Collisionless $m=1$ Tearing Mode

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The instability of a magnetically confined plasma against macroscopic modes is analyzed in collisionless regimes where magnetic reconnection occurs because of finite electron inertia and the ion gyroradius ρ_i replaces the skin depth d as the width of the mode boundary layer. Growth rates $\gamma/\omega_A \sim (d/r_s)(\rho_i/d)^{2/3}$ are found, with ω_A the Alfvén frequency and r_s the radius of the reconnecting surface. For typical JET parameters, $\gamma^{-1} \sim 50\text{–}100\ \mu\text{s}$, which compares favorably with the observed instability growth time of internal plasma relaxations.

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Internal relaxation oscillations of the central electron temperature and soft-x-ray emissivity (so-called[1] "sawtooth oscillations") are a well-known instability in magnetically confined, axisymmetric toroidal plasmas where the poloidal magnetic field B_ϑ is produced by a toroidal current carried by the plasma itself. The relaxation is initiated by a helical mode with toroidal $n=1$ and dominant poloidal $m=1$ wave numbers. This mode displaces the equilibrium magnetic axis, where $q(0)<1$, and causes magnetic surfaces to reconnect around the region where $q(r_s)=1$, which is also the region where the pitch of the equilibrium field equals that of the perturbation. Here, $q(r) \approx rB_\varphi/RB_\vartheta$, with B_φ the toroidal field, R the torus major radius, and r the mean distance of a magnetic surface from the magnetic axis.

Plasma discharges produced by the JET tokamak are in a high-temperature regime where the electron-ion collision time τ_{ei} is comparable to, and sometimes exceeds, that of sawtooth relaxations (typical values are discussed at the end of this Letter). In this regime, the rate at which magnetic reconnection can occur is determined by electron inertia and by the ion gyroradius, rather than by collisional effects. This rate is found in this Letter to be significantly larger than that of the well-known[2] resistive internal kink mode.

In a collisionless plasma, electron inertia can be responsible for the decoupling of the plasma motion from that of the magnetic field, allowing magnetic-field line reconnection. This possibility was first considered by Furth,[3] and analyzed in detail by Coppi,[4] by Pellat, Laval, and Vuillemin,[5] and by Cross and Van Hoven,[6] within the context of magnetospheric and solar physics as well as of laboratory plasmas, for modes with a nearly constant perturbed radial magnetic field across the reconnecting layer. Later, Basu and Coppi[7] extended the analysis to $m=1$ modes whose perturbed $\tilde{B}_r$ generally varies rapidly across the $q=1$ surface. Near ideal magnetohydrodynamic (MHD) marginal stability conditions, their analysis led to an exponential growth rate γ in the linear instability stage given by

$$\gamma/\omega_A \sim d/r_s \,, \tag{1}$$

where $\omega_A \equiv V_A/L_s$ is the relevant Alfvén frequency, with $V_A \equiv B/(4\pi m_i n_i)^{1/2}_{r_s}$, $L_s \equiv R/s$, $s \equiv r_s q'(r_s)$, and $d \equiv c/\omega_{pe}$, with $\omega_{pe} \equiv (4\pi n_e e^2/m_e)^{1/2}_{r_s}$. The width δ of the reconnecting layer was found to be determined by the plasma skin depth $\delta \sim d$. Their analysis adopted a kinetic treatment for the electrons and a fluid description of the ions. Therefore it was limited to values of the ion gyroradius $\rho_i \equiv (T_i m_i c^2/e^2B^2)^{1/2}_{r_s} < d$, corresponding to values of the local ion beta, $\beta_i = (8\pi n_i T_i/B^2)_{r_s} < 2m_e/m_i$, that are unrealistically low in most situations of interest.

In this Letter, we are interested in the modification of the result (1) in the limit $\rho_i > d$. Therefore, a kinetic treatment for the ions that is valid for arbitrary ion gyroradii is adopted, following the analysis of Pegoraro, Porcelli, and Schep.[8] Since we look for modes whose growth rates are determined by a macroscopic process, rather than by a selected number of particles in resonance with the mode, a fluid treatment for the electrons is adequate. The relevant Ohm law is

$$\mathbf{E} + \frac{\mathbf{V}_e \times \mathbf{B}}{c} = \eta \mathbf{J} + \left(\frac{m_e}{n_e e^2}\right)\left(\frac{\partial}{\partial t} + \mathbf{V}_e \cdot \nabla\right)\mathbf{J} - \left(\frac{\nabla \cdot \overleftrightarrow{\mathbf{P}}_e}{e n_e}\right), \tag{2}$$

where $\mathbf{V}_e$ is the electron fluid velocity, η is the electrical resistivity, and $\overleftrightarrow{\mathbf{P}}_e$ is the electron pressure tensor. Eventually we are interested in the collisionless limit where $\eta \to 0$. The resistive term is retained in order to make contact with the collisional theory of Refs. 2 and 8.

Assuming a q profile such that $1-q \gtrsim r/R$ in the central region of the plasma column and finite magnetic shear at the $q=1$ surface, we can use the ideal MHD analysis of Ref. 9 to describe the plasma motion everywhere except in a layer around the $q=1$ surface where nonideal effects become important. Perturbed quantities are assumed to vary as $\xi(\mathbf{r},t) \approx \tilde{\xi}(r)\exp[\gamma t + i(\vartheta - \varphi)]$, with $m \neq 1$ satellite poloidal components retained in the MHD region but neglected in the layer. In order to treat layer widths of the order of the ion gyroradius, it is convenient to use a generalized Fourier representation of the mode amplitude, where k is the Fourier variable conjugate to the layer variable $x=(r-r_s)/r_s$. An extensive treatment of this approach is given in Refs. 8 and 10, where the perturbed ion and electron responses are derived. Here, we focus on the modification of the electron response resulting from the inertial term in Ohm's law. The analysis is carried out in a frame of reference where the equilibrium radial electric field at the $q=1$ surface vanishes. The perturbed electron continuity equation is

$$\tilde{n}_e/n_e = -i(\hat{\omega}_{*e}/\hat{\gamma})\phi + (1/\hat{\gamma})dJ/dk ,$$

where $\phi \equiv e\tilde{\phi}/T_e$ and $J \equiv \tilde{J}_{\parallel}/en_eV_A$ are the normalized perturbed electrostatic potential and parallel current density, respectively, $\hat{\gamma} \equiv \gamma/\omega_A$, $\hat{\omega}_{*e} = \omega_{*e}/\omega_A$, and $\omega_{*e} = [(cT_e/eBn_er)(dn_e/dr)]_{r_s}$ is the electron drift frequency. The perturbed ion density is obtained from Vlasov's equation and, for modes with parallel phase velocity larger than the ion thermal velocity, is given by

$$\tilde{n}_i/n_i = [i(\hat{\omega}_{*i}/\hat{\gamma}) - L]\tau\phi ,$$

with $\omega_{*i} = -\omega_{*e}/\tau$ and $\tau = (T_e/T_i)_{r_s}$. The function L includes gyroradius effects to all orders,

$$L = i(\hat{\omega}_{*i}/\hat{\gamma}) + (1-\Gamma_0) - i(\hat{\omega}_{*i}/\hat{\gamma})(1-\eta_i M)\Gamma_0 ,$$

with $\Gamma_{0,1}(b) = I_{0,1}(b)\exp(-b)$, $M = b(1-\Gamma_1/\Gamma_0)$, $I_{0,1}$ being modified Bessel functions of the first kind, $b \equiv (\hat{\rho}k)^2$, $\hat{\rho} \equiv \rho_i/r_s$, and $\eta_i \equiv (d\ln T_i/d\ln n_i)_{r_s}$. The quasineutrality condition gives $\hat{\gamma}\tau L\phi = -dJ/dk$. Ampère's law takes the form $J = \tau\hat{\rho}^2k^2A$, where $A \equiv (ev_A/cT_e)\tilde{A}_{\parallel}$ is the normalized perturbed parallel vector potential. The linearized Ohm's law depends on the perturbed electron temperature through the pressure term in Eq. (2). For modes with $\gamma^2 \gg k_{\parallel}^2 v_{\text{the}}^2$, where $v_{\text{the}}^2 = T_e/m_e$ is the electron thermal velocity, an adiabatic equation of state applies,

$$\tilde{T}_e/T_e = -i\eta_e(\hat{\omega}_{*e}/\hat{\gamma})\phi + (2/3\hat{\gamma})dJ/dk ,$$

where $\eta_e \equiv (d\ln T_e/d\ln n_e)_{r_s}$. In the opposite limit $\gamma^2 \ll k_{\parallel}^2 v_{\text{the}}^2$, the electron response is isothermal. In the latter case, the electrons equalize their temperature along perturbed field lines, leading to $(d/dk)(\tilde{T}_e/T_e) = \eta_e\hat{\omega}_{*e}A$. It can be shown *a posteriori* that, in the regimes of interest here, the electron response within the layer is adiabatic in the small-gyroradius limit, $\rho_i < d$, and mainly isothermal in the opposite limit, $\rho_i > d$. In both limits, Ohm's law can be reduced to[4,10]

$$[(\hat{d}/\hat{\rho})^2(\hat{\gamma} + \hat{\nu}_{ei}/2) - (\tau\mu/\hat{\gamma})d^2/dk^2]J = \tau g E ,$$

where $\hat{\nu}_{ei} \equiv \nu_{ei}/\omega_A$, $\nu_{ei} \equiv \tau_{ei}^{-1}$, $\mu \equiv 1 + 2\alpha/3$, $g \equiv 1 + i(\hat{\omega}_{*e}/\hat{\gamma})(1+\alpha\eta_e)$, with $\alpha = 1$ in the adiabatic limit and $\alpha = 0$ in the isothermal limit, and $E = -d\phi/dk - \hat{\gamma}A$ is the parallel electric field. We have used $\eta_{\parallel} = m_e\nu_{ei}/2n_ee^2$ for the parallel resistivity. A term involving the gradient of the parallel component of the equilibrium current density[4] has been neglected. After straightforward algebra we obtain the dispersion equation

$$\frac{d}{dk}\left[\tau\mu + \frac{g}{L}\right]\frac{dJ}{dk} - \left[\frac{\hat{\gamma}}{\hat{\rho}}\right]^2\left[\Delta^2 + \frac{g}{k^2}\right]J = 0 , \qquad (3)$$

where $\Delta^2 \equiv \hat{d}^2(1+\hat{\nu}_{ei}/2\hat{\gamma})$. The current density J must vanish for $|k| \to \infty$ so that the mode amplitude in x space is regular. For small values of k, J must satisfy the boundary condition[10]

$$J \sim 1 - (\delta_{\text{in}}^2 k^2/2) + (\delta_{\text{in}}^2\lambda_H|k|^3/3) , \qquad (4)$$

which reflects the global $m=1$ mode properties. The first two contributions to (4) are related to the radially constant part of the ideal MHD displacement in the region $q<1$; in particular, the second term is related to the ion inertia, with $\delta_{\text{in}}^2 \equiv \hat{\gamma}(\hat{\gamma} + i\hat{\omega}_{di})$ and $\hat{\omega}_{di} \equiv \hat{\omega}_{*i}(1+\eta_i)$. The last term arises from the x^{-1} correction to the displacement approaching the singular layer from outside. The MHD driving parameter $\lambda_H \sim (r_s/R)^2 \times (\beta_p^2 - \beta_{p,\text{cr}}^2)$ is a measure of the potential energy that is available outside the layer, with[9] $\beta_p(r_s)$ the relevant poloidal β parameter and $\beta_{p,\text{cr}} \sim 0.1$–$0.3$.

First, we consider briefly the small-gyroradius limit, $\rho_i < \delta$. For the sake of simplicity, we assume that $\tau \sim 1$ and $\hat{\omega}_{*e}/\hat{\gamma} \to 0$. For $\hat{\rho}k \ll 1$, we can approximate $L \approx (\hat{\rho}k)^2$. Equation (3) reduces to

$$(d/dk)(k^{-2}dJ/dk) - \hat{\gamma}^2(\Delta^2 + k^{-2})J = 0 ,$$

whose exact analytic solution in terms of confluent hypergeometric functions is known.[8,10] Using the boundary condition (4), we obtain the dispersion relation

$$\hat{\gamma} = \lambda_H(\tfrac{1}{4}Q)^{3/2}\Gamma(\tfrac{1}{4}(Q-1))/\Gamma(\tfrac{1}{4}(Q+5)) , \qquad (5)$$

where $Q \equiv \hat{\gamma}/\Delta$ and $\Gamma(z)$ is the gamma function. For $\hat{\nu}_{ei}/\hat{\gamma} \gg 1$, this dispersion relation reduces to that of Ref. 2. In particular, near ideal MHD marginal stability, i.e., $|\lambda_H| < \Delta$, Eq. (5) yields (reintroducing dimensional quantities)

$$\gamma/\omega_A \approx (d/r_s)(1 + \nu_{ei}/2\gamma)^{1/2} . \qquad (6)$$

The eigenfunction is $J(k) \approx \exp(-\Delta^2k^2/2)$. When $\nu_{ei} \gg \gamma$, Eq. (6) gives the well-known[2] resistive internal kink growth rate, $\gamma/\omega_A \approx \varepsilon_\eta^{1/3}$, with $\varepsilon_\eta \equiv \hat{d}^2\hat{\nu}_{ei}/2$, the inverse magnetic Reynolds number. In the collisionless limit, the mentioned result by Basu and Coppi[7] in Eq. (1) is recovered. The small-gyroradius condition, $\rho_i < \delta \sim d$, also justifies the use of the electron adiabatic equation of state. In fact, within the layer, $x \lesssim \hat{d}$, we have $(\gamma/k_{\parallel}v_{\text{the}})^2 = (\hat{\gamma}\hat{d}/\hat{\rho}x)^2\tau^{-1} \gg 1$.

Interestingly, Wesson[11] has recently obtained a result similar to that in Eq. (6), but following a different approach. Considering the convective inertial term, $\mathbf{V}_e \cdot \nabla \mathbf{J}$, in the generalized Ohm's law, and following a Sweet-Parker[12] type of analysis, he has found a nonlinear reconnection layer $\delta \sim d$ and a reconnection time $\tau \sim (r_s/d)\tau_A$ in collisionless regimes, with $\tau_A = \omega_A^{-1}$ (the same type of analysis leads to Kadomtsev's reconnection time[13] $\tau \sim \tau_A\varepsilon_\eta^{-1/2}$ in the collisional limit). The term $\mathbf{V}_e \cdot \nabla \mathbf{J}$ can become large nonlinearly because of current sheets developing near the X point of the $m=1$ island, enhancing the local value of $dJ_{\parallel}/dr$.

We now consider the more realistic limit $\rho_i > d$ and $\nu_{ei} \lesssim \gamma$. Analytic progress can be made by adopting a Padé approximation of the ion response function in Fourier space,

$$L^{-1} \approx (1+i\hat{\omega}_{*i}/\hat{\gamma})^{-1} + (1+i\hat{\omega}_{di}/\hat{\gamma})^{-1}(\hat{\rho}k)^{-2},$$

which is an interpolation formula between the fluid and the large-gyroradius responses. Then, the differential equation (3) can be solved by a double asymptotic matching technique. We neglect, at first, diamagnetic effects. We identify two overlapping intervals in k, $(\Delta k)^2 < 1$ and $(\hat{\rho}k)^2 > 1$, corresponding in x space to an inner sublayer of width $\delta_{\rm inner} \sim d$, where electron inertia is important, nested into a broader layer of width $\delta \sim \rho_i$. An isothermal equation of state for the electrons is assumed. For $(\Delta k)^2 < 1$, Eq. (3) reduces to

$$(d/dk)[(\hat{\rho}_\tau^2 + k^{-2})dJ/dk] - (\hat{\gamma}/k)^2 J = 0,$$

with $\hat{\rho}_\tau = (1+\tau)^{1/2}\hat{\rho}$, which can be solved exactly in terms of a combination of hypergeometric functions of $-(\hat{\rho}_\tau k)^2$ with the constants of integration fixed by the boundary condition (4). For $(\hat{\rho}k)^2 > 1$, Eq. (3) reduces to

$$(1+\tau)d^2J/dk^2 - (\hat{\gamma}/\hat{\rho})^2(\Delta^2 + k^{-2})J = 0.$$

The solution of this equation that behaves well at $|k| \to \infty$ is $J = C(\sigma|k|)^{1/2}K_\nu(\sigma|k|)$, where $\sigma = \hat{\gamma}\Delta/\hat{\rho}_\tau$, $\nu^2 = \frac{1}{4} + (\hat{\gamma}/\hat{\rho}_\tau)^2$, and $K_\nu(z)$ is a modified Bessel function of the second type. Matching the two solutions in the interval $\hat{\rho}^{-1} < k < \sigma^{-1}$ determines the constant C and leads to the eigenvalue condition. In particular, in the relevant limit $\hat{\rho}_\tau > \hat{\gamma}$ and $\nu_{ei} < \gamma$, the eigenvalue condition reduces to

$$(\pi/2)\hat{\gamma}^2 = \hat{\rho}_\tau\lambda_H + \hat{\rho}_\tau^2\Delta/\hat{\gamma}. \tag{7}$$

For $|\lambda_H| < \hat{\rho}_\tau^{1/3}\Delta^{2/3}$, we obtain the growth rate

$$\frac{\gamma}{\omega_A} \approx c_0 \frac{d}{r_s}\left(\frac{\rho_i}{d}\right)^{2/3} \equiv \frac{\gamma_0}{\omega_A}, \tag{8}$$

with $c_0 = [2(1+\tau)/\pi]^{1/3}$, which is higher than the growth rate in (1). We refer to this as the collisionless $m=1$ tearing regime. Note that

$$\frac{\nu_{ei}}{2\gamma_0} = \epsilon_\eta \left(\frac{r_s}{d}\right)^3 \left(\frac{2m_e}{m_i\beta_i}\right)^{1/3}.$$

A modest enhancement of the growth rate (8), by a factor $(1+\nu_{ei}/2\gamma)^{1/6}$, is found for $\nu_{ei} \lesssim \gamma$.

For $\lambda_H < -\hat{\rho}_\tau^{1/3}\Delta^{2/3}$, Eq. (7) yields $\hat{\gamma} \sim \hat{\rho}_\tau\Delta/|\lambda_H|$, so that, for sufficiently large and negative λ_H, γ drops below ν_{ei} and the (semi) collisional regime is recovered. In this regime $\tilde{B}_r$ is nearly constant across the layer, and the $m=1$ mode is expected to become fully stable or to saturate at a very modest amplitude. In the limit $(\hat{\rho}_\tau\Delta^2)^{1/3} < \lambda_H < \hat{\rho}_\tau$, the growth rate[8] $\hat{\gamma} \sim (\lambda_H\hat{\rho}_\tau)^{1/2}$ is obtained from Eq. (7). These results, however, are valid as long as diamagnetic effects can be neglected. The diamagnetic modification of the dispersion relation (8) can be easily evaluated. If $|\lambda_H| < \hat{\rho}_\tau^{1/3}\Delta^{2/3}$, we find [cf. Eq. (18) of Ref. 8 with $\lambda_H = 0$ and $\epsilon_\eta \to \hat{\gamma}d^2$]

$$(\gamma + i\omega_{*e})(\gamma + i\omega_{*i})^{2/3}(\gamma + i\omega_{di})^{1/3} \approx \gamma_0^2. \tag{9}$$

Thus, for

$$\frac{\omega_{*e}}{\gamma_0} = \tau^{1/3}\left(\frac{m_i}{m_e}\right)^{1/6}\left(\frac{\beta_e}{2}\right)^{2/3}\frac{L_s}{L_n}$$

larger than unity, with $L_n \equiv |d\ln n_e/dr|^{-1}$, diamagnetic effects fully stabilize the collisionless $m=1$ tearing mode. The Padé approximation for the ion response function fails[8] when $\gamma \sim \omega_{*i}$ and η_i becomes large. A residual growth rate is expected to persist when[8] $\eta_i > \eta_{i,\rm cr} \approx 1.6$.

We check the consistency of the obtained results with the assumed model equations. Since in the collisionless $m=1$ tearing regime, characterized by the inequalities $\nu_{ei} \lesssim \gamma_0$, $\rho_i > d$, $|\lambda_H| < \hat{\rho}_\tau^{1/3}\hat{d}^{2/3}$, and $\omega_{*e} < \gamma_0$, the bulk of the electron population is involved in the instability process, a fluid, as opposed to kinetic, treatment of the electrons is justified. The mode is localized in Fourier space over a distance $k \lesssim \hat{\rho}_\tau/\hat{\gamma}\Delta = \sigma^{-1}$. In real space this corresponds to values of $x \gtrsim \Delta\hat{\gamma}/\hat{\rho}$, where the isothermal equation of state applies. In particular, the two asymptotic solutions of Eq. (3), one obtained for $\hat{\rho}k > 1$ and the other for $\hat{d}k < 1$, can be matched entirely in the isothermal domain (since $\hat{d} < \sigma$). Finite thermal conductivity alters the rate of decay of the eigenfunction at large $k \gtrsim \sigma^{-1}$, but the eigenvalue condition is not affected to leading order in the asymptotic parameter $d/\rho_i \ll 1$.

Clearly, all different aspects of the sawtooth relaxation phenomenon cannot be explained by the linear analysis presented here. Nevertheless, a comparison between our results and experimental observations points to an important role played by electron inertia for the plasma regimes attained in recent JET experiments. This comparison also suggests that the new time scale given by Eq. (8) may indeed have been observed, even though a clear confirmation must await a full nonlinear analysis taking into account various factors such as the possibility of low-shear q profiles in the central plasma region, of enhanced dissipative effects from microturbulence, neoclassical MHD effects, etc. In fact, two phases of the sawtooth relaxation can be identified.[14] The first phase, lasting typically $\tau_{\rm displ} \sim 100$–$300$ μs, is characterized by the displacement of the magnetic axis, exponentially growing with time from its equilibrium position to a final position $r_{\rm final} \sim (0.8 \pm 0.2)r_{\rm inv}$, with $r_{\rm inv}$ the sawtooth inversion radius. Thus it is possible to define an experimental growth rate $\gamma_{\rm expt}$; typically, $\gamma_{\rm expt}^{-1} \sim \frac{1}{3}\tau_{\rm displ}$. In the second phase, the peak electron temperature drops on a time scale $\tau_{\rm diff} \sim 100$–200 μs. Clearly, the linear stability analysis cannot address this second phase. The two phases sometimes overlap.

Let us now evaluate the relevant stability parameters for JET high-temperature discharges. We choose the reference values $R=3$ m, $B=3$ T, $n_e(r_s)=3\times10^{19}$ m^{-3} and $T_e(r_s)=5$ keV. With $Z_{\rm eff}=2$, we find $\tau_{ei}\approx130$ μs. Thus ν_{ei} is 1 to 4 times smaller than $\gamma_{\rm expt}$. The plasma skin depth is $d\approx1$ mm. Considering a deuterium plasma with $T_i(r_s)\approx T_e(r_s)$, the ion gyroradius is $\rho_i\approx\frac{1}{3}$ cm. Assuming a parabolic q profile with $q_0=0.7$ and $r_s=0.3$ m, we find $\gamma_0^{-1}\approx70$ μs. Thus γ_0^{-1} agrees within a factor of 2 with the observed growth time, $\gamma_{\rm expt}^{-1}$. Finally, $\beta_e(r_s)\approx0.6\%$ and, assuming $L_n\sim1$ m, we have $\omega_{*e}/\gamma_0\approx0.36$.

On the basis of Eq. (9), diamagnetic effects will eventually prevail as the temperature is increased; however, the condition for diamagnetic stabilization of the $m=1$ mode is more stringent in the collisionless regime than predicted by the two-fluid model. Thus, the main conclusion from the present analysis is that the $m=1$ tearing instability can remain virulent at high temperatures, contrary to expectations based on collisional models.

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