usual tensor gravitational field couples to antimatter with its normal strength, but in addition there is a hypothetical new tensor field $h'_{\mu\nu}$ of finite range, which acts only on antimatter, with the form of Eq. (6). (A scalar field could also be considered.) The potential U , which is now of Yukawa form, has a value that depends upon the assumed range of this new interaction, the parameter α determines its strength relative to conventional gravity, and the frequency anomaly becomes

$$(\bar{\omega}_c - \omega_c)/\omega_c = 3\alpha U/c^2. \quad (18)$$

(With a scalar interaction the magnitude of the frequency anomaly would be a factor of 3 smaller than for a tensor.) For instance, if the range is large compared to the size of our Galaxy, but small relative to the separation of galaxies we should replace U by the value of our Galaxy's Newtonian potential at the surface of the Earth, $|U_{\text{Galaxy}}/c^2| \sim 10^{-6}$. In this case, the existing result for the electron-positron mass ratio⁵ limits a violation of weak equivalence for the positron in the gravitational field of the galaxy to have a strength $|\alpha| < 0.03$, while the result of Gabrielse *et al.*⁴ constrains such an interaction for the antiproton to have a strength which is

less than 1% of gravitational ($|\alpha| < 0.01$).

A stronger violation of the equivalence principle by antiprotons would have to have a correspondingly shorter range because the size of the potential decreases with decreasing range. Therefore, these results do not rule out a measurable (few percent) gravitational acceleration difference between antiprotons and matter. However, in the next few years improvements in trapping, cooling, and detection techniques for single charged particles could provide particle-antiparticle cyclotron-frequency comparisons which might be better than 1 part in 10^{11} .^{18,19} (A frequency comparison on single ions has already been achieved with a precision of 4×10^{-10} .¹⁹) For an anomalous interaction with a range which is large compared to the radius of the Earth, but small compared to the Earth-Sun distance we should use the Earth's Newtonian potential, $|U_{\oplus}/c^2| \approx 6 \times 10^{-10}$, in Eq. (18). A null result in such an experiment would then constrain the gravitational acceleration of positrons or antiprotons to differ from that of matter by less than 1 part in a 100,¹² which is comparable with the precision hoped for in a direct measurement.³

Other clocks which could offer potentially much higher-precision tests of weak equivalence for positrons and antiprotons are the hyperfine or $1s$ - $2s$ transitions in antihydrogen.²⁰ (Antihydrogen has yet to be produced in the laboratory, although several formation experiments are under discussion.²¹)

In this Letter we have shown that if there is a gravitational coupling to the energy of antimatter which violates the weak equivalence principle, there will be a frequency difference between a clock and its *CPT* conjugate, at the same height in a gravitational field. Existing experimental results already provide a strong test of the equivalence principle for antiprotons and positrons if the gravitational interaction is mediated by a single, essentially infinite-range tensor field. However, these results cannot yet significantly constrain an anomalous tensor (or scalar) gravitational interaction with these antiparticles if its range is less than the size of our Galaxy. Improvements in the precision spectroscopy of trapped single particles of antimatter and antihydrogen would allow these equivalence-principle tests to be extended to shorter ranges, thereby complementing direct measurements of the gravitational accelerations of antiprotons³ and positrons by constraining the range of an equivalence-principle-violating interaction from above. These ideas will be developed further in a future paper.¹⁵

Finally, there is the question of how to interpret a nonequality of proton and antiproton cyclotron frequencies should one arise in such a comparison. Obviously, this could be a signal of *CPT* violation instead of a violation of equivalence, and to distinguish the two possibilities one could compare the redshifts between two different heights in the *CPT*-conjugate clocks. A violation of equivalence would then show up as an anomalous

redshift in the antiproton cyclotron frequency. This would be extremely difficult in the Earth's gravitational field where the redshift has a magnitude of $g/c^2 \sim 10^{-16} \text{ m}^{-1}$, and an absolute maximum value of only 6×10^{-10} . Therefore, very large separations would be needed before a direct redshift measurement became feasible. Alternatively, this question could be decided with the direct measurement of the antiproton's gravitational acceleration.³

It is a pleasure to acknowledge helpful conversations with J. S. Bell, G. Gabrielse, and D. Wineland.

¹T. Goldman and M. M. Nieto, Phys. Lett. **112B**, 437 (1982); E. Segre, Annu. Rev. Nucl. Sci. **8**, 127 (1958).

²T. Goldman, R. J. Hughes, and M. M. Nieto, Phys. Lett. B **171**, 217 (1986); Sci. Am. **258**, No. 3, 48 (1988).

³N. Beverini *et al.*, CERN Report No. CERN/PSCC/86-2/P-94, 1986 (unpublished); Los Alamos Report No. LA-UR-86-260, 1986 (unpublished).

⁴G. Gabrielse *et al.*, Phys. Rev. Lett. **63**, 1360 (1989); **65**, 1317 (1990).

⁵P. B. Schwinberg, R. S. VanDyck, Jr., and H. G. Dehmelt, Phys. Lett. **81A**, 119 (1981).

⁶K. Nortvedt, Phys. Rev. D **11**, 245 (1975).

⁷R. J. Hughes, Am. J. Phys. **58**, 826 (1990).

⁸A. Einstein, Ann. Phys. **35**, 898 (1911).

⁹J. S. Bell, in *Fundamental Symmetries*, edited by P. Bloch

et al. (Plenum, New York, 1987).

¹⁰R. J. Hughes, Phys. Rev. D **41**, 2367 (1990).

¹¹R. J. Hughes, in *New and Exotic Phenomena '90*, edited by O. Fackler and J. Trinh Thanh Van (Editions Frontieres, Gif-sur-Yvette, 1990).

¹²R. J. Hughes, in Proceedings of the First Biennial Conference on Low-Energy Antiproton Physics, Stockholm, Sweden, July 1990 [Los Alamos Report No. LA-UR-90-2945 (to be published)].

¹³C. W. Stubbs, E. G. Adelberger, and E. C. Gregory, Phys. Rev. Lett. **61**, 2409 (1988).

¹⁴W. E. Thirring, Ann. Phys. (N.Y.) **16**, 96 (1961).

¹⁵R. J. Hughes and M. H. Holzschneider (to be published).

¹⁶M. L. Good, Phys. Rev. **121**, 311 (1961); W. E. Thirring, in *Essays in Physics*, edited by G. K. T. Conn and G. N. Fowler (Academic, London, 1972), Vol. 4, p. 125; I. R. Kenyon, Phys. Lett. B **237**, 274 (1990).

¹⁷A. S. Goldhaber and M. M. Nieto, Phys. Rev. D **9**, 1119 (1974).

¹⁸D. J. Heinzen and D. J. Wineland, Phys. Rev. A **42**, 2977 (1990).

¹⁹E. A. Cornell *et al.*, Phys. Rev. Lett. **63**, 1674 (1989).

²⁰T. W. Hansch, in *The Hydrogen Atom*, edited by G. F. Bassani *et al.* (Springer, New York, 1989).

²¹H. Poth, Phys. Scr. **T22**, 806 (1988); B. I. Deutsch *et al.*, Hyperfine Interact. **44**, 271 (1988); G. Gabrielse *et al.*, Hyperfine Interact. **44**, 349 (1988); D. B. Holtkamp, M. H. Holzschneider, and R. J. Hughes, in *Physics at Fermilab in the 1990's*, edited by D. Green and H. Lubbatl (World Scientific, Singapore, 1990).