Fractional Quantum Hall Effect and Multiple Aharonov-Bohm Periods

D. J. Thouless
Department of Physics FM-15, University of Washington, Seattle, Washington 98195

Y. Gefen
Department of Physics, Weizmann Institute of Science, Rehovoth, 76100 Israel

(Received 19 November 1990)

An arrangement for obtaining Aharonov-Bohm oscillations of basic periodicity qh/e $(q>1)$ is discussed. The relaxation towards h/e periodicity is characterized by a decay time exponential in the system size at zero temperature, and linear in the size at finite temperature.

PACS numbers: 73.20.Dx, 72.20.My, 73.20.Mf

Laughlin's explanation of the fractional quantum Hall effect[1] (FQHE) combines two rather different features. First, he proposed an approximate ground state which gave a downward-pointing cusp of the energy when the lowest Landau level was filled to a proportion of exactly $1/q$ of its maximum capacity. Second, the excitations from this ground state, separated from the ground state by an energy gap, carry charge $\pm e^* = e/q$. Although explanations of the FQHE have been given in terms of this fractional electron charge e^*, this does not seem to be necessary for the explanation of the effect, which is a consequence of the cusp in the ground-state energy at the appropriate filling factor, and would be observed even if the fractional charges were confined by the forces between them.[2] One likely consequence of fractionally charged particles is that the Aharonov-Bohm (AB) period, as detected, for example, by resistance oscillations in a ring with a varying flux through it, should be modified, and have the value h/e^* rather than h/e. Observation of this has been reported by Simmons *et al.*[3] Lee[4] has suggested that there is some doubt about whether the data really show this effect, and has proposed that a Coulomb blockade is responsible for the observation.

In this paper we discuss a possible arrangement for obtaining these multiple AB periods and analyze the conditions necessary to observe them. We find that the period qh/e is closely related to the q-fold broken symmetry associated with the FQHE which one of us discussed in a recent paper,[5] but that there is a relaxation towards the period h/e which is due to the motion of fractionally charged quasiparticles. Thus, it is not the observation of a higher AB period but its quenching that demonstrates the existence of free fractionally charged quasiparticles; a similar observation is made in the paper by Lee.[4] At very low temperatures we relate the decay time of the higher period to a quasiparticle tunneling time, while at higher temperatures it can be estimated in terms of the longitudinal conductivity of the system.

We also show that the decay times involved in this process represent maximum times for which the FQHE itself is stable. A very slow rate of change of flux corresponds to a very low Hall voltage. We find that in the zero-temperature limit the FQHE relaxes towards the integer quantum Hall effect, but this decay time increases exponentially with the size of the system. However, at nonzero temperatures the decay time is exponential in temperature but increases linearly with the size of the system. We have not shown this for the standard geometry of a quantum Hall measurement, but only for our special geometry. Nevertheless, it fits in neatly with the idea that the transition to a fractional quantum Hall state is a zero-temperature phase transition to a phase with broken symmetry.

Aharanov-Bohm oscillations at zero temperature. —The argument we use in this paper is closely related to the argument developed in Ref. 5. The electrons in the present geometry are confined to an annulus in a uniform perpendicular magnetic field B. The annulus has inner radius R_1 and outer radius R_2. A solenoid passes through the center of the annulus and carries magnetic flux $\Phi = \phi h/e$, which can be continuously varied. The two edges of the system are each connected through a tunneling barrier to a reservoir (large capacitor) that maintains a constant and equal electrochemical potential μ at the edges. This tunneling barrier must be large enough that the system is not strongly perturbed by the reservoirs, but small enough that the barrier penetration time is small compared with the times for other relaxation processes that we are interested in. There is an ammeter that monitors the current flowing from one edge to the other through the reservoirs. To monitor the AB period of the resistance fluctuations we suppose that there are two additional leads attached to one of the edges, so that the edge resistance can be measured. The reservoirs, ammeter, and leads are all supposed to contain ordinary electrons, but the annulus, which has some substrate disorder, is supposed to be in a fractional quantum Hall state. For definiteness we suppose that the lowest Landau level is close to $\frac{1}{3}$ filling, so that the $\frac{1}{3}$ FQHE is obtained; in this case $q=3$.

The Laughlin wave function for an ideal annulus has the form

$$\Psi=\prod_{i=1}^{N}|z_i|^{\phi}z_i^n\exp\left(-\frac{|z_i|^2}{4l_0^2}\right)\prod_{j=1}^{i-1}(z_i-z_j)^q\,, \qquad (1)$$

where $z_j=x_j+iy_j$, l_0 is the magnetic length, and the wave function is confined to the region $R_1<|z|<R_2$ if the parameters n and N are given by

$$n\approx R_1^2/2l_0^2-\phi,\quad n+qN\approx R_2^2/2l_0^2-\phi\,. \qquad (2)$$

Because of the disorder this wave function will be perturbed, so that, for example, total angular momentum is no longer a good quantum number, and if the filling factor is not exactly $\frac{1}{3}$ there will be some localized quasiparticle excitations added to this state. When the flux, proportional to ϕ, is increased, the wave function changes adiabatically, and, because of the factors $|z_i|^{\phi}$, the distribution of electrons moves outwards. The energy changes because the electrons are moving relative to the positive background and relative to the confining potential at the edges of the annulus. Eventually, at some value of ϕ, it will be energetically favorable for an electron at the outer edge to tunnel into the reservoir, leaving the annulus in a Laughlin state with one less electron, and for a different value of ϕ an electron can tunnel from the other reservoir to the inner edge.[6] Finally, when ϕ has increased by q, the system is restored to its initial state, apart from a phase factor resulting from the increase of ϕ and the decrease of n by q. This is the basis of the period qh/e in the flux for this system.[7]

In the absence of substrate disorder there is a smooth periodic dependence of the energy on the flux ϕ. We can assume without loss of generality that there is a minimum of the energy at $\phi=0$. The displacement of the electron charge density given approximately by the wave function of Eq. (1) gives rise to a force due to the confining potential, which is electrostatic in origin. To a reasonable approximation the energy change can be taken to be the sum of the changes in energy due to the displacements of the centers of the single-particle wave functions from which Ψ is composed, weighted with the occupancy $\nu=1/q$ of these wave functions. If the energy of such a single-particle state as a function of the center coordinate r is $U(r)$, then the sum over all states with centers between R_1 and R_2 gives

$$\frac{\partial^2E}{\partial\phi^2}=\nu l_0^2\left(\frac{U'(R_2)}{R_2}-\frac{U'(R_1)}{R_1}\right), \qquad (3)$$

where the relation between ϕ and the center coordinate implied by Eq. (2) has been used. Higher derivatives of the potential give contributions which are down by further factors of l_0/R. The energy scale of these parabolic variations in energy is l_0/R times some macroscopic energy scale U_0 of the order of the distance of the Fermi energy from the Landau level; this will be reduced if the confining potential is soft rather than hard.[8] On top of this smooth variation of the energy there are fluctuations due to the disorder, similar to the fluctuations of resistance. These and all the related fluctuations in the mesoscopic persistent current and the magnetic susceptibility are periodic functions of the flux ϕ with period q.

When the energy of this state is sufficiently high it will be energetically favorable for an electron to pass between the annulus and one of the reservoirs, and the system of $N\pm1$ electrons has an energy which is also parabolic, with its minimum as a function of ϕ close to $q/2$ (cf. Fig. 1).[6]

It is clear, as discussed in Ref. 5, that there are $q-1$ other equivalent sets of states which might have been involved, which can be obtained from Eq. (1) simply by changing n by an integer. For any given value of ϕ these typically differ in energy from one another by a term of order U_0l_0/R. We assume, as Tao and Haldane[9] did, that these states are nonoverlapping. This appears to contradict the von Neuman-Wigner theorem[10] that the levels of a closed system should not cross when a single parameter is varied, but the gap which opens up will be exceedingly narrow because the overlap is so small. One can describe the process which leads from one of these sets of levels to another in terms of the spontaneous creation of a pair of quasiparticles with charges $\pm e/q$, followed by the tunneling of the members of the pair to the two different edges. If this process occurs at half-integer values of ϕ it leads to the gap which prevents the crossing of the levels, but it can also occur at other values of the flux with the emission of phonons or other forms of energy to carry the system from a higher to a lower branch of the curves shown in Fig. 1. The quasiparticles are exponentially localized in the presence of

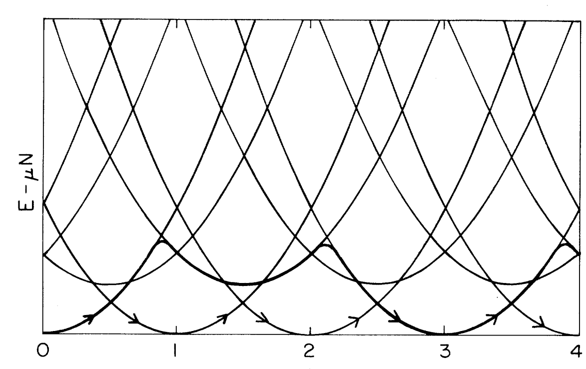

FIG. 1. Dependence of $E-\mu N$ (in arbitrary units) for the annulus at zero temperature on the flux ϕ which threads the annulus, in the case of the $\frac{1}{3}$ FQHE. Here E and N are the energy and number of the electrons, and μ is the chemical potential. The higher set of parabolas represents a change in N of ±1 over the lower set. The system normally follows the path shown by the thickened curve, but very slow adiabatic changes allow the system to follow the lower path shown by the arrows.

disorder, with a localization length which, at fixed energy, increases as the disorder increases.[11,12] To obtain a quantitative estimate of the crossover from multiple to single AB periods, we have considered a toy model for which the dynamics is governed by a single relaxation time τ for transitions between one of the three states which is instantaneously higher in energy to one of the lower-energy states. Analysis of this model shows that there is a steady-state solution in which all three levels are equally occupied on the average, so that the AB period is h/e, but there are long-term transients with periodic qh/e which decay to the equilibrium state in a time of order τ. The time for tunneling increases exponentially with the ratio of the width of the system to the localization length, and at very low temperatures the AB period is qh/e except for very long time scales or very small systems.

Aharonov-Bohm oscillations at nonzero temperatures.— At nonzero temperatures the situation is very different. At low temperatures pairs of oppositely charged quasiparticles can be created by thermal fluctuations, and these pairs should not be confined, since there is a three-dimensional Coulomb interaction ($1/r$ potential) between the quasiparticles. They therefore form a dilute plasma which can drift in response to any applied field, and, in particular, they can respond to the energy differences between the q different ground states. A total charge transfer of e/q across the system leads from one of these states to another. There is a driving voltage of order $qU_0 l_0/Re$ when the system is in one of the higher states which drives it towards the lower state, and the conductance across the annulus is $R\sigma_{xx}/(R_2-R_1)$, so the time for the system to relax to its lowest-energy state is of order

$$\tau \approx \frac{R_2-R_1}{l_0}\frac{e^2}{q^2\sigma_{xx}}\frac{1}{U_0}. \tag{4}$$

If the longitudinal conductivity σ_{xx} is activated, this time will increase exponentially as the temperature goes to zero. At any nonzero temperature there is a finite relaxation time that increases linearly with the width of the system.

The dynamics of the system can be analyzed similarly to the $T=0$ scenario alluded to above. Within the same single-relaxation-time picture we find an eigenvalue 1 of the density matrix. It corresponds to a stationary state where all the q states (time averaged) are equally likely. Selecting an initial condition for which one of these states is preferred, one should expect to see AB periods qh/e for time scales short compared with τ, but these would be damped, and the true period h/e should be observed over longer time scales.

To monitor the periodic variation of the state of the system with flux, leads attached to the edge states can be used. It is not possible to use a current between the two edges, as that will be carried by the fractionally charged quasiparticles moving across the ring, and these will themselves produce a major disturbance of the effect we would like to measure. The damping of the qh/e oscillations should provide a method of studying the temperature dependence of the longitudinal resistance under conditions in which it is too low to measure directly.

Quantum Hall effect at low voltages.— As was discussed in Refs. 5 and 16, at very low temperatures truly adiabatic motion of the system would involve motion along the lowest branch of each parabola with no Hall current. At each "crossing" of the parabolas a quasiparticle of charge e/q would tunnel within the system from one edge to the other, thus canceling the average current induced by the slowly changing flux. This rate has to be compared with the temporal frequency of the AB oscillations, and, because the tunneling rate depends exponentially on the size, it is only for very small systems or ridiculously low Hall voltages that this adiabatic process would occur; otherwise the system follows the path which leads to the FQHE. For different filling factors, such as $\frac{2}{3}$, it may be that the total charge transfer upon variation of the flux by h/e would be e. Consequently, there should be no similar quenching of the integer quantum Hall effect.

At nonzero temperatures the situation is less clear, but the conditions for observing a crossover between the two regimes are much less stringent. If the temporal period of the Aharonov-Bohm oscillations, which in the case of $1/q$ filling is the electron charge divided by the Hall current, is much less than the τ given by Eq. (4), the usual FQHE plateau should be observed. If this period is greater than the relaxation time τ the energy differences between the different parabolas shown in Fig. 1 should produce a varying bias between the edges, and these will induce a radial current between the edges which should make a substantial change in the plateau. Our guess is that it will lead to a more or less smooth variation of the Hall current with magnetic field instead of the plateau which is observed at larger values of the Hall voltage.

The relaxation time that comes into this discussion, Eq. (4), gives some basic time, of the order of 1 psec, multiplied both by the width of the annulus in magnetic lengths and by the exponential factor in the reciprocal of the longitudinal conductance. The range of times available may be quite convenient for measurement of the damping of the multiple Aharonov-Bohm periods, but observation of any loss of the FQHE plateaus must involve a combination of small samples, low current, and relatively high temperature.

This work was supported in part by the U.S. National Science Foundation under Grant No. DMR 89-16052, by the U.S.-Israel Binational Science Foundation, by the Minerva Foundation, Munich, Germany, and by the German-Israeli Foundation for Scientific Research and Development. One of us (D.J.T.) would like to acknowledge the hospitality of the Aspen Center for Physics. We have been greatly helped by discussions with B. Altshuler, L. I. Glazman, B. I. Halperin, Y. Imry, P. A. Lee, A. MacDonald, B. I. Shklovskii, N. Sivan, U. Sivan, A. D. Stone, and P. Wiegmann.

[1]R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[2]S. A. Kivelson and V. I. Pokrovsky, Phys. Rev. B **40**, 1373 (1989).

[3]J. A. Simmons, H. P. Wei, L. W. Engel, D. C. Tsui, and M. Shayegan, Phys. Rev. Lett. **63**, 1731 (1989).

[4]P. A. Lee, Phys. Rev. Lett. **65**, 2206 (1990).

[5]D. J. Thouless, Phys. Rev. B **40**, 12034 (1989).

[6]The order in which these processes occur depends on the precise position of the chemical potential μ relative to the energies of the edge states. If the Coulomb blockade is too strong it may be more favorable for the loss of an electron at one edge and the gain at the other edge to occur simultaneously, in which case only one set of parabolas is involved.

[7]It is not self-evident that there is a large matrix element for the transition between two Laughlin states by the removal of one electron from the boundary. Following the discussion of order parameters for the FQHE [S. M. Girvin and A. H. MacDonald, Phys. Rev. Lett. **58**, 1252 (1987); N. Read, Phys. Rev. Lett. **62**, 86 (1989)] as well as numerical evidence [E. H. Rezayi and F. D. M. Haldane, Phys. Rev. Lett. **61**, 1985 (1988)] that there is a big matrix element for the removal or addition of an electron at a point, combined with an appropriate gauge transformation, we assume here that the same result holds when the boundary is a circle rather than a point.

[8]There is a recent study of the confining potential by L. I. Glazman and I. A. Larkin, Semicond. Sci. Technol. (to be published).

[9]R. Tao and F. D. M. Haldane, Phys. Rev. B **33**, 3844 (1986).

[10]J. von Neumann and E. P. Wigner, Z. Phys. **30**, 467 (1929).

[11]B. I. Shklovskii and A. L. Efros, Zh. Eksp. Teor. Fiz. **84**, 811 (1983) [Sov. Phys. JETP **57**, 470 (1983)].

[12]Q. Li and D. J. Thouless, Phys. Rev. B **40**, 9738 (1989).

[13]O. Viehweger, W. Pook, M. Janssen, and J. Hajdu, Z. Phys. B **78**, 11 (1990).