Duality-Symmetric String Theory and the Cosmological-Constant Problem

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We propose an ansatz for a low-energy effective action $\bar{S}$ for a gravity-plus-matter system which is given by the usual action S (integral over space-time) *divided* by the space-time volume, $\bar{S}=S/V$. The resulting gravitational equation is independent of the value of the cosmological constant that is present in S. We suggest that $\bar{S}$ can be considered as an approximation for the effective action in the "duality-symmetric" closed-string theory in which the string coordinates x and their "duals" $\tilde{x}$ are treated as independent.

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The smallness of the cosmological constant remains one of the main problems of a theory of fundamental interactions (see Ref. 1 for a review). It appears unlikely that it can be resolved by a mechanism that operates at a small (Planck) scale level. An explanation of why the cosmological constant is so small *now* should probably involve a modification of the low-energy effective gravitational action (introducing some sort of "nonlocality," cf., e.g., Refs. 2 and 3). Below we shall suggest an ansatz for the action which apparently resolves the cosmological-constant problem: The resulting field equations are independent of the cosmological constant that might be present in the action. We shall then explain how this ansatz can be motivated by starting from the "duality-symmetric" closed-string theory[4] in which the "left" and "right" parts of the string coordinates are treated in a more independent way than in the standard formulation of string theory.

Our basic suggestion is that at sufficiently large distances the effective gravitational action is not the usual "extensive" quantity

$$S=\int d^Dx\sqrt{g}\,(R+L_m)\,, \tag{1}$$

but the "intensive" one,

$$\bar{S}=\frac{S}{V}=\frac{\int d^Dx\sqrt{g}\,(R+L_m)}{\int d^Dx\sqrt{g}}\,. \tag{2}$$

Here L_m is the matter Lagrangian, D is the space-time dimension (e.g., 4) and V is the space-time volume (for notational simplicity we have set the gravitational constant equal to 1). The idea to use (2) as an action may seem implausible. For example, it apparently contradicts the principle of additiveness of actions which is important in quantum mechanics, etc. Let us stress, however, that (2) should be considered as a classical effective action and *not* as a fundamental action which should be quantized (note that the presence of a $1/V$ factor does not influence the standard nongravitational physics; one may, for example, absorb V into a renormalization of the Planck constant). As we shall discuss later, (2) is closely related to the "usual" effective action in the duality-symmetric string theory of Ref. 4.

Let us first formally study the classical dynamics corresponding to the action (2) leaving aside the question of how (2) appears from a complete quantum theory. While the equations of motion for the matter fields are obviously not modified by the presence of the $1/V$ factor in (2), the equation for the metric takes the form[5]

$$R_{\mu\nu}-\tfrac{1}{2}Rg_{\mu\nu}+T_{\mu\nu}+\tfrac{1}{2}\bar{S}g_{\mu\nu}=0\,, \tag{3}$$

$$T_{\mu\nu}\equiv\partial L_m/\partial g^{\mu\nu}-\tfrac{1}{2}L_m g_{\mu\nu}\,, \tag{4}$$

$$\bar{S}=\langle R+L_m\rangle,\quad \langle\cdots\rangle=\frac{\int d^Dx\sqrt{g}\,\cdots}{\int d^Dx\sqrt{g}}\,. \tag{5}$$

Equation (3) differs from the usual Einstein equation in the "cosmological term" $\frac{1}{2}\bar{S}g_{\mu\nu}$. The "cosmological constant" $\bar{S}$ is not, however, arbitrary, but is determined by the dynamics (which introduces a nonlocality in the field equation). As is clear from (2) and (3) any constant cosmological term (corresponding, e.g., to a classical vacuum energy or a contribution of the zero-point vacuum fluctuations) that may be present in L_m gives a constant $g_{\mu\nu}$-independent term in the action (2) and hence drops out from the gravitational equation (3).

Taking the trace of (3) and integrating over the space-time we find

$$\left\langle(1-\tfrac{1}{2}D)R+g^{\mu\nu}\frac{\partial L_m}{\partial g^{\mu\nu}}-\tfrac{1}{2}DL_m+\tfrac{1}{2}D(R+L_m)\right\rangle=0\,, \tag{6}$$

i.e.,

$$\langle R+g^{\mu\nu}\partial L_m/\partial g^{\mu\nu}\rangle=0,\quad \bar{S}=\langle L_m-g^{\mu\nu}\partial L_m/\partial g^{\mu\nu}\rangle\,, \tag{7}$$

so that (3) can be rewritten as

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}+\frac{\partial L_m}{\partial g^{\mu\nu}}-\frac{1}{2}\left[(L_m-\langle L_m\rangle)+\left\langle g^{\alpha\beta}\frac{\partial L_m}{\partial g^{\alpha\beta}}\right\rangle\right]g_{\mu\nu}=0\,. \tag{8}$$

Hence the vacuum Einstein equations are not modified: If $L_m = \lambda_0 = \text{const}$, Eq. (8) reduces to

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = 0, \quad R = 0, \quad \bar{S} = \lambda_0 , \tag{9}$$

and the standard tests of a gravitational theory which rely upon the vacuum Schwarzshild solution are satisfied. Moreover, one may argue that the $\langle \cdots \rangle$ correction terms in (8) vanish for a localized (static) matter distribution (a finite space integral of matter density is divided by an infinite space volume).

When the matter is represented by a scalar field with the Lagrangian

$$\begin{aligned} L_m &= \tfrac{1}{2}\, \partial_\mu \varphi\, \partial^\mu \varphi + U(\varphi), \\ U(\varphi) &= \lambda_0 + \tfrac{1}{2} m^2 \varphi^2 + g \varphi^4 + \cdots , \end{aligned} \tag{10}$$

Eq. (8) takes the form

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} + T^{(0)}_{\mu\nu} - \tfrac{1}{2} (U - \langle U \rangle) g_{\mu\nu} = 0 , \tag{11}$$

$$T^{(0)}_{\mu\nu} \equiv \tfrac{1}{2}\, \partial_\mu \varphi\, \partial_\nu \varphi - \tfrac{1}{4} g_{\mu\nu} \partial_\alpha \varphi\, \partial^\alpha \varphi . \tag{12}$$

Equation (11) derived from the ansatz (2) thus differs from the usual gravity-plus-scalar-matter Einstein equation in that the scalar potential U is replaced by $\hat{U} = U - \langle U \rangle$. While the scalar-field equation implies that φ "rolls down" into a minimum of U, the gravitational equation (11) is insensitive to the value of U at the minimum: If $\varphi = \varphi_0 + \xi$, $U'(\varphi_0) = 0$,

$$\hat{U} = \tfrac{1}{2} U''(\varphi_0) (\xi^2 - \langle \xi^2 \rangle) + O(\xi^3) . \tag{13}$$

One may further expect that the $\langle \xi^n \rangle$ terms in (11) can be neglected if the fluctuations of the scalar field are sufficiently localized. This suggests a resolution of the problem of the cosmological term generated during a phase transition.

In the case of the electromagnetic matter field ($L_m = \frac{1}{4} F^2$), Eq. (8) reduces to

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} + T^{(1)}_{\mu\nu} - \tfrac{1}{8} \langle F^2 \rangle g_{\mu\nu} = 0 , \tag{14}$$

$$T^{(1)}_{\mu\nu} \equiv \tfrac{1}{2} F_{\mu\alpha} F_{\nu\alpha} - \tfrac{1}{8} F^2 g_{\mu\nu} . \tag{15}$$

The $\langle F^2 \rangle$ correction term vanishes, e.g., for a plane-wave background or for localized field configurations with finite matter action (assuming that the space-time volume is infinite).

The cosmological consequences of Eq. (3) are more difficult to analyze directly. It is not clear, in particular, how to compute an average action corresponding to an isotropic and homogeneous matter distribution. It seems plausible that the $\langle \cdots \rangle$ correction terms in (8) will vanish for an open universe and one will be able to reproduce the Friedman solution at a sufficiently late time of evolution. To substantiate our expectation that Eq. (3) is consistent with the standard (inflationary) cosmology we can apply the same qualitative argument as used in Refs. 2 and 6. Let us first recall that we are assuming (2) and (3) to be valid only at rather large distances, i.e., we are not excluding the possibility of a Planck scale cosmological constant and some kind of (e.g., chaotic) inflation scenario. After the inflation has taken place the scalar fields have the values corresponding to a minimum of their potential U and the dominant contribution to $\bar{S} = \langle R + L_m \rangle$ comes from the late stage of evolution of the Universe. Then the only effect of the $\bar{S}$ term in (3) is to cancel the minimum value of U at the late stage of evolution (thus resolving the fine-tuning problem). This is consistent with the fact that the Universe should become almost flat after inflation.

Now let us try to motivate the ansatz (2) starting with the duality-symmetric closed-string theory of Ref. 4. A fundamental property of two dimensions is that the usual scalar field is not a "smallest" building block—its left and right chiral parts are independent off-shell 2D fields and hence should be treated as such in a construction of a string theory.[7] This suggests the replacement of the usual $\partial x\, \partial x$ term in the free-string action by the sum of the actions[8] for the left and right chiral fields x_+ and x_- and consideration of world-sheet interactions which depend not only on $x = x_+ + x_-$ but also on the "dual" coordinates $\tilde{x} = x_+ - x_-$. The 2D couplings which represent the string vacua are then determined from the conditions of Weyl and local 2D Lorentz invariance of the string σ model. The simplest vacuum corresponds to x compactified on a torus of radius r and $\tilde{x}$ compactified on a dual torus of radius $\tilde{r} = \alpha'/r$. The simplest σ-model Lagrangian with the constant couplings $g_{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ is given by[4]

$$\begin{aligned} L = \tfrac{1}{2} (& \partial_0 x^\mu\, \partial_1 \tilde{x}_\mu + \partial_1 x^\mu\, \partial_0 \tilde{x}_\mu \\ & - g_{\mu\nu} \partial_1 x^\mu\, \partial_1 x^\nu - \tilde{g}^{\mu\nu} \partial_1 \tilde{x}_\mu\, \partial_1 \tilde{x}_\nu) . \end{aligned} \tag{16}$$

The condition of local 2D Lorentz invariance implies that $\tilde{g} = g^{-1}$ [for the torus vacuum $g_{\mu\nu} = (r^2/\alpha') \delta_{\mu\nu}$, $\tilde{g}^{\mu\nu} = (\tilde{r}^2/\alpha') \delta^{\mu\nu}$]. In that case the Gaussian integration over $\tilde{x}$ gives back the usual action $g_{\mu\nu} \partial_a x^\mu\, \partial^a x^\nu$. In general, there are many more σ-model couplings than in the usual formulation (most of them correspond to massive states) and they depend on both x and $\tilde{x}$. The theory is manifestly duality symmetric, i.e., is invariant under $x \leftrightarrow \tilde{x}$ accompanied by a transformation of 2D couplings (space-time fields), e.g., $g \leftrightarrow \tilde{g}$, etc.

In addition to the zero mode of x there is now the zero mode of $\tilde{x}$ so that the partition function and effective action are given by integrals over the "doubled" set of coordinates $(x, \tilde{x})$ (the usual space-time and the dual space-time). The effective action which reproduces the graviton scattering amplitudes has the following structure:[4]

$$\hat{S} = \int d^D x\, d^D \tilde{x} \sqrt{g} \sqrt{\tilde{g}}\, [R(g, \partial) + R(\tilde{g}, \tilde{\partial}) + \cdots] , \tag{17}$$

where $g_{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ depend on both x and $\tilde{x}$, $\partial \equiv \partial/\partial x$, $\tilde{\partial} \equiv \partial/\partial \tilde{x}$, and $R(g, \partial)$ is the usual curvature scalar constructed from the metric g using the derivative ∂. The action (17) is manifestly duality symmetric under $x \leftrightarrow \tilde x$, $g \leftrightarrow \tilde g$. Thus the number of space-time coordinates is effectively doubled at the Planck ($\sqrt{\alpha'}$) scale.[9]

In order to establish a correspondence with the usual low-energy effective action, let us assume that we are perturbing near the vacuum in which x "lives" in a periodic "box" of a large dimension $r \gg \sqrt{\alpha'}$, so that $\tilde x$ is confined to a small "box" of scale $\tilde r = \alpha'/r \ll \sqrt{\alpha'}$. Then the $\tilde x$ dependence of the usual fields ($g_{\mu\nu}$, etc.) can be treated in a Kaluza-Klein fashion (the winding modes are very massive). In the leading approximation $g(x,\tilde x) \simeq g(x)$, and the integral over $\tilde x$ decouples $[R \equiv R(g(x),\partial)]$,

$$\hat S \simeq \tilde V \int d^D x \sqrt{g} R + \cdots , \quad \tilde V = \int d^D \tilde x \sqrt{\tilde g(\tilde x)} . \tag{18}$$

In the case of the torus vacuum ($g = \tilde g^{-1} = \text{const}$) the dual volume $\tilde V$ is simply the *inverse* of the usual volume $V = \int d^D x \sqrt{g}$. It seems natural to expect that the relation $\tilde V \simeq V^{-1}$ is approximately true also for x-dependent perturbation of the torus vacuum (this is suggested, e.g., by the general covariance property of the low-energy x space action; a similar argument applies in the case of the matter action, cf. Ref. 4). Then Eq. (18) takes the form

$$\tilde S \simeq \bar S = (1/V) \int d^D x \sqrt{g} R + \cdots , \tag{19}$$

i.e., we get the ansatz of Eq. (2). Let us stress that though the above remarks should not be considered as a rigorous derivation of (2) from the duality-symmetric string theory, it appears likely that (19) qualitatively correctly accounts for the effect of the dual $\tilde x$ space on the low-energy gravitational dynamics in the x space.

We finish with a remark about another possible interpretation of the action (2). Consider a theory in which one is averaging over the values of the gravitational and cosmological coupling constants (cf. Refs. 3 and 10),

$$Z = \int d\kappa \, d\lambda \int [dg][d\varphi] \mu(\kappa,\lambda) \exp \left(-\kappa \int d^D x \sqrt{g} [R + L_m(\varphi) + \lambda] + \cdots \right) . \tag{20}$$

Here $\mu(\kappa,\lambda)$ is some measure determined by a particular dynamical model. Suppose now that in the low-energy semiclassical limit μ is approximated by e^{λ}. Then λ plays the role of the Lagrange multiplier imposing the constraint $\int d^D x \sqrt{g} = \kappa^{-1}$ so that the effective action which governs (20) in this limit is given by (2).

It is not clear how to directly construct a locally supersymmetric analog of (2). This, however, might not be necessary since (2) is supposed to be valid only in the low-energy region where the local 4D supersymmetry should already be broken. Note also that there is no problem with supersymmetrization if the interpretation based on Eq. (20) is supposed to apply.

One interesting aspect of the ansatz (2) is that in the case of a compactification of some of the D space-time dimensions the corresponding volume of the "internal" space will cancel out and hence will not rescale the effective coupling constants. This may resolve some problems with compactifications on "large" internal spaces.

The idea to use (2) in order to resolve the cosmological-constant problem was inspired by a particular "renormalization-group" representation[11] for the tree-level effective action in the closed-string theory[12] and also by Linde's proposal.[2] The author is grateful to F. Quevedo, A. Linde, L. Rozansky, J. Russo, and M. Vasiliev for useful remarks on the subject of this paper. He would like also to acknowledge the hospitality of the Aspen Center for Physics, Los Alamos National Laboratory, and the Department of Physics and Astronomy of Johns Hopkins University. This work was supported in part by the National Science Foundation Grant No. PHY90-96198.

Note added.—The idea to use S/V as an action may suffer from a problem of large quantum corrections (large effective Planck constant). I am grateful to W. Fischler, V. Kaplunovsky, and J. Polchinski for a discussion of that point. Remarks on the possible relevance of a duality-symmetric action for the cosmological-constant problem were also made in Ref. 13.

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[1] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[2] A. D. Linde, Phys. Lett. B **200**, 272 (1988).

[3] S. Coleman, Nucl. Phys. **B307**, 864 (1988); **B310**, 643 (1988); S. Giddings and A. Strominger, Nucl. Phys. **B307**, 854 (1988); T. Banks, Nucl. Phys. **B309**, 493 (1988); I. Klebanov, L. Susskind, and T. Banks, Nucl. Phys. **B317**, 665 (1989).

[4] A. A. Tseytlin, Phys. Lett. B **242**, 163 (1990); Nucl. Phys. **B350**, 395 (1991).

[5] Equation (3) is very similar to the (one of the two) basic equations of Ref. 2 where the role of $\bar S$ was played by the average action of the "antipodal" world (which in the present case is identical to our world).

[6] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, Chur, 1990).

[7] This was, in fact, one of the basic ideas of the original formulation of the heterotic string theory [D. Gross, J. Harvey, E. Martinec, and R. Rohm, Nucl. Phys. **B256**, 253 (1985)]. Here we apply it to the "nonchiral" closed bosonic string with one of the motivations coming from the fact that the world-sheet interactions (vertex operators) corresponding to the winding string states in general depend not only on the string coordinates but also on their "duals."

[8] R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59**, 1873 (1987); W. Siegel, Nucl. Phys. **B238**, 307 (1984).

[9] It is interesting to note that some features of Linde's model (Ref. 2) are quite similar to these of the action (17). In particular, the "antipodal symmetry" (Ref. 2) is a simplified analog of duality. However, the action (17) does not have the "difference" structure postulated in Ref. 2 in order to provide a cancellation of the cosmological constant. Our mechanism of resolution of the cosmological-constant problem is based on Eqs. (2) and (19) with the necessary "cancellation term" originating from the variation of the $1/V$ factor.

[10] S. W. Hawking, Phys. Lett. **134B**, 403 (1984); E. Baum, Phys. Lett. **133B**, 185 (1983).

[11] A. A. Tseytlin, Phys. Lett. B **194**, 63 (1987); **208**, 221 (1988).

[12] Let $g(t)$ and $\phi(t)$ be the "running" metric and dilaton coupling ($t = \ln\epsilon$, ϵ is a 2D UV cutoff). Then the closed-string effective action can be represented in the form $S = (dV/dt)_{t=0}$, where $V = \int d^Dx \sqrt{g} e^{-2\phi}$ is the space-time volume. A formal replacement of V by $\ln V$ in this representation corresponds to the replacement of S by $\bar{S} = S/V$. The effective action which reproduces the standard closed-string tree-level scattering amplitudes is, of course, S, not $\bar{S}$.

[13] N. P. Chan and X. Li (unpublished); M. J. Duff, Nucl. Phys. **B235**, 610 (1990).