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Weinberg's Nonlinear Quantum Mechanics and the Einstein-Podolsky-Rosen Paradox

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I show that Weinberg's nonlinear quantum mechanics leads either to communication via Einstein-Podolsky-Rosen correlations, or to communication between branches of the wave function.

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Weinberg[1,2] has proposed a formalism for testing nonlinear extensions of quantum mechanics. In this framework, there are nonlinear observables in addition to the usual linear ones. Heuristically, the greater number of observables suggests that there is more information in the wave function than in the usual linear theory. This in turn raises the possibility that the fictitious violation of locality that occurs in the Einstein-Podolsky-Rosen (EPR) experiment in linear quantum mechanics[3] might become a real violation in the nonlinear theory. That is, the EPR apparatus might be used to send instantaneous signals.[4]

In this Letter I determine the constraints imposed upon observables by the requirement that transmission not occur in the EPR experiment. This leads to a different treatment of separated systems than that originally proposed by Weinberg. I find that forbidding EPR communication in nonlinear quantum mechanics necessarily leads to another sort of unusual communication: that between different branches of the wave function.[5] Gisin[6,7] and Czachor[8] have also discussed EPR communication in nonlinear quantum mechanics; their results are discussed at the end of the paper.

Consider two widely separated systems I and II. The wave function is Ψ_{ij}, where the indices i and j refer to the two systems, running over $1,\ldots,M$ and $1,\ldots,N$, respectively. At time $t=0$ the wave function has been prepared, with correlations between the two systems. At some later time, an observable $a_{\rm II}$ will be measured in the receiving system II. It is sufficient to consider ordinary linear observables in system II. In the formalism of Weinberg, linear observables are bilinear functions of Ψ and Ψ^*,

$$a_{\rm II}=\Psi^*_{ij}A_{jl}\Psi_{il}\,, \tag{1}$$

with A_{jl} a Hermitian matrix (repeated indices are summed). At the receiving end I will therefore assume that the usual idea of measurement applies. In system I, a signal is sent by turning on a field $\epsilon(t)$ which couples to a (possibly nonlinear) observable $a_{\rm I}(\Psi_{ij},\Psi^*_{kl})$. For simplicity, suppose that this is the whole Hamiltonian:

$$h(t,\Psi_{ij},\Psi^*_{kl})=\epsilon(t)a_{\rm I}(\Psi_{ij},\Psi^*_{kl})\,. \tag{2}$$

This gives the equation of motion[1,2]

$$da_{\rm II}/dt=-i\{a_{\rm II},h\}\,, \tag{3}$$

where the Poisson bracket is

$$\{a,b\}=\frac{\partial a}{\partial\Psi_{ij}}\frac{\partial b}{\partial\Psi^*_{ij}}-\frac{\partial a}{\partial\Psi^*_{ij}}\frac{\partial b}{\partial\Psi_{ij}}\,. \tag{4}$$

It is sufficient to consider a weak field, integrating the equation of motion to first order in ϵ:

$$a_{\rm II}(t)=a_{\rm II}(0)-i\{a_{\rm II}(0),a_{\rm I}(0)\}\int_0^t dt'\,\epsilon(t')\,. \tag{5}$$

In this framework,[1] the expectation value of $a_{\rm II}$ is simply the value of $a_{\rm II}$, and so the condition that the measurement made in system II not depend on the field applied in system I is

$$\{a_{\rm II},a_{\rm I}\}=0\,. \tag{6}$$

The Poisson bracket $\{a_{\rm II},\cdot\}$ generates unitary rotations on the second index of Ψ_{ij}. By considering all linear observables $a_{\rm II}$—that is, all Hermitian matrices A_{jl}—the condition (6) is precisely the statement that $a_{\rm I}(\Psi_{ij},\Psi^*_{kl})$ is invariant under general unitary rotations of the second index. Thus, this index must always be contracted between Ψ and Ψ^*, and $a_{\rm I}$ is a function only of the density matrix

$$\rho^{\rm I}_{ik}=\sum_m \Psi_{im}\Psi^*_{km}\,. \tag{7}$$

Now it is possible to go back and enlarge the set of observables in system II, subject to the no-signal condition (6). The density matrices for systems I and II commute,

$$\{\rho^{\rm I}_{ik},\rho^{\rm II}_{jl}\}=0\,, \tag{8}$$

where

$$\rho^{\rm II}_{jl}=\sum_n \Psi_{nj}\Psi^*_{nl}\,. \tag{9}$$

Thus, the no-signal condition allows observation of any function $a_{\rm II}$ of $\rho^{\rm II}$.

This is the first main result: Assuming only that the observables include all the usual linear Hermitian observables, the necessary and sufficient condition for an isolated system not to receive information via EPR correlations is that all observables in the system depend only on the density matrix for the system:

$$a_{\rm I}=h_{\rm I}\left(\sum_m \Psi_{im}\Psi^*_{km}\right). \tag{10}$$

The observables proposed by Weinberg[1] for separated systems,

$$a'_{\rm I}=\sum_m h_{\rm I}(\Psi_{im},\Psi^*_{km})\,, \tag{11}$$

are not of this form (except when linear) and therefore allow EPR communication.

I now show that nonlinear observables of the form (10) lead to unusual communication of another sort. Consider a process involving four steps.

(1) A spin-$\frac{1}{2}$ ion enters a Stern-Gerlach device, which couples to the linear spin component $\langle\psi|\sigma^3|\psi\rangle$. In the device the beam splits, a macroscopic observer notes the direction taken, and then the two paths are rejoined.

(2) After the Stern-Gerlach device: If the observer saw $\langle\psi|\sigma^3|\psi\rangle=\frac{1}{2}$ he does nothing. If he saw $\langle\psi|\sigma^3|\psi\rangle=-\frac{1}{2}$ he takes one of two actions: (a) nothing, or (b) rotates the spin into the $+\hat{1}$ direction with a magnetic field coupled to the linear observable $\langle\psi|\sigma^2|\psi\rangle$.

(3) The ion enters a region of field coupled to the nonlinear observable[9]

$$h_3=f\frac{\langle\psi|\sigma^1|\psi\rangle\langle\psi|\sigma^1|\psi\rangle}{\langle\psi|\psi\rangle}\,. \tag{12}$$

(4) If the observer originally saw $\langle\psi|\sigma^3|\psi\rangle=\frac{1}{2}$ he again measures the spin with a Stern-Gerlach device coupled to the linear spin component $\langle\psi|\sigma^3|\psi\rangle$; otherwise, he does nothing.

Now follow the evolution of the wave function Ψ_{ij}, where the indices refer respectively to the ion and to the observer. It is useful to focus on the two partial density matrices

$$\begin{aligned}\rho^{{\rm I}(+)}_{ik}&=\sum_{m\in S^+}\Psi_{im}\Psi^*_{km}\,,\\ \rho^{{\rm I}(-)}_{ik}&=\sum_{m\in S^-}\Psi_{im}\Psi^*_{km}\,,\end{aligned} \tag{13}$$

where S^+ is the set of all observer states in which $\langle\psi|\sigma^3|\psi\rangle=\frac{1}{2}$ was seen, and S^- is the set of all observer states in which $\langle\psi|\sigma^3|\psi\rangle=-\frac{1}{2}$ was seen. After step (1),

$$\rho^{{\rm I}(+)}=\frac{1+\sigma^3}{4},\quad \rho^{{\rm I}(-)}=\frac{1-\sigma^3}{4}\,. \tag{14}$$

After step (2), action (a) leaves the density matrices (14) unchanged, while action (b) gives rise to

$$\rho^{{\rm I}(+)}=\frac{1+\sigma^3}{4},\quad \rho^{{\rm I}(-)}=\frac{1+\sigma^1}{4}\,. \tag{15}$$

To study step (3) it is necessary to be precise about the form of the nonlinear interaction. Here, the result of the EPR analysis enters, requiring that the nonlinear observable (12), when extended to the two-system wave function, must be a function only of the total $\rho^{\rm I}$ and not of the separate $\rho^{{\rm I}(\pm)}$. Note that one could arrange for the ion to propagate a long distance between steps (2) and (3), and then to be reflected back for step (4), making it clear that the observer and ion are separated systems. Knowing the form (12) of h_3 on pure states does not fully determine it as a function of density matrices. Rather,

$$h_3=f_1\frac{{\rm Tr}\rho^{\rm I}\sigma^1{\rm Tr}\rho^{\rm I}\sigma^1}{{\rm Tr}\rho^{\rm I}}+f_2\frac{{\rm Tr}\rho^{\rm I}\sigma^1\rho^{\rm I}\sigma^1}{{\rm Tr}\rho^{\rm I}}\,, \tag{16}$$

with $f_1+f_2=f$. Expanding out the sums in the density matrices, there are cross terms between states in S^+ and S^-, which will lead to cross terms between the evolution of different macroscopic states of the observer. It may be that a particular ratio of f_1 and f_2 is always present, or it may be that there are independent fields which couple to each interaction.

It is convenient to analyze first the case that $f_1=f$ and $f_2=0$. The time τ spent in the field, and the strength f of the field, will be chosen to satisfy $\tau f=\frac{1}{2}\pi$. The equations of motion in step (3) are then

$$\frac{d\rho^{{\rm I}(\pm)}}{dt}=2if[\rho^{{\rm I}(\pm)},\sigma^1]\frac{{\rm Tr}\rho^{\rm I}\sigma^1}{{\rm Tr}\rho^{\rm I}}\,. \tag{17}$$

This integrates easily to give the partial density matrices after step (3). If action (a) was taken, these do not change,

$$\rho^{\mathrm{I}(+)}=\frac{1+\sigma^3}{4},\quad \rho^{\mathrm{I}(-)}=\frac{1-\sigma^3}{4}, \tag{18}$$

while if action (b) was taken they become

$$\rho^{\mathrm{I}(+)}=\frac{1-\sigma^3}{4},\quad \rho^{\mathrm{I}(-)}=\frac{1+\sigma^1}{4}. \tag{19}$$

Finally, in step (4), it follows from Eqs. (18) and (19) that if the observer measures the spin, he obtains $+\frac{1}{2}$ if action (a) was taken, and $-\frac{1}{2}$ if action (b) was taken. *But the action and observation are in two different branches of the wave function,* in which the original spin was measured to be $-\frac{1}{2}$ and $+\frac{1}{2}$, respectively. The method by which the choice of action is to be made has not been discussed, and there are various interesting possibilities to contemplate, including classical and quantum random-number generators. For the present purpose it is sufficient to imagine that the observer has made some firm choice prior to the experiment. If he then sees the original spin as $+\frac{1}{2}$, he does nothing but make a second measurement, but the outcome depends on what he *would have done* had he originally seen $-\frac{1}{2}$. In effect, the apparatus reads the observer's mind.

Note that in steps (1), (2), and (4), in which the observer participates, only linear observables are involved, and so the usual quantum-mechanical idea of measurement has been used. The experiment has been designed to produce no further bifurcation of the wave function, so that by iterating steps (2), (3), and (4), and allowing the two branches to switch roles, the observers in the two branches of the wave function may exchange binary messages of arbitrary length. This is an Everett phone, in contrast to the EPR phone designed above.

In the event that f_2 is nonzero, the coupling between branches remains but the evolution and outcome are no longer as simple for the particular setup above. It is convenient to alter the conditions of the experiment so as to obtain a simple outcome. There are many ways to do this. One is the following: action (a) is now to rotate the field into the $+\hat{2}$ direction, action (b) is now to rotate the field into the $-\hat{2}$ direction, the time interval is $tf_2=\pi/\sqrt{2}$, and in step (4) it is the $\hat{2}$ component that is measured. After step (3), if action (a) was taken at step (2), the partial density matrices are

$$\rho^{\mathrm{I}(+)}=\frac{1+\sigma^2}{4},\quad \rho^{\mathrm{I}(-)}=\frac{1+\sigma^3}{4}, \tag{20}$$

while if action (b) was taken they are

$$\rho^{\mathrm{I}(+)}=\frac{1-\sigma^2}{4},\quad \rho^{\mathrm{I}(-)}=\frac{1+\sigma^3}{4}. \tag{21}$$

Again the observation made in step (4) depends on the action taken in the other branch in step (2).

The alternative formulation of separated systems, Eq. (11), allows the construction of an EPR phone, but at first sight appears to be free of the Everett-phone phenomenon. This is because the evolution for different states of the observer, different m values, separates. However, the form (11) is basis dependent, and it is not clear how a basis is to be singled out. It is necessary to give a prescription for choosing the basis in which Eq. (11) holds. In order to forbid the Everett phone, it is necessary that this basis never involve superpositions of macroscopic systems (observers) in different states.

It is important now to note that the Everett phone may not actually work in practice. I have ignored *previous* branchings of the wave function, describing the macroscopic observer and apparatus as though they started in a definite state, as would be acceptable in the linear theory. However, the analysis of the Everett phone shows this assumption to be self-inconsistent: The evolution of the wave function will be coupled to all other possible states. Thus, while the analysis does show that branches are coupled to one another, practical communication between branches may be drowned out by the coupling to all the other branches of the wave function of the Universe.

Do the results imply that nonlinear quantum mechanics is inconsistent, and thus "explain" the linearity of the theory?[10] Communication between branches of the wave function seems even more bizarre than faster-than-light communication and consequent loss of Lorentz invariance, but it is not clear that it represents an actual inconsistency. It means that reduction of the wave function never occurs, so that the standard Copenhagen interpretation of quantum mechanics no longer applies. The many-worlds interpretation of quantum mechanics[5] becomes the natural one, with communication between the worlds now possible. I do not know whether this allows a completely consistent interpretation of the nonlinear theory—this is a far-reaching question—but can only note that in the present thought experiments the evolution equations are mathematically consistent and allow a consistent interpretation.

Gisin has argued that nonlinear quantum mechanics leads to EPR communication, both in Weinberg's theory[7] and more generally.[6] He assumes reduction of the wave packet—that is, the projection postulate. As we have seen, without the projection postulate EPR communication need no longer occur,[11] provided that the observables (11) for separated systems are replaced with the form (10). Also, Czachor[8] shows by explicit calculation, using the form (11), that EPR communication occurs in Weinberg's theory.

To what extent do the results apply to more general nonlinear quantum theories? Weinberg's theory is rather general, carrying over two main assumptions from the linear theory. The first is that the equation of motion can be put in the form (3). I have used this assumption extensively. In particular, "observables" have been defined pragmatically, as any quantities which can be added to $h(\Psi,\Psi^*)$, Eq. (2). This assumption in any case seems very well motivated, as it leads to the usual connection between symmetries and conservation laws, and aids in the interpretation in other ways. The second assumption, which may be less well motivated, is that observables are homogeneous of degree (1,1) in Ψ and Ψ^*. This played no role in the analysis of the EPR phone, and only a minor role in the analysis of the Everett phone (it allowed the separation of the space and spin wave functions[1]).

Finally, what are the experimental implications of these results? It is important to note that communication between branches of the wave function invalidates most previous attempts to analyze the experimental consequences of nonlinearities, such as those in Refs. 1, 2, and 12. This is because these analyses ignore previous branchings of the wave function and treat macroscopic systems as though they begin in definite macroscopic states. A complete analysis requires consideration of the entire wave function of the Universe and is therefore rather complicated. Naively, it would seem that nonlinear effects will be very much diluted by the enormous number of branches, since the amplitude for any given branch of the wave function is exceedingly small. This leads to the discouraging conclusion that nonlinearities could be of order 1 in a fundamental theory and yet the effective nonlinearity measurable experimentally would still be unobservably small.

If quantum nonlinearities are observed nonetheless, the thought experiments described herein would seem to be simple enough to carry out in practice, thereby determining which of EPR communication and Everett communication is actually realized.

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[1]S. Weinberg, Ann. Phys. (N.Y.) **194**, 336 (1989).

[2]S. Weinberg, Phys. Rev. Lett. **62**, 485 (1989).

[3]A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[4]This issue of information in nonlinear quantum mechanics has been discussed from a different (thermodynamic) point of view by A. Peres, Phys. Rev. Lett. **63**, 1114 (1989), and S. Weinberg, Phys. Rev. Lett. **63**, 1115 (1989).

[5]*The Many-Worlds Interpretation of Quantum Mechanics,* edited by B. S. DeWitt and N. Graham (Princeton Univ. Press, Princeton, 1973). In this interpretation, a branch of the wave function is a subspace in which a macroscopic system is in a definite macroscopic state.

[6]N. Gisin, Helv. Phys. Acta **62**, 363 (1989).

[7]N. Gisin, Phys. Lett. A **143**, 1 (1990).

[8]M. Czachor, "Mobility in Weinberg's Non-linear Quantum Mechanics" (unpublished).

[9]The state $|\psi\rangle$ refers only to the spin wave function. It is shown in Ref. 1 that the nonlinear equation admits solutions in which the wave function is a product of spin and space wave functions. For simplicity we have implicitly assumed this in step (3).

[10]Jordan has recently discussed general grounds from which the linearity of quantum mechanics might be derived in T. F. Jordan, Am. J. Phys. (to be published).

[11]In the many-worlds interpretation (Ref. 5), projection is a derived consequence of the (linear) theory, rather than a postulate.

[12]J. J. Bollinger, D. J. Heinzen, W. M. Itano, S. L. Gilbert, and D. J. Wineland, Phys. Rev. Lett. **63**, 1031 (1989).