Comment on Nondiffracting Beams

Nondiffracting directed beams have been discussed by several authors.[1-7] Our intention is to comment on (i) the Bessel beam[2-4] which has been called[3] "remarkably resistant to the diffractive spreading" and (ii) the electromagnetic directed energy pulse train[5-7] (EDEPT) which is claimed[7] to be "significantly improved over conventional, diffraction-limited beams," and to[5] "defeat diffraction." We find that diffraction is not reduced in either case and that conventional Gaussian beams will propagate at least as far for a given transmitting antenna dimension.

Durnin, Miceli, and Eberly[3] have studied the field $\psi(r,z,t)=J_0(k_\perp r)\exp[i(k_z z-\omega t)]$, where J_0 is the Bessel function, $k_\perp$ is the transverse wave number, ω is the frequency, and k_z is the axial wave number. Two properties of ψ are as follows: (i) The power contained in each lobe, between the adjacent zeros of J_0, is of the same order, and (ii) J_0 is a superposition of plane waves propagating at an angle $\sim k_\perp/k_z$ to the z axis.[8]

Based on geometric optics, Durnin, Miceli, and Eberly[3] found the propagation distance of the *central* lobe of an apertured Bessel beam of radius R to be $\sim 2Rr_0/\lambda$, where $r_0 \simeq \pi/k_\perp$ is the spacing between zeros of J_0 and λ is the wavelength. They compare the propagation distance of the apertured Bessel beam with a Gaussian beam of spot size r_0. The diffraction distance of the Gaussian beam is $Z_G \simeq \pi r_0^2/\lambda$. Since $R \gg r_0$, they observed that the Bessel beam propagated $\sim(2/\pi)R/r_0$ times further than the Gaussian beam.

Our interpretation differs in a number of fundamental ways and shows that the comparison between the Bessel beam of radius R and the Gaussian beam of spot size r_0 is not appropriate. If $N \gg 1$ is the number of lobes, then $R \simeq Nr_0$. The diffraction length associated with the central lobe is

$$Z_B \simeq R/\theta_B = 2Nr_0^2/\lambda = 2Rr_0/\lambda = (2/\pi)NZ_G\,,$$

where $\theta_B \simeq k_\perp/k_z \simeq \lambda/2r_0$ is the diffraction angle. The N lobes diffract away sequentially starting with the outermost one. The outermost lobe diffracts after a distance $\sim \pi r_0^2/\lambda$, the next one diffracts after a distance $2\pi r_0^2/\lambda$, and so on until the central lobe diffracts away after a distance $\sim N\pi r_0^2/\lambda \simeq Z_B$. The central lobe persists as long as there are off-axis lobes to replenish its diffraction losses.

If we take a Gaussian beam having a spot size equal to the aperture R, it will propagate *N times further* than the apertured Bessel beam. By focusing the Gaussian beam, nearly all the power can be focused on a target of dimension r_0 in a distance Z_B. For the same power through the aperture, the focused Gaussian beam delivers $\sim$*N times more power* than the Bessel beam.

Another solution to the wave equation which has been studied for its diffractive properties is the EDEPT.[5-7] Ziolkowski, Lewis, and Cook[6] have examined a particular pulse form both numerically and experimentally. The dominant radial profile of the modified-power-spectrum (MPS) pulse amplitude is $\exp[-br^2/\beta(z_0+i\xi)]$, where $\xi=z-ct$, and b, β, and z_0 are constants. Ziolkowski, Lewis, and Cook use the minimum spot size, $w_0=(\beta z_0/b)^{1/2}$, at the pulse center $\xi=0$, to obtain the diffraction length $Z \simeq \pi w_0^2/\lambda = \pi\beta z_0/b\lambda$. Their results indicate that the pulse propagates significantly further than $Z \simeq \pi w_0^2/\lambda$.

In our interpretation the diffraction length is not $\sim \pi w_0^2/\lambda$, but is $Z_{\mathrm{MPS}} = R/\theta_{\mathrm{MPS}} \simeq \pi w_0 R/\lambda$, where R is the transmitting antenna dimension and $\theta_{\mathrm{MPS}} \simeq \lambda/\pi w_0$ is the diffraction angle associated with a pulse having a typical transverse variation of $\sim w_0 < R$. As in the Bessel beam, the energy in the MPS pulse is radially spread out, typically over the full width R of the aperture. The scale length Z_{MPS}, derived here by a consistent application of diffraction theory to the MPS pulse, is fully consistent with the experimental and numerical results.[6,7]

Utilizing the entire transmitting antenna radius R, an unfocused Gaussian beam would propagate a distance $\sim \pi R^2/\lambda$; this is greater than the MPS pulse propagation distance. A Gaussian beam can be focused to a dimension $\sim w_0$ in the distance $\sim Z_{\mathrm{MPS}}$. Such a Gaussian beam focuses more power on the target than a corresponding MPS pulse.

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[1] See, for example, *Proceedings of the SPIE Conference on Microwave and Particle Beam Sources and Propagation, Los Angeles, 1988,* edited by N. Rostoker, SPIE Proceedings No. 873 (Society of Photo-Optical Instrumentation Engineers, Bellingham, WA, 1988).

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[8] See also D. DeBeer, S. R. Hartmann, and R. Friedberg, Phys. Rev. Lett. **59**, 2611 (1987).