Critical Properties of a Randomly Driven Diffusive System

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We consider a system of interacting particles, diffusing under the influence of both thermal noise and a random, external electric field which acts in a subspace of m dimensions. In the nonequilibrium steady state, the net current is zero. When the interparticle interaction is short ranged and attractive, a second-order phase transition is expected. Analyzing this system in field-theoretic terms, we find the upper critical dimension to be $4-m$ and its behavior to fall outside the universality classes of the equilibrium Ising model and the usual driven diffusive system. A new fixed point and critical exponents are computed.

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Recently, considerable effort[1-10] has been devoted to the study of phase transitions in a driven lattice-gas system, which was originally introduced as a model for superionic conductors.[1] In this system, particles hop from site to nearest-neighbor site under the influence of a short-ranged interaction between the particles, a simulated thermal bath, and an external "driving" field E. Most attention was directed to models in which the interaction is attractive ("ferromagnetic," if the lattice gas is phrased in the language of Ising spins), while some authors[1,9,10] considered repulsive ("antiferromagnetic") interactions. In all cases, the E field is a *constant,* in both space and time, so that, with periodic boundary conditions, the system eventually settles into a steady state with a positive average current.

In the absence of E, these systems are well known to undergo a second-order transition. Near this critical point, nonclassical singularities, belonging to the Ising universality class, are present in the thermodynamics if d, the spatial dimension, is below 4. When driven, such a critical point survives for a range of E. However, the critical properties are very different, depending on whether the interaction is attractive or repulsive. All results pertaining to criticality are obtained by one of the three following methods: (a) Monte Carlo simulations[1,6,7] in $d \leq 3$, (b) mean-field approaches,[2] and (c) field-theoretic renormalization-group calculations.[3,4] Though not all approaches agree on the details, there is little doubt that driven diffusive systems with attractive interactions belong to a *new* universality class, while those with repulsive interactions are Ising-like.[9]

In this Letter, we extend these investigations by studying the effects of a *random* driving field on critical properties of this system. More precisely, we have in mind annealed randomness, i.e., an $E_\alpha(x,t)$ with the following properties: (a) It acts in an m-dimensional subspace of a d-dimensional system; (b) $\langle E_\alpha(x,t)\rangle=0$, $\alpha=1,\ldots,m$; and (c) $\langle E_\alpha(x,t)E_\beta(x',t')\rangle=\Delta\delta(x-x')\delta(t-t')\delta_{\alpha\beta}$. Clearly, $m=0$ is the usual equilibrium case while $m=1$ corresponds to the standard driven diffusive system with a randomly fluctuating field amplitude. Borrowing the language of the constant-E case, we will use the adjectives "parallel" and "transverse" when referring to the m- and $(d-m)$-dimensional subspaces, respectively.

Since we are interested in the universal, critical properties, we will focus only on the low-frequency and long-wavelength limits of the system. To study such infrared behavior, it is customary to consider a coarse-grained, continuum version, regardless of the microscopic discreteness. In principle, one can perform such a coarse-graining operation, starting from the microscopic transition probabilities and the master equation, arriving at the Langevin equation. In practice, this route has rarely been taken. Instead, one tries to identify the slow variables relevant to the critical point and postulate a Langevin equation, guided by considerations such as symmetries and conservation laws.

Following this route, we propose a set of Langevin equations. We will argue that these equations display all the features necessary for an adequate description of the system. Then, we exploit standard techniques[11] of field-theoretic renormalization-group analysis for studying dynamics. For the attractive case, the upper critical dimension, above which critical behavior is mean-field-like, is $d_c=4-m$. This result is in stark contrast to[3-5] $d_c=5$, which holds for a system driven by a constant E. The fixed point governing this nonequilibrium transition is quite *distinct* from both the Wilson-Fisher[12] (for equilibrium systems) and the standard driven diffusive fixed points.[3,4] For the repulsive case, we find little difference between systems driven by random or constant fields, so that the (leading) singularities are again Ising-like.

In the remainder of this Letter, we describe the system and our methods briefly. More details will be published elsewhere. We will present the main results, i.e., some of the exponents associated with the new fixed point. We conclude with some remarks on possible observations and related issues.

Consider a lattice gas on, say, a square lattice. To denote the configurations, we use the set of occupation numbers $\{n_i\}$, where $n_i=1$ or 0 depending on whether the site i is occupied or not. The short-ranged interaction is modeled by the Hamiltonian $H=-J\sum n_i n_j$, the sum being over nearest-neighbor pairs. For the attractive case, $J>0$; otherwise, it is negative. Since we wish to conserve particle number, $\sum n_i$, and to study criticality, we consider only half-filled systems. For equilibrium properties, we let the system evolve stochastically under Kawasaki[13] dynamics, i.e., the particles may only jump into a neighboring hole. Coupling to a thermal bath, at temperature T, can be simulated by having the jump rates depend on the change in the internal energy, ΔH, through the ratio $\Delta H/T$ (Boltzmann's constant is set to unity).

To investigate nonequilibrium steady states due to a *constant* E, we may simply modify the jump rates by adding a term ϵE to ΔH, where $\epsilon=1$, -1, and 0 for jumps against, along, or transverse to E, respectively. With periodic boundary conditions, the system settles into a steady state eventually. These *periodic* boundary conditions are crucial in producing a true non-Hamiltonian system, since the driving force can no longer be written as gradient of a well-defined potential.

In a field-theoretic analysis of such systems, we start with a continuum version of the dynamics, in terms of a Langevin equation for an order parameter $\phi(x,t)$ with an appropriate noise term. For the $J>0$ case, ϕ denotes the coarse-grained local density fluctuations $n(x,t)-\frac{1}{2}$, while for $J<0$, ϕ is the difference between particle densities on the two checkerboard sublattices. As we mentioned, although such a coarse-graining operation is possible in principle, we follow the customary practice by postulating Langevin equations directly. Since most of the novel properties appear in the attractive case, we now focus on that and defer discussions on the $J<0$ system until the end.

One way to arrive at the desired equations is to start with the ones for a constant driving field. Replacing the constant by $E(x,t)$ and assuming a Gaussian distribution with the properties stated above, we can integrate out E to give new effective parameters. Another way is to argue that the extra randomness associated with exchanges in the parallel directions has three important effects: (a) The diffusion constant, or the effective temperature, is higher for this subspace and (b) the noise correlation matrix is not proportional to the diffusion matrix, due to violation of the fluctuation-dissipation theorem. While both of these effects are present already in the constant-E model, the key distinction here is that (c) the term $g\nabla^2\phi^3$ replaces $\partial E\phi^2$ as the most relevant nonlinearity. The latter must be absent, since our model is symmetric under $x\to -x$. Taking either route leads us to the following Langevin equation:

$$\lambda^{-1}\partial_t\phi(x,t)=\rho\partial^2\phi+\tau\nabla^2\phi-\nabla^4\phi + g\nabla^2\phi^3/3!-(\boldsymbol{\partial}\cdot\xi+\boldsymbol{\nabla}\cdot\zeta)\,, \qquad (1)$$

where ∂ (∇) stands for gradients in the parallel (transverse) subspace. For economy, we left out all terms which are irrelevant in the renormalization-group sense and scaled the coefficient of $-\nabla^4\phi$ to unity. As in the constant-E system, strong anisotropy leads us to expect the critical point at $\tau=0$ with $\rho>0$, the effective temperature being higher in the parallel subspace. Our noise is distributed according to $\exp\{-\int dx\,dt[\xi^2/4\sigma' + \zeta^2/4\sigma]\}$. However, $\sigma'/\rho\neq\sigma/\tau$, since we have a nonequilibrium system. Finally, note that all parameters are functions (assumed analytic) of the microscopic T, J, and Δ. But the detailed dependence is not crucial to universal properties. We only need $\tau(T_c,J,\Delta)=0$, which defines $T_c(J,\Delta)$, while $\tau\approx 0$ is the critical region.

Next, we follow a standard procedure, casting (1) in the dynamic functional formalism[11] and studying the critical theory (renormalized $\tau_R=0$). Denoting the external momentum scale by μ, we see that, naively, parallel and transverse momenta scale as μ^2 and μ, respectively. As a result, σ' is naively irrelevant, compared to σ. Further, the dimension of g is $4-m-d$, giving us the upper critical dimension $d_c=4-m$. In contrast, the same analysis leads to $(5-d)/2$ for the naive dimension of E in the constant-drive model.[3-5] Thus, we set up an expansion in powers of $\epsilon\equiv 4-m-d$ and find a nontrivial fixed point $g^*=O(\epsilon)$. In the sense that $m=0$ *is* the equilibrium system, this expansion is similar to the Wilson-Fisher case.

Before we present the results, we should note that only the $m=1$ case is "interesting," in the following sense. Since the (infrared) divergences are associated with the transverse $(d-m)$-dimensional subspace, we keep m a positive integer. If $m=2$, we see that $d_c=2$, meaning that such models are well described by classical theories for $d>2$. On the other hand, for $d\leq 2$, $d-m$ itself is no longer positive. Thus, we restrict ourselves to studying $m=1$ below.

To obtain nonclassical exponents of our theory, we investigate divergences in $d=3-\epsilon$. As in model B,[14] the integrals associated with λ and σ are convergent. Apart from these, ρ clearly needs no renormalization, since the nonlinear term has no ∂'s. On the other hand, renormalization of ϕ is nontrivial, giving an anomalous dimension $\eta/2$. Similarly τ also requires renormalization, leading to ν, which may be translated into the exponent for an appropriately defined correlation length. Note that our correlation function will also have r^{-d} decays[7] for all $T>T_c$. Thus some care in data fitting must be exercised before ν can be extracted. Besides these dominant singularities, the renormalization of g automatically gives ω, a correction-to-scaling exponent associated with

$g-g^*$. To lowest nonvanishing order in ϵ, we find

$$\eta=4\epsilon^2/243, \quad \nu=\tfrac{1}{2}(1+\epsilon/6), \quad \omega=\epsilon. \tag{2}$$

To obtain other exponents, we cannot simply use the static scaling laws. Instead, scaling analysis[3] of the equation of state gives $\beta=\frac{1}{2}\nu(d-1+3\eta/2)$ exactly. We caution that, unlike ordinary critical systems, the exponent η does not enter naively into the correlation function. Since the latter is very anisotropic in momentum and real spaces (MS and RS), we may define four different η-like exponents:[4]

$$\eta_{\parallel}^{\mathrm{MS}}=\frac{4}{4-\eta}, \quad \eta_{\perp}^{\mathrm{MS}}=\eta, \quad \eta_{\parallel}^{\mathrm{RS}}=\frac{2\epsilon+4\eta}{4-\eta}, \quad \eta_{\perp}^{RS}=1+\tfrac{1}{2}\eta. \tag{3}$$

Other exponents can be obtained in a similar manner.

Finally, we study the critical behavior of a system with repulsive interparticle interactions. Our analysis leads to the same conclusion as for a system driven by a uniform E,[9] i.e., the leading singularities are Ising-like. For completeness, we mention here the main points. There are two "slow" variables: the ordering field ("staggered magnetization") $\phi(x,t)$ and the "magnetization" $m(x,t)$, which is the coarse-grained $n_i-\frac{1}{2}$. Though nonordering and being zero on the average, the latter is essential since E has an effect on the phase transition only via m. The setup and analysis for the new, random-field model again follows closely the constant-E case,[9] the major difference here being the absence of the $\partial E m^2$ term in the Langevin equations. Then, by using either the arguments of Grinstein, Jayaprakash, and He[15] or straightforward naive dimensional analysis, we arrive at the same conclusion, i.e., that our system belongs to the universality class of the ordinary Ising model. Of course, major differences exist, but they will appear only at the corrections-to-scaling level. Before concluding, we note that, like in the uniform-E case, we also expect first-order transitions for strong fields, when E simply overwhelms the ordering mechanism, even at $T=0$.

We conclude with some remarks on observability and related open questions. The easiest way to observe the new critical behavior of $J>0$ systems is through Monte Carlo simulations, as in the uniform-E case.[1,6,7] We are encouraged by preliminary data for $d=2$ systems[16] indicating $\beta\sim0.2$, which is consistent with our prediction of 0.29 (by naively setting $\epsilon=1$). Clearly more work is needed, on both fronts. In addition to simulations, we propose the following experiment for testing these predictions. Adsorb a thin film of a fast ionic conductor on a cylindrical substrate. Applying a random (in time) magnetic field through the cylinder will induce a random electric field on the sample. Further, we argue that, with (annealed) impurities distributed randomly in space, such a system is well modeled by our equations. Note that the uniform-drive case cannot be realized so simply, since a magnetic field having a *linear* time dependence is necessary. An experimentalist may naturally ask if our model can be extended to include ac fields. However, such fields produce periodic, if not chaotic, statistical distributions, which are completely outside the scope of present analytical techniques.

Although we presented our model in terms of a randomly driven diffusive system, we believe that it is also appropriate for a lattice gas under two temperatures,[17] in which particle hops in the m-dimensional subspace are coupled to a higher-temperature bath than in the complementary subspace. Clearly, such a system is not expected to satisfy the fluctuation-dissipation theorem. By setting the parallel temperature to be higher, we expect τ to vanish before ρ. Studies are in progress to elucidate the differences between these models by, for example, identifying appropriate irrelevant operators.

Another issue deserving further study is the stability of our fixed point against the constant-E interaction. Although it is easy to see that $\partial E\phi^2$ is naively more relevant than $g\nabla^2\phi^3$, we must be cautious, bearing in mind the many surprises associated with driven systems; e.g., E itself needs no renormalization. To end, we point to an interesting generalization of these models, involving a uniform E set off axis, e.g., $E_x=E_y>0$, $E_z=0$. Microscopically, a particle may hop along either the x or the y axis, mimicking a random drive in the $\hat{\mathbf{x}}-\hat{\mathbf{y}}$ direction. On the other hand, we have essentially a uniform drive in the $\hat{\mathbf{x}}+\hat{\mathbf{y}}$ direction, while jumps along the $\hat{\mathbf{z}}$ direction are dictated only by internal energetics. Thus, we have a combination of both types of drive, giving rise to a threefold anisotropy. Clearly, new theoretical vistas abound. It would be most gratifying if corresponding physical materials and experimental realizations were found.

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[1]S. Katz, J. L. Lebowitz, and H. Spohn, Phys. Rev. B **28**, 1655 (1983); J. Stat. Phys. **34**, 497 (1984).

[2]H. van Beijeren and L. S. Schulman, Phys. Rev. Lett. **53**, 806 (1984); J. Krug, J. L. Lebowitz, H. Spohn, and M. Q. Zhang, J. Stat. Phys. **44**, 535 (1986); R. Dickman, Phys. Rev. B **38**, 2588 (1988).

[3]H. K. Janssen and B. Schmittmann, Z. Phys. B **64**, 503 (1986).

[4]K.-t. Leung and J. L. Cardy, J. Stat. Phys. **44**, 567 (1986).

[5]K. Gawedzki and A. Kupiainen, Nucl. Phys. **B269**, 45

(1986).

[6]J. Marro, J. L. Lebowitz, H. Spohn, and M. H. Kalos, J. Stat. Phys. **38**, 725 (1985); J. L. Vallés and J. Marro, J. Stat. Phys. **43**, 441 (1986); **49**, 89 (1987); J. Marro and J. L. Vallés, J. Stat. Phys. **49**, 121 (1987).

[7]M. Q. Zhang, J.-S. Wang, J. L. Lebowitz, and J. L. Vallés, J. Stat. Phys. **52**, 1461 (1988).

[8]K.-t. Leung, J. Stat. Phys. **50**, 405 (1988); A. Hernandez-Machado and D. Jasnow, Phys. Rev. A **37**, 656 (1988); K.-t. Leung, K. K. Mon, J. L. Vallés, and R. K. P. Zia, Phys. Rev. Lett. **61**, 1744 (1988); J. L. Vallés, K.-t. Leung, and R. K. P. Zia, J. Stat. Phys. **56**, 43 (1989).

[9]K.-t. Leung, B. Schmittmann, and R. K. P. Zia, Phys. Rev. Lett. **62**, 1772 (1989).

[10]R. Dickman, Phys. Rev. A **41**, 2192 (1990).

[11]H. K. Janssen, Z. Phys. B **23**, 377 (1976); R. Bausch, H. K. Janssen, and H. Wagner, Z. Phys. B **24**, 113 (1976); C. DeDominicis, J. Phys. (Paris), Colloq. **37**, C1-247 (1976).

[12]K. G. Wilson and M. E. Fisher, Phys. Rev. Lett. **28**, 240 (1972). For a review, see K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975).

[13]K. Kawasaki, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and M. S. Green (Academic, New York, 1972), Vol. 2.

[14]B. I. Halperin, P. C. Hohenberg, and S.-k. Ma, Phys. Rev. B **10**, 139 (1974). For a review, see P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[15]G. Grinstein, C. Jayaprakash, and Y. He, Phys. Rev. Lett. **55**, 2527 (1985).

[16]K. Hwang (to be published).

[17]This is distinct from a variety of other models with competing dynamics; e.g., P. L. Garrido, A. Labarta, and J. Marro, J. Stat. Phys. **49**, 551 (1987); J. M. Gonzalez-Miranda, P. L. Garrido, J. Marro, and J. L. Lebowitz, Phys. Rev. Lett. **59**, 1934 (1987); P. L. Garrido, J. M. Gonzalez-Miranda, and J. Marro, Phys. Rev. A **40**, 5802 (1989); H. W. J. Blöte, J. R. Heringa, A. Hoogland, and R. K. P. Zia, J. Phys. A (to be published).