Stability of a Free-Electron-Laser Spectrum in the Continuous-Beam Limit

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A multifrequency model is developed for a Compton free-electron laser and is solved to investigate the continuous-electron-beam limit. Both numerical simulations and perturbation expansions allow us to exhibit a mechanism leading to the broadening of the spectrum. Then, the nonlinear asymptotic behavior of the spectrum is analyzed.

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One of the most challenging questions in free-electron-laser (FEL) physics is related to the nonlinear evolution of the spectrum, including the sideband instability[1-3] and mode competition (MC).[4]

New nonlinear effects can alter the further evolution of a FEL spectrum, especially in the asymptotic regime which corresponds to a large number of round trips for an oscillator. Of course, optical elements can select a single frequency,[5] but we are concerned here with the natural physical evolution of the spectrum in the absence of frequency discrimination. Moreover, we focus on the limit of a continuous electron beam, which means that finite-pulse issues, such as the natural Fourier spread or temporal overlap effects, are disregarded.

In the continuous-beam limit (CBL), our numerical simulations exhibit the following essential property: Any spectral pattern, for instance, two frequencies with the same amplitude, can duplicate itself. We confirm this numerical prediction by expanding the multifrequency model with two different perturbation theories. Then, the nonlinear evolution of the spectrum is investigated by means of computer simulations. We observe some expected results, such as the transition between narrow and broad spectra depending upon the electronic current or the cavity losses. A more challenging numerical prediction in the saturation regime is a scaling law between the spectral width and the extracted efficiency.

To investigate the FEL spectral dynamics, we use a specific model for the CBL which is appropriate for electronic pulses much longer than the slippage distance.[6] To work out this model, we expand the laser field A_L as the product of rapid phases and slowly varying envelopes $\mathcal{E}_n(z)$:

$$A_L = \frac{mc}{e} \mathrm{Re}\left[\sum_{n \ll N} \frac{\mathcal{E}_n(z)}{k_n} e^{ik_n(z-ct)} \right], \tag{1}$$

where $k_n = (1+n/N)k_L$ is a wave number close to the central mode $k_L = 2\pi/\lambda_L$. Each complex laser field $\mathcal{E}_n$ satisfies a paraxial equation:

$$i\partial_z \mathcal{E}_n = \mu_0 \frac{e^2 c}{2\pi} a_w e^{i(n/N)k_w z} \int_1^{+\infty} d\gamma \frac{1}{N} \int_0^{2\pi N} d\psi\, g(z,\psi,\gamma) \frac{e^{-i(1+n/N)\psi}}{2\gamma}, \tag{2}$$

where a_w and k_w are the normalized amplitude and the wave number of the magnetic field. The longitudinal electronic distribution $g(z,\psi,\gamma)$ satisfies the associated Vlasov equations:

$$\begin{gathered}
[\partial_z + v\partial_\psi + \Gamma(\partial_\gamma - 1/\gamma)] g(z,\psi,\gamma) = 0\,, \\
v = k_w - (k_L/2\gamma^2)(1+\tfrac{1}{2}a_w^2)\,, \\
\Gamma = \tfrac{1}{2} a_w \mathrm{Im}\left[\sum_{m \ll N} \mathcal{E}_m e^{-i(m/N)k_w z} \frac{e^{i(1+m/N)\psi}}{2\gamma} \right], \\
\frac{1}{2\pi N} \int_0^{2\pi N} d\psi \int_1^{+\infty} d\gamma\, g(z,\psi,\gamma) mc = \rho_e\,,
\end{gathered} \tag{3}$$

where ρ_e is the electron number per unit of volume.

The electron phase space is characterized[1] by the resonant phase $\psi = (k_L + k_w)z - \omega_L t$ and the kinetic energy γ normalized to the electronic mass m. The variable ψ depends on the central frequency $\omega_L = ck_L$, but Eqs. (1)-(3) are invariant under the choice of ω_L when we consider the limit of the continuous Fourier expansion in Eq. (1). Indeed, the phase $(1+n/N)\psi - (n/N)k_w z$, in Eqs. (2) and (3), is precisely the resonant phase $\psi_n = (k_n + k_w)z - \omega_n t$ for the frequency $\omega_n = ck_n$. The term $(1/\gamma)g$ in Eq. (3) is required for the conservation of the phase-space volume and for the equivalence with the usual equations for the one-electron dynamics.

The models usually used for multimode simulations[5,6] work in position space,[7] possibly with periodic boundary conditions.[8] In the limit of the continuous Fourier expansion, Eqs. (1)-(3) are physically equivalent to these models. This technical choice of the Fourier representation is well suited to study mechanisms involving frequency generation since it allows a fine control of the initial spectrum and a precise tracking of each frequency.

For computational reasons, the laser field is expanded as a finite sequence of discrete Fourier modes rather than a continuous Fourier expansion. Therefore, the whole model, Eqs. (1)-(3), in periodic with a period $N\lambda_L$, which implies that the minimal distance between two laser frequencies is $\delta\omega = \omega_L/N$. Typically, by using $N \geq 500$, we can resolve spectrum details with relative fractional widths $\geq 10^{-3}$. To investigate new frequency generation mechanisms, it is necessary to perform relevant simulations for low-amplitude frequencies. So, we have taken advantage of the Fourier representation to achieve very-low-noise simulations, which precisely satisfy the energy-conservation law.

In particular, one can consider a one-pass simulation corresponding to a particular round-trip where the fundamental mode and the sideband (centered, respectively, on ω_f and ω_s) are both present. We assume that the initial laser energy E_l is shared equally between ω_f and ω_s. The results of the numerical simulation for the energy of each frequency versus the z position are plotted in Fig. 1. The low noise level of this CBL simulation allows us to prove that all of the modes $\mathcal{E}_n$, with a frequency given by $\omega_n = \omega_f + n(\omega_f - \omega_s)$, where n is an integer, are amplified. This mechanism is a difference frequency generation (DFG) around ω_f. If we extend this one-pass simulation to a few passes, the new frequencies $\mathcal{E}_n$ are amplified and can even dominate the initial modes ω_f and ω_s. So, we propose that the spectrum can be spoiled in two steps: First, by the well-known sideband instability which causes the emergence of a second mode ω_s near the fundamental ω_f and, second, by the generation of a sequence of harmonics of the difference $\omega_f - \omega_s$.

The origin of this frequency generation can be clarified with perturbation expansions. We consider an initial condition given by a nonbunched electron beam and a two-frequency laser field. In this case, the driving electronic force Γ depends on the phases ψ_f and ψ_s, respectively, resonant with the two frequencies ω_f and ω_s. Then, any perturbation theory will associate the perturbation order $2m+1$ with Γ^{2m+1} the expansion of which includes the phases $\psi_m = \psi_s \pm m(\psi_s - \psi_f)$ resonant with the frequencies $\omega_m = \omega_s \pm m(\omega_s - \omega_f)$.

To test the DFG mechanism, Eqs. (2) and (3) can be solved with a time-dependent perturbation procedure where the laser field is computed as a Taylor expansion in the variable z. The first nontrivial term is the third-order perturbation of the electronic current. This term confirms that the harmonics $2\omega_f - \omega_s$ and $2\omega_s - \omega_f$ are amplified and shows that the starting-energy growth rate is proportional to $z^8 E_l^3$.

The comparison between the z expansion and a fully numerical simulation exhibits a large discrepancy, which shows that the validity range of the z expansion is very small (a few mm). The scale of this drastic limitation is the wiggler period λ_w. Indeed, this scale appears in high perturbation orders through the variation of the longitudinal speed of the electrons with their energy: $\partial_\gamma v$. It should be emphasized that the function vz is a slowly varying function compared to λ_w, but this is not the case for its derivative: $\gamma\partial_\gamma vz \simeq k_w z$.

Assuming a weak-gain regime, one can expand the solution in powers of the laser energy E_l, up to the third order. This expansion, compared with numerical simulations, is very accurate for realistic physical parameters, showing that the DFG mechanism is not a numerical artifact. Moreover, the E_l expansion shows that the efficiency of the DFG mechanism depends on ω_f and ω_s (Fig. 2). This expected property is not given by the third order of the z expansion and would require a high-order-term summation in z. Then, E_l is the most pertinent ex-

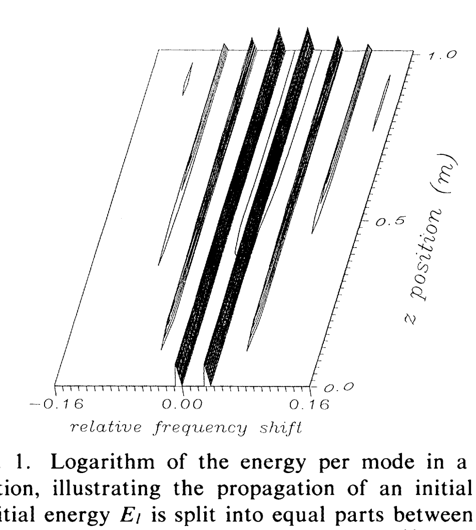

FIG. 1. Logarithm of the energy per mode in a one-pass simulation, illustrating the propagation of an initial pattern. The initial energy E_l is split into equal parts between two frequencies. This low-noise simulation ($\simeq 10^{-16}E_l$) illustrates the DFG from noise up to a level of $10^{-6}E_l$ for the first generation and of $10^{-11}E_l$ for the second generation.

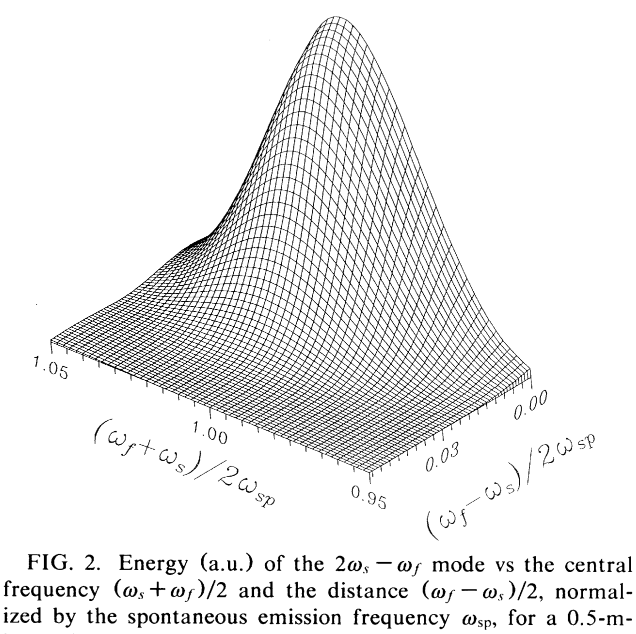

FIG. 2. Energy (a.u.) of the $2\omega_s - \omega_f$ mode vs the central frequency $(\omega_s + \omega_f)/2$ and the distance $(\omega_f - \omega_s)/2$, normalized by the spontaneous emission frequency ω_{sp}, for a 0.5-m-long wiggler. This plot is deduced from a direct computation of the third-order terms in the E_l expansion. It shows that the effective interaction between frequencies is a rapidly-falling-off function.

pansion variable for a perturbation theory which investigates the first nonlinear orders of the multifrequency mechanisms. This third-order computation provides an effective interaction between laser frequencies and it gives the appearance of a Φ^4 field theory to the nonlinear Schrödinger equation governing the laser dynamics. Although the E_l expansion is more suitable than the z expansion, it is also much more involved and detailed calculations and results will be presented in a forthcoming paper. The E_l expansion indicates that the DFG scheme is not a generation of the sideband since it is not related with synchrotron movements.

These perturbative expansions establish the existence of a mechanism able to generate new frequencies. Our full calculation of the third-order terms in the E_l expansion is a generalization of the analysis developed in Ref. 4 since (i) we do not suppose that ω_s and ω_f are close together and (ii) we compute the evolution of frequencies possibly different from ω_s and ω_f. So, it must be noted that, if the third perturbation order leads to saturation and to MC between ω_s and ω_f, it is also responsible for the DFG and for the generation of new frequencies. The understanding of basic mechanisms, such as the DFG, allows us to control the FEL dynamics. For example, by lowering Q, the laser energy at saturation decreases, which inhibits the sideband generation.[9] Then, this restrains the spectrum broadening since the DFG depends on the energy level of the fundamental mode and its sideband. The DFG processes are a complementary interpretation to the usual assumption of additional second-order sidebands to justify the expected transition[9] from chaotic to nonchaotic spectra when cavity losses increase.

The above basic nonlinear mechanism has been established from perturbative expansions for the particular case of a Compton FEL. A different approach exhibits a similar instability for a two-frequency system in the Raman regime.[10] More generally, these nonlinear couplings belong to a large class of mechanisms where an electron beam is self-consistently coupled with strong electromagnetic fields. It is the case for the interaction of Langmuir waves with electrons in a one-dimensional plasma or for the intermodulation mechanism in traveling-wave-tube amplifiers.

Computer simulations allow us to gain some insight into the further nonlinear evolution of these new frequencies for long interaction times. We use the average parameters of the experiment,[11] where an efficiency of 4% is obtained for a 10-μm laser wavelength, with electron pulses characterized by an energy $E_{\mathrm{elec}}=0.06$ J and a 10-ps pulse duration. The average current (cavity loss factor) is roughly $J_e=200$ A ($Q=20$). The simulations presented in Figs. 3 and 4 assume different values of J_e and Q in the CBL approach. Figure 3 shows different asymptotic evolutions which converge toward an average equilibrium. A low current [Fig. 3(a)] leads to a narrow spectrum. For a high current [Fig. 3(b)], the spectrum is more complicated since the energy of a single frequency follows large oscillations. This is essentially due to the stochasticlike behavior of the electronic trajectories in the ponderomotive wells created by the different frequencies. Nevertheless, the main features of the spectral dynamics, such as the total laser energy E_l or the relative spectral width $\Sigma=(\langle k^2\rangle-\langle k\rangle^2)^{1/2}/k_L$, are stable. Simulations show that at saturation the spectrum exhibits a width Σ which increases with the electronic current J_e and the cavity quality factor Q.

As no long-pulse high-power FEL experimental data are presently available, we test the relevance of the CBL

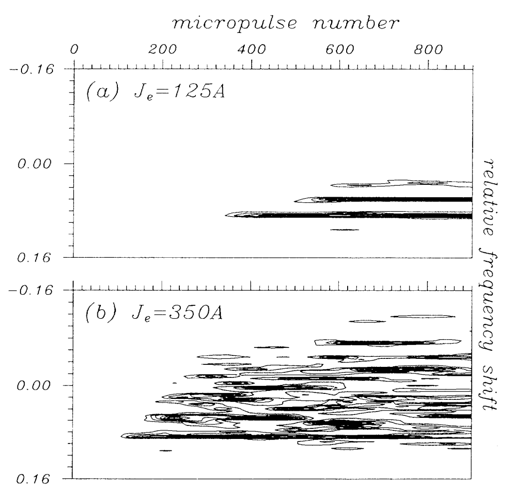

FIG. 3. Plot of the energy per mode, to illustrate the transition between narrow and broad spectra depending on the electronic current J_e for $Q=25$. In (a) [(b)] the efficiency is 1.8% [4.4%] for a current of $J_e=125$ A [$J_e=350$ A].

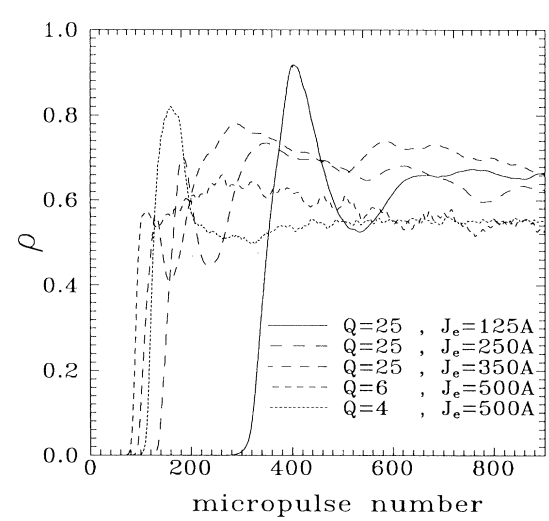

FIG. 4. The ratio ρ of the extracted efficiency to the relative width for different values of the electronic current J_e and the cavity losses Q.

model results with the short-pulse experiment.[11] Accurate comparisons would require some phenomenological lowering of the Q factor to take into account the variation of the longitudinal overlap $\mathcal{O}$ between the laser and electron beams (the CBL model assumes $\mathcal{O}=1$). Indeed, in a finite-pulse FEL experiment, the spectral width can be controlled by the slippage effects and the optical cavity detuning[11] since the interaction time depends on the longitudinal overlap $\mathcal{O}$ between the two beams. In the CBL, an equivalent control is performed by means of the optical-cavity quality factor Q because the average number of round-trips for a photon is proportional to Q.

Nevertheless, the pertinence of the CBL approach to investigate basic behaviors of the asymptotic spectral equilibrium in high-power FEL experiments can be established by the following result. Figure 4 shows that the saturation is characterized by a constant ratio $\rho=E_l/\Sigma Q E_{\text{elec}}$ (i.e., the ratio of the extracted efficiency to the relative width) which is almost independent of Q and J_e, and hence E_{elec}. Such a behavior does not occur in monofrequency simulations. For different values of J_e and Q, we get a predicted constant ratio $\rho \simeq 0.7$, which is in agreement with the experiment.[11] Indeed, when $\mathcal{O}$ is maximized, the measured width is large (about 5%) for an efficiency of 4%, leading to a ratio of $\rho \simeq 0.8$.

In summary, we have investigated the CBL of a Compton FEL. Numerical simulations and perturbation expansions have proved the startup of an instability of the laser spectrum from a two-frequency initial condition. Then, because the sideband effect generates such a two-frequency spectrum, a realistic FEL simulation can present a very broad spectrum depending upon the electronic current, the cavity losses, and so on. An analysis of the asymptotic behavior of the spectrum is proposed from computer simulations with a large number of round-trips. These full simulations illustrate that the DFG can overcome the MC and lead ultimately to a spectrum broadening. Some important numerical predictions, such as the fact that the laser energy is distributed on a stable plateau, lead to challenging questions for long-pulse high-power future experiments.

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