The Cauchy Problem for the Scalar Wave Equation is Well Defined on a Class of Spacetimes with Closed Timelike Curves

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We study the massless scalar wave equation on a class of asymptotically flat, static spacetimes with closed timelike curves (CTC's), in which all future-directed CTC's traverse one end of a handle (wormhole) and emerge from the other end at an earlier time. Existence of smooth, asymptotically regular solutions is proved for smooth data with finite energy given on $\mathcal{I}^-$. The proof requires a generalized spectral decomposition for a non-Hermitian operator. We also prove that this solution is unique among solutions that die off in time.

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It has commonly been thought that in spacetimes with closed timelike curves one cannot find consistent time evolutions of classical fields for generic initial data—that the Cauchy problem is not well defined. Recently, however, Morris and Thorne[1] introduced a class of wormhole spacetimes in which, although there are many closed timelike curves, the set of closed timelike and null geodesics has measure zero. For these spacetimes, Morris, Thorne, and Yurtsever[2] noted that the evolution of free fields is well defined in the limit of geometrical optics; and this in turn makes it seem likely that a multiple-scattering series converges to a solution for arbitrary initial data.[1-3]

We consider here the simplest of the wormhole spacetimes of Ref. 3. These have a timelike Killing vector and are constructed from Minkowski spacetime by removing two timelike solid cylinders (the histories of the handles' mouths) and identifying the $\mathcal{S}^2\times\Re$ boundaries after a time translation (Fig. 1). For these spacetimes, there is no spacelike Cauchy surface, but initial data for the scalar wave equation can be given at past null infinity $\mathcal{I}^{-1}$. Because the spacetime is static, a quantum test field would have no particle production, and the existence of a solution to the classical Cauchy problem implies a well-defined solution to the free-field quantum scattering problem as well.

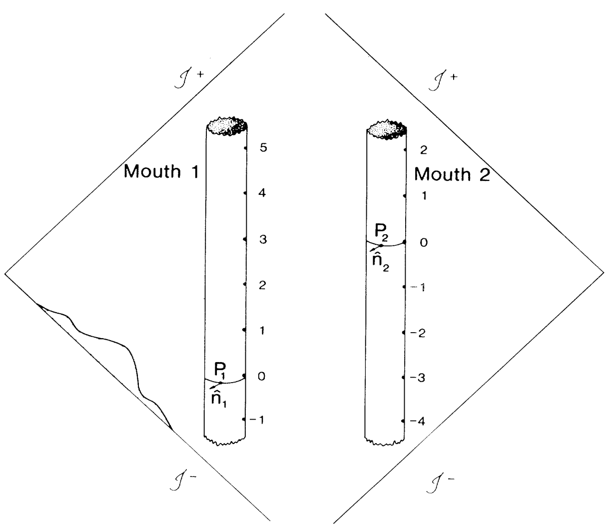

FIG. 1. A wormhole spacetime with closed timelike curves is constructed by removing two cylinders from R^4 and identifying their boundaries after a time translation. Points labeled by the same numbers (indicating proper time along the wormhole) are identified, as are points P_1 and P_2. The identification maps the outward normal $\hat{\mathbf{n}}_1$ to the inward normal $-\hat{\mathbf{n}}_2$.

In a Lorentzian path-integral approach to quantum gravity, any topology change—evolution from one (nonempty) spatial three-manifold to one that is inequivalent—requires closed timelike curves[4] (CTC's); evaluating the path integral over metrics near one with CTC's is roughly equivalent to finding the quantum scalar field on such a background spacetime. Even in the approximation of classical gravity with quantum matter fields, it is conceivable that physics allows one to construct and maintain wormholes; and if there is a well-defined initial-value problem for a spacetime with CTC's, one might be able to create a spacetime with macroscopic CTC's in this way, by accelerating one end of a wormhole.

The spacetimes of Refs. 2 and 3 are flat outside the two identified cylinders, for ease of analyzing the geodesics. This simplification is not needed for our proof, and we shall assume that the metric is smooth and static (see Ref. 1). It does, however, simplify the asymptotic analysis to assume that outside a compact region of some large radius R the spacetime has a flat metric η_{ab}. Let (t,x) be a natural chart of the flat metric η_{ab} extended to $\Re^4$ and let the removed cylinders having radius a, centers at $z=\pm d/2$, and thus boundaries Σ_A, $A=1,2$ given by $|\mathbf{x}\pm\hat{\mathbf{z}}d/2|=a$. Here $\mathbf{x}$ is the three-vector associated with the point (t,x), and we shall write $r=|\mathbf{x}|$, $\hat{\mathbf{r}}=\mathbf{x}/|\mathbf{x}|$. Similarly, in describing Fourier components of a scalar field Φ we write k to denote a point of the three-dimensional k space, $\mathbf{k}$ when the vector character of k is explicitly used, and we set $\omega=|\mathbf{k}|$. The manifold $\mathcal{N}$ is constructed by identifying the cylindrical boundaries according to a translation $\mathcal{T}$, given by $t\rightarrow t+\tau$, $\mathbf{r}\rightarrow \mathcal{R}_{\pm}\mathbf{x}+\hat{\mathbf{z}}d$, where $\mathcal{R}_+$ is a rotation of Σ_1, yielding a nonorientable handle, and $\mathcal{R}_-$ is a rotation-reflection of Σ_1, yielding an orientable handle. The metric, $g_{ab} = -\partial_a t\, \partial_b t + h_{ab}$, is static, with timelike Killing vector ∂_t orthogonal to h_{ab}. When τ exceeds some τ_0, $\mathcal{N}$ has CTC's that traverse the handle and are present at all times. Although $\mathcal{N}$ has no spacelike hypersurface which could play the role of a Cauchy surface, one can pose initial data at past null infinity, and our goal will be to show that for all such data with finite energy, there is a solution to the scalar wave equation on $\mathcal{N}$. A simple heuristic argument based on geometrical optics is given in Refs. 2 and 3.

Because the removed cylinders are timelike, null infinity $\mathcal{J}$ is the Minkowski space $\mathcal{J}$. In the null chart $(v=t+r,\mathbf{x})$, $\mathcal{J}^-$ has coordinates $(v,\hat{\mathbf{r}})$, and a solution Φ to the scalar wave equation on Minkowski space has as initial data on $\mathcal{J}^-$ the single function

$$\psi(v,\hat{\mathbf{r}}) = \lim_{r\to\infty} r\Phi(v,r\hat{\mathbf{r}}) .$$

For our metric, the scalar wave equation has the form

$$(-\partial_t^2+\nabla^2)\Phi=0 , \tag{1}$$

where ∇ is the covariant derivative of the spatial metric h_{ab}. It will be convenient to regard the spacetime as a manifold with the boundary given by the two cylinders. Continuity of Φ and its normal derivative at identified spacetime points $P_1=(t,x_1)$, $P_2=(t+\tau,x_2)$ is then expressed by the boundary conditions

$$\Phi(P_2)=\Phi(P_1) \text{ and } \hat{\mathbf{n}}_2\cdot\nabla\Phi(P_2)=-\hat{\mathbf{n}}_1\cdot\nabla\Phi(P_1) , \tag{2}$$

where $\hat{\mathbf{n}}$ is the unit outward normal to the boundary Σ_1 (or Σ_2)$\times\Re$ (and $\hat{\mathbf{n}}\perp\partial_t$).

Because the spacetime is static, a solution Φ can be expressed as a superposition of the form

$$\Phi(t,x)=\int_{-\infty}^{\infty}\phi(\omega,x)e^{-i\omega t}d\omega , \tag{3}$$

where ϕ (in general, a distribution) satisfies

$$\begin{aligned} &\phi(\omega,x_2)=e^{i\eta}\phi(\omega,x_1) ,\\ &\hat{\mathbf{n}}_2\cdot\nabla\phi(\omega,x_2)=-e^{i\eta}\hat{\mathbf{n}}_1\cdot\nabla\phi(\omega,x_1) , \end{aligned} \tag{4}$$

with $\eta=\omega\tau$. Finding solutions with harmonic time dependence to the scalar wave equation on $\mathcal{N}$ is equivalent to solving the elliptic equation

$$(\nabla^2+\omega^2)\phi=0 , \tag{5}$$

with boundary conditions (4), on a three-manifold $\mathcal{M}\cong\Re^3-$(the balls enclosed by Σ_A).

We can now state our main result.

Theorem.—Let ψ be smooth initial data on $\mathcal{J}^-$ with finite energy. Then there exists a solution Φ to the scalar wave equation which is smooth and asymptotically regular on $\mathcal{N}$ and which has ψ as initial data.

This existence theorem will be proved by showing that a spectral decomposition of the form

$$\Phi(x,t)=\int E(k,x)[e^{-i\omega t}a(k)+e^{i\omega t}a^*(k)]d^3k \tag{6}$$

exists ($\omega=|\mathbf{k}|$), where $E(k,x)$ is the result of scattering a plane wave, $e^{i\mathbf{k}\cdot\mathbf{x}}$, off of the wormhole geometry. That is, $E(k,x)$ is the unique solution of the form $e^{i\mathbf{k}\cdot\mathbf{x}}$ plus purely outgoing waves. For fixed k, the function $E(k,x)$ is a solution on the spatial manifold $\mathcal{M}$ to Eq. (5), with boundary conditions (4). Because the boundary conditions (2) involve a time translation, the corresponding boundary conditions (4) depend on the frequency ω. Thus $E(k,x)$, $E(k',x)$ are, for $|\mathbf{k}|\neq|\mathbf{k}'|$, eigenfunctions of different operators; they are not orthogonal and their completeness is not guaranteed by the spectral theorem. Our task is to show that the solution to the scalar wave equation for arbitrary initial data on $\mathcal{J}^-$ can nevertheless be constructed as a spectral integral of the form (6).

The proof is given as a series of lemmas. For boundary conditions (4) specified by a fixed phase η, we show that the operator $\mathcal{L}_\eta := \nabla^2+\omega^2$ is self-adjoint on a dense subspace of $L_2(\mathcal{M})$ and that its eigenfunctions $F(\eta,k,x)$ (smooth solutions in a weighted L_2) are complete and orthonormal. This part of the proof is patterned on lectures of Wilcox.[5] The next step is a major departure from standard scattering theory, because one must piece together eigenfunctions corresponding to a phase η that depends on ω in accordance with $\eta=\omega\tau$. That is, the solution is of the form (6), where $E(k,x)=F(\eta=\tau\omega,k,x)$. We show that Eq. (6) is in fact well defined and gives a smooth solution to the scalar wave equation. We then verify that the solution is asymptotically regular and that it does have ψ as initial data on $\mathcal{J}^-$. Finally we prove uniqueness in a form somewhat weaker than we would like, but as strong as is available for fields with bounded source on Minkowski space.

We shall need several standard properties of Sobolev spaces, including the Sobolev embedding and trace theorems. These may be found in Reed and Simon.[6] Denote by $H_s(\mathcal{M})$ the Sobolev space on $\mathcal{M}$, so that for s a positive integer, $H_s(\mathcal{M})$ is the space of functions on $\mathcal{M}$ for which the function and its first s derivatives are square integrable.

Lemma 1.—The operator ∇^2 with boundary conditions (4) is self-adjoint on $L_2(\mathcal{M})$ with domain $\tilde{H}_2$, where $\tilde{H}_2 := \{f\in H_2(\mathcal{M}),$ satisfying Eq. (4) with fixed $\eta\}$.

Proof of lemma 1.—First note that ∇^2 is symmetric on the space of smooth functions satisfying the η boundary conditions:

$$\begin{aligned}\langle\psi|\nabla^2\phi\rangle &= \langle\nabla^2\psi|\phi\rangle + \left(\int_{\Sigma_1}+\int_{\Sigma_2}\right)(\psi^*\mathbf{n}\cdot\overleftrightarrow{\nabla}\phi)dS\\ &=\langle\nabla^2\psi|\phi\rangle , \end{aligned} \tag{7}$$

where the surface terms cancel by Eqs. (4). The operator ∇^2 on smooth functions in $L_s(\Re^3)$ has a self-adjoint extension with domain $H_2(\Re^3)$ (Ref. 7). The only difference here is that by imposing the η boundary conditions on the domain of ∇^2, one also imposes them on the domain of the adjoint operator.

From lemma 1, the analogous result for $\mathcal{L}_\eta$ immediately follows as a corollary.

Corollary.— $\mathcal{L}_\eta$ is self-adjoint on $L_2(\mathcal{M})$ with domain $\tilde{H}_2$.

We now characterize the eigenfunctions of $\mathcal{L}_\eta$.

Lemma 2.—There is a unique solution, $F(\eta,k,x)$, to the equation $\mathcal{L}_\eta F=0$, for which $F=(2\pi)^{-3/2}e^{i\mathbf{k}\cdot\mathbf{x}}$ plus outgoing waves. The functions $F(\eta,k,x)$ are complete and orthonormal.

To prove existence, one first recasts the problem for the existence of a solution to Eq. (5) as the existence of a solution to an inhomogeneous equation, $(\nabla^2+\omega^2)\varphi=\rho$, with ρ having compact support and φ purely outgoing. To do this, let $\chi(r)$ be a smooth steplike function of a radial coordinate r for which $\chi(r)=0$, $r\le R$; $\chi(r)=1$, $r\ge R+\epsilon$, for some $\epsilon>0$. Write $\phi=\chi(2\pi)^{-3/2}e^{i\mathbf{k}\cdot\mathbf{x}}+\varphi$. Then, if we set $\rho=-(2\pi)^{-3/2}e^{i\mathbf{k}\cdot\mathbf{x}}(\nabla^2+2i\mathbf{k}\cdot\nabla)\chi$, ρ has support (is nonvanishing) only in the annulus $R-\epsilon<r<R$, and outside $r=R$, φ is the outgoing scattered wave. Existence is now proved by adding a small imaginary part to the frequency and taking the limit as the imaginary part goes to zero. This "limiting absorption" method selects the outgoing solution in the real-frequency limit by the choice of sign for the imaginary part of the frequency. The key to the proof is the fact that, because $\mathcal{L}_\eta$ is self-adjoint, $\mathcal{L}_\eta+i\epsilon$ is invertible in L_2. The proof of lemma 2 requires lemma 3.

Lemma 3.—Let λ_k be a sequence of complex numbers with positive imaginary part, such that $\lambda_k\to\omega^2$. Consider a family $\{\varphi_k,\rho_k\}$ of smooth functions on $\mathcal{M}$, where, for each k, ρ_k has compact support, and φ_k is the unique asymptotically regular solution to the equation

$$(\nabla^2+\lambda_k)\varphi_k=\rho_k\,. \tag{8}$$

If $\rho_k\to\rho$ in $L_2(\mathcal{M})$, then a subsequence φ_m converges in an $H_1(\mathcal{M})$ norm to a smooth outgoing solution φ to $(\nabla^2+\omega^2)\varphi=\rho$.

Proof of lemma 3.—Let $\mathcal{M}_R=\{x\in\mathcal{M},\ r\le R\}$, with R large enough to enclose the handle and the support of $\{\varphi_k\}$. Denote by $\|\ \|_{2,R}$ the norm of $H_2(\mathcal{M}_R)$. One first shows that if $\|\varphi_k\|_{2,R}$ has a bound independent of k, then by the Sobolev embedding theorem, a subsequence φ_m converges in $H_1(\mathcal{M}_R)$ to φ satisfying $(\nabla^2+\omega^2)\varphi=\rho$ on $\mathcal{M}_R$. Outside $\mathcal{M}_R$, φ_m is given in terms of its values inside $\mathcal{M}_R$ (at $R'<R$) by

$$\varphi_m(x)=\int_{|y|=R'}dS\,\varphi(y)\overleftrightarrow{\partial}_{|y|}\frac{e^{i(\lambda_m)^{1/2}|\mathbf{x}-\mathbf{y}|}}{|\mathbf{x}-\mathbf{y}|}\,. \tag{9}$$

Then the Sobolev trace theorem and convergence of φ_m in $H_1(\mathcal{M}_R)$ imply that φ_m converges pointwise to φ outside $\mathcal{M}_R$. Because $\mathrm{Im}(\lambda_w)^{1/2}>0$, we have for $r>R$,

$$\varphi(x)=\int_{|y|=R'}dS\,\varphi(y)\overleftrightarrow{\partial}_{|y|}\frac{e^{i\omega|\mathbf{x}-\mathbf{y}|}}{|\mathbf{x}-\mathbf{y}|}\,, \tag{10}$$

an outgoing solution.

This relied on the assumption that $\|\varphi_k\|_{2,R}$ was bounded. If it is not, then the sequence $\hat{\varphi}_k=\varphi_k/\|\varphi_k\|_{2,R}$ has both unit norm and a source $\hat{\rho}_k=\rho_k/\|\varphi_k\|_{2,R}$ whose norm converges to zero. This easily leads to a contradiction:

$$\|\hat{\rho}_k\|\to 0\Longrightarrow\|\hat{\varphi}_k\|_{0,R}\to 0\Longrightarrow\|\hat{\varphi}_k\|_{2,R}\to 0\,,$$

because $\|\hat{\varphi}_k\|_{2,R}\le C\|\hat{\varphi}_k\|_{0,R}+\|\hat{\rho}_k\|_0$.

Finally, to verify the second boundary condition, one again uses convergence of φ_m in $H_{1,R}$ to show that for any φ' in $H_{2,R}$,

$$\langle\nabla\varphi'|\nabla\varphi\rangle-\omega^2\langle\varphi'|\varphi\rangle-\langle\varphi'|\rho\rangle=0\,.$$

Then the second boundary condition of Eq. (4) follows from the first. Elliptic regularity implies smoothness.

Proof of lemma 2.—Existence of the solution $F(\eta,k,x)$ is an immediate consequence of lemma 3. Uniqueness is automatic from Rellich's theorem. The orthonormality relations essentially express the fact that $\mathcal{L}_\eta$ is self-adjoint; an explicit proof of orthogonality, needed because the eigenfunctions do not belong to L_2, is given by Wilcox.[7]

Proof of theorem.—The proof shows that for $a(k)$ corresponding to smooth, asymptotically regular data for Minkowski space, the right-hand side of Eq. (6) exists and is both smooth and asymptotically regular. Although, for different frequencies the functions $E_\theta(k,x)=F(\eta=\omega\tan\theta,\ k,\ x)$ are not orthonormal, one can use the orthonormality of $F(\eta,k,x)$ for fixed η to bound an integral norm of $E_\theta(k,x)$. One integrates a norm of F over all values of η, and then rewrites the integral in terms of θ, where $\eta=\omega\tan\theta$: The fact that F itself is norm preserving implies (after some algebra) the inequality

$$\int_{\theta=0}^{\pi/2-\delta}d\theta\int dk\,dy\frac{|\omega^n\hat{E}_\theta(k,y)|^2}{(1+\omega^2)^{n+\epsilon}(1+y^2)^{3/2+\epsilon}}<C\,, \tag{11}$$

where

$$\hat{E}_\theta(k,y)=\int dx\,E_\theta(k,x)\frac{e^{i\mathbf{x}\cdot\mathbf{y}}}{(1+x^2)^{3/2+\epsilon}}\,, \tag{12}$$

and where C is some positive real constant. From Eq. (11) and smoothness of F in θ, it follows that for any n,

$$\frac{E_\theta(k,x)}{(1+x^2)^{3/2+\epsilon}}=L_{2,n+\epsilon}(\Re^3)\otimes H_{n-3/2-\epsilon}(\mathcal{M})\,. \tag{13}$$

Then we have $\int dk\,a(k)E_\theta(k,x)e^{-i\omega t}\in C^\infty$ when the initial data are smooth, because $a(k)$ is a function of rapid decrease.

Asymptotic regularity is proved as follows. Outside $r=R$ the spacetime is flat, and the field Φ can be written in the form, $\Phi^{\text{free}}_{-}+\Phi^{\text{out}}$, the sum of the Minkowski space solution with initial data ψ on $\mathcal{I}^-$ and an outgoing solution. With χ given by (13), $\chi\Phi^{\text{out}}$ is the outgoing solution to the flat-space wave equation with a smooth, bounded source, whence it is regular at $\mathcal{I}^+$; and self-adjointness of $\mathcal{L}_{\omega\tau}$ implies that the data at $\mathcal{I}^+$ have finite energy. The same argument with time reversed implies regularity at $\mathcal{I}^-$ with data for Φ agreeing with ψ. Finally, regularity at spatial infinity (finiteness of the field's energy on spacelike hypersurfaces outside the handle) follows from an argument analogous to that leading to Eqs. (11) and (12) above, but with a bound on a Fourier transform of E with respect to k and η, instead of x.

Because the field equation is linear, proving uniqueness is equivalent to showing that $\Phi=0$ is the only smooth solution with vanishing initial data at $\mathcal{I}^-$. Note that uniqueness does *not* hold for distributional (i.e., weak) solutions to $\Box\Phi=0$. That is, if $c(\lambda)$ is a null geodesic of the spacetime $\mathcal{N}$, then $\Phi=\int d\lambda\,\delta(x-c(\lambda))$ is a solution in the distribution space H_{-2}. There are null geodesics that never hit $\mathcal{I}^-$, but loop through the handle an infinite number of times, and the corresponding distributional solution has zero data on $\mathcal{I}^-$. Our proof also assumes that $\Phi\in L_2(\mathcal{N})$; the condition on falloff in time is stronger than one would like (it fails to rule out smoothed versions of the looping null rays). The proof is straightforward: Integrating the local equation $\nabla_a T^a_b t^b=0$ (where T^a_b is the stress-energy tensor of the scalar field and t^b is the timelike Killing vector field) over the spacetime implies that incoming flux at $\mathcal{I}^-$ is equal to outgoing flux at $\mathcal{I}^+$, vanishing when there is zero data on $\mathcal{I}^-$. Outside a spatially compact region $\mathcal{D}$, the metric is flat, and one can use the flat-space result for each harmonic, that zero data on $\mathcal{I}^+$ and $\mathcal{I}^-$ implies $\phi_{lm\omega}=0$. Finally, a solution vanishing on the timelike boundary $\partial\mathcal{D}$ must vanish on $\mathcal{D}$ by Calderón's[8] timelike uniqueness theorem.

The explicit eigenfunctions can be constructed iteratively as a multiple scattering series if the metric is chosen to be exactly flat outside the removed cylinders. A proof of convergence for wavelength longer than $\sqrt{2}e\pi a$ and $a/d<1/2e$ and a numerical verification of convergence for shorter wavelengths will be presented elsewhere,[9] together with details of the work outlined above.

In conclusion, we claim to have proved a surprising result—a result that provides some foundation for the suggestions of Ref. 3 that CTC's may not be as nasty as some have been inclined to think. Given the positive step that this is, it is important to emphasize what we have not done. First, our uniqueness proof is too weak to rule out solutions that do not die off in time. It is likely that the only such solutions are distributional, but this is a conjecture, not a theorem. By restricting consideration to a test field on a background spacetime, we avoid the deeper question of consistency of a quantum or classical spacetime in which CTC's arise dynamically and in which the topology is not trivial. In particular, it is not yet clear whether the material stress tensor of a realistic quantum field can keep a wormhole open. The creation of a time-tunnel spacetime from a causal wormhole spacetime takes place across a Cauchy horizon. We have not proved the stability of this horizon to classical nonzero initial data given on a spacelike hypersurface before it, although our result lends credence to the physical argument recounted in Ref. 3 that this Cauchy horizon is, in fact, classically stable. For a quantum field, the results of Kim and Thorne[10] indicate that divergences of the vacuum polarization are to be expected on surfaces of closed-null crossing points in our spacetime. However, it also seems that these divergences can be small in the sense that, within a Planck length of the surface of crossing-null geodesics, the vacuum polarization stress tensor can be very much less than the Planck density. Left unexplored are the difficult problems of nonlinear, self-interacting fields, where the grandfather paradox arises in a stronger form than it does here. First steps at looking into the nonlinear problem have been taken by Klinkhammer and Thorne[11] and by Novikov.[12]

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