Finite-Temperature Transport of a Pinned 2D Electron Lattice

S. T. Chui and K. Esfarjani
Bartol Research Institute, University of Delaware, Newark, Delaware 19716
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We argue that for a pinned 2D electron lattice in a magnetic field thermally activated bound dislocation pairs contribute to the conductivity from which an activation energy E_{dp} can be deduced. Near the melting point, this activation energy E_{dp} approaches zero as $E_{dp}=E_0(1-\nu/\nu_0)$, where ν is the filling factor. We calculate E_0 from the change in the zero-point energy of the phonons as a dislocation pair is created and find agreement to within 30% with the experimental result of Willet *et al.*

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Recently, there has been much interest in the properties of 2D electrons in GaAs heterojunctions in an external magnetic field in the low-density limit where the transition to a solid with long-range order is expected to occur. Willett *et al.*[1] found that the conductivity is activated in character and that the activation energy E_{dp} depends on the filling factor ν as $E_0(1-\nu/\nu_0)$. They propose that this type of behavior is related to the solid transition. Andrei *et al.*[2] observed anomalies in long-wavelength absorption around the same filling factor. Jiang *et al.*[3] observed reentrant melting behavior close to a filling factor of $\frac{1}{5}$. Goldman *et al.*[4] and Andrei *et al.*[5] also observed nonlinear conductivity similar to charge-density-wave systems in the low-filling-factor phase.

On the theoretical side, motivated by work on the 2D dislocation-mediated finite-temperature classical melting,[6,7] Chui and Esfarjani recently[8] calculated the change in zero-point energy of the phonons as a *single* dislocation is created. They found that the elastic energy and this quantum correction are both of the order of log of the area of the system; the sum of these two becomes zero very close to the experimental and simulation solid-liquid transition point for the 2D electron gas (2DEG) with[9] and without an external magnetic field and suggest a mechanism of melting due to the creation of dislocations. Energetics of the solid-Laughlin-fluid transition have also been calculated by Lam and Girvin[10] with improvement given by Esfarjani and Chui.[11]

In the solid phase dislocations occur as *bound pairs* at finite temperatures but they can affect the physical properties of the system close to the melting point. In this paper we investigate the effect of these bound dislocation pairs on the finite-temperature transport of the 2D electron solid. In the presence of impurities an electron lattice will most likely be pinned.[12,13] At a finite temperature dislocation pairs will be thermally activated and will not be pinned by impurities. We found that it is not necessary for the dislocation pairs to become unbound to carry a current and provide for a conductivity that is thermally activated in character. Near the melting filling factor at low temperatures, we calculated the change in zero-point energy of the phonons as a *bound dislocation pair* is created and found the activation energy $E_{dp}=0.048e^2(1-\nu/0.2)/\epsilon a+2E_c$, where E_c is the core energy of the dislocation. For the three densities investigated by Willet *et al.* the dependence on the filling factor, E_0/ν_0, is equal to $0.24e^2/\epsilon a$ and is in good agreement with the experiment of Willet *et al.* given in Table I. This result provides support that defects play an important part in the fundamental mechanism of quantum melting in two dimensions.

In 2D, a solid will be pinned at zero temperature. This comes about because the solid tends to form domains of size L_0 so that the energy gained from the pinning potential $L_0^{d/2}$ more than outweighs the elastic energy expanded which is of the order of L_0^{d-2}.[12,13] In 2D, at a finite temperature, it is possible that a solid can be depinned.[14] Here, we shall assume that the temperature is low enough that this does not happen.

One might have thought that dislocation pairs need to be unbound to carry a current. This is not so. Corresponding to any finite displacement of a dislocation pair, there is a net total displacement of the electrons. A dislocation situated at the origin with the Burgers vector along the **x** direction creates a long-range static displacement of the lattice positions given by

$$u_x(\mathbf{r})=a[\tan^{-1}(y/x)+xy/r^2]/2\pi,$$

$$u_y(\mathbf{r})=-\frac{ax^2/r^2}{2\pi},$$

for a system with zero Poisson ratio in 2D. A bound pair implies that two dislocations with opposite Burgers vector move by the same amount. Thus we consider a dislo-

TABLE I. The theoretical and experimental slope E_0/ν_0, the experimental estimate for the core energy; the experimental intercept ν_0 for three samples with densities ρ.

E_0/ν_0(theor) ($e^2/\epsilon a$)		0.24	
E_0/ν_0(expt) ($e^2/\epsilon a$)	0.21	0.23	0.32
E_c(expt) ($e^2/\epsilon a$)	0.021	0.019	0.017
ν_0	0.3	0.274	0.253
ρ (10^{10} cm^{-2})	3	4	4.5

cation pair located at $\mathbf{c},\mathbf{c}'$ and move them by the same amount $\delta\mathbf{r}$. Corresponding to the motion of the dislocation pair, the associated motion of an electron at position $\mathbf{r}$ is $\delta\mathbf{u}(r)=\delta\mathbf{r}\cdot\nabla[\mathbf{u}(\mathbf{r}-\mathbf{c})-\mathbf{u}(\mathbf{r}-\mathbf{c}')]$. The net movement of the electrons is $\mathbf{s}=\int d^2r\,\delta\mathbf{u}(\mathbf{r})/a_c$, where a_c is the area per unit particle of the lattice. We evaluate this explicitly and show that it is nonzero. In the integral, the dominant contribution comes from those $\mathbf{r}\gg\mathbf{c},\mathbf{c}'$, we get

$$s_k=\sum_i \delta r_i \int d^2r(\mathbf{c}-\mathbf{c}')\cdot\nabla\partial_i u_k/a_c .$$

The 2D integral can be done by converting it to a surface integral

$$s_k=\sum_i \delta r_i \int r\,d\theta(\mathbf{c}-\mathbf{c}')\cdot\mathbf{r}\partial_i u_k/ra_c .$$

It is obvious that this quantity is nonzero. For example, if one writes $s_x=\delta r_x r_{xx}+\delta r_y r_{xy}$, then $r_{xx}=b[(c_y-c_y')/4+(c_x-c_x')/8]/a_c$. Thus it is not necessary for the dislocations to unbind to carry a finite current. Physically, a bound dislocation pair create a nonuniform density distribution of the particles which are now charged. Thus when the pair moves the nonuniform charge distribution will move with it and current is created.

To investigate whether dislocations are pinned or not, we asked whether in moving the dislocation pair the energy gained from the external field can be compensated for by the loss from the impurity potential. If the energy gained from the external field is larger, the dislocations are not pinned. We show below that the energy change in moving the dislocations by any distance is finite when the external field is absent. Thus provided the distance moved is large enough, the energy gained from the external field will always be larger.

The energy change ΔE_i in moving the dislocation pair by a distance d is equal to

$$\int d^2q(c-c')q(\nabla u)_q[\exp(iqd)-1]U(q) ,$$

where $U(q)$ is the Fourier transform of the impurity potential. We shall assume a random distribution of the impurities R_i in a spacer layer at a distance l from the 2DEG plane as in most experimental samples. Then $U_q=\sum_i v_q\exp(iqR_i)$, where $v_q=\exp(-ql)2\pi/q$. The average of ΔE_i is zero. Its root mean squared is equal to

$$\int d^2q\,2[(c-c')q(\nabla u)_q]^2[\cos(qd)-1]v_q^2 .$$

Because $(\nabla u)_q$ is inversely proportional to q, the above integral is finite for all values of d. Thus proving our assertion. We next discuss the calculation of the dependence of the energy of the dislocation pair on the filling factor.

First, a clarification of the effect of magnetic field on dislocations. The displacement field $\mathbf{u}$ of a dislocation is determined by the condition that $\sum_i'\nabla V(\mathbf{R}_{ij}+\mathbf{u}_{ij})=0$ for all j subjected to the constraint that the circulation of u equals the Burgers vector. The strain energy is just the change in the potential energy with and without u. The effect of the magnetic field comes not in the potential energy but in the kinetic energy $(p-eA/c)^2$ with $A=\mathbf{r}\times\mathbf{B}/2$. There is no additional contribution to the dislocation energy from the kinetic energy in the magnetic field which can be expressed in terms of fluctuations from local static positions with $A=\delta\mathbf{r}\times\mathbf{B}/2$ after a gauge transformation so that a phase factor $i\sum_j\mathbf{r}_j\times\mathbf{R}_j\cdot\mathbf{B}/2$ is introduced into the wave function.

Chui and Esfarjani discuss the energy of a single dislocation but their calculation can be easily carried over for dislocation pairs. Briefly, the change in the Hamiltonian as the dislocation pair with total strain field u_{ij} is created is given by

$$\delta H=\sum_{ij}[u_{ij}\delta r_{ij}\delta r_{ij}\nabla\nabla\nabla V(r_{ij})]/4 . \quad (1)$$

The first-order correction due to this term is zero. The change in the zero-point energy of the phonons can be obtained from second-order perturbation as

$$\sum_i |\langle i|\delta H|0\rangle|^2/(E_0-E_i) .$$

ΔE is thus of the order of $\int[qu(q)]^2$. Writing u in terms of its Fourier transform $u_q[\exp(iqc)-\exp(iqc')]$, it is easy to see that this energy is of the order of $\ln|c-c'|$. We have performed numerical calculations using the Ewald sum technique[15] to deal with the long-range Coulomb interaction V. The magnetophonon energy E_i and wave function first discussed by Chaplik[16] and simplified by Chui, Haken, and Ma[17] was used. Finally, we performed a thermal average over the possible separations and orientation of $\mathbf{c},\mathbf{c}'$. The dependences are shown in Table I. Good agreement with experimental results is found.

Chui and Esfarjani[8] considered the transition to the fluid by considering generation of unbound dislocations; they focus on terms that are of the order of $\ln A$ and thus need not worry about core energies. In the solid phase dislocation occurs as pairs at a finite temperature, the dislocations may not be very far apart; the core energy may not be much smaller than the logarithmic terms. Estimates from the experimental data suggest that the core energies are of the order of 0.02 and quite small.

In our picture melting occurs when the energies of a *single* dislocation are zero. For bound dislocation pairs $E_{dp}=2E_c+\mu a^2(1-\nu/0.2)/2\pi$. ($\mu$ is the shear modulus.) The y intercept of the activation energy occurs at $E_0=2E_c+\mu a^2/2\pi$. From this we obtain estimates for the core energy shown in Table I. Thus this experimental estimate of the core energy is indeed quite small.

This estimate of the core energy is much smaller than the *classical* core energy of dislocations in the Coulomb solid at zero magnetic field (0.11) calculated by Fisher, Morf, and Halperin.[18] However, two corrections come

in to make the theoretical number much smaller.

The first correction comes from the quantum-mechanical hybridization of states with the dislocations situated at different positions within the unit cell of the lattice. This correction is similar to the formation of a vacancy band in solid He as first discussed by Hetherington.[19,20] We have performed a preliminary estimate of this energy by ignoring the Peierls potential on the dislocations and treating the interparticle potential in the harmonic approximation. We found that this correction is approximately $-0.17e^2/\epsilon a$, indicating that this mechanism can provide for a substantial reduction of the classical core energy.

The core energy is dominated by the short-range interaction between the electrons. Because of the finite extent of the wave function perpendicular to the planes, their short-range interaction is less than the $1/r$ interaction used by Fisher, Morf, and Halperin. For example, if one approximates the electrons by rods of length b, the Coulomb interaction between two rods a distance r apart is given by

$$V(r)=2(z-\ln\{z/[(1+z^2)^{1/2}+1]\}-(z^2+1)^{1/2})/d\,,$$

where $z=r/b$. At small distances, this potential is logarithmic in character. If b is comparable to a, we expect a substantial reduction in the core energy. According to the Fang-Howard model[21] the length b depends on the charge density as $(n_{\rm dep}+\rho)^{-1/3}$, where $n_{\rm dep}$ is the depletion charge density. Since $a=(0.866\rho)^{-1/2}$, b/a becomes larger the higher the density. Thus this mechanism produces a smaller core energy as the density becomes higher; consistent with the trend seen experimentally.

We have focused on dislocation pairs as one type of defects that contribute to the conductivity because they are intimately connected with melting. It is possible that other types of defects with similar defect energies also contribute to the conductivity.

In summary, we have calculated a quantum-mechanical correction to the energy of a bound dislocation pair and found the ratio of this energy to the filling factor agrees to within 30% with the corresponding experimental value for the activation energy measured in transport measurements. Our interpretation provides a direct link between the transport anomalies observed experimentally and its implication on the melting transition. It provides the first explicit evidence that defects play a significant role on melting and are not just convenient intellectual artifacts.

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