Quantum Collective Creep

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We study the phenomenon of quantum creep in bulk superconductors within the framework of weak-collective-pinning theory. Single-vortex pinning at moderate magnetic fields as well as creep of vortex bundles at high fields are discussed for the case of current densities near critical. In the limit of strong dissipation and for moderate magnetic fields the basic parameter determining the tunneling rate is the ratio ρ_n/ξ (ρ_n =normal-state resistivity, ξ =coherence length), with strong tunneling realized for $\rho_n/\xi \gtrsim 1$ kΩ.

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The classical model of flux creep in type-II superconductors predicts a linearly vanishing magnetic relaxation rate at low temperatures. However, in a number of recent experiments on oxide,[1,2] heavy-fermion,[3] chevrel,[4] and organic[5] superconductors, the low-temperature magnetic relaxation rate has been found not to extrapolate to zero, suggesting the existence of vortex motion by quantum tunneling. In this Letter we describe the process of quantum tunneling within the framework of collective-pinning theory[6,7] and determine the tunneling rates of single vortices and of vortex bundles out of metastable states.

Macroscopic quantum tunneling (MQT) has been studied by Caldeira and Leggett[8] and by Eckern, Schön, and Ambegaokar[9] for the case of an (rf) superconducting quantum interference device (SQUID). Macroscopic systems are inherently dissipative and the application of quantum mechanics to a macroscopic variable may be questioned. Caldeira and Leggett have shown that the quantum description can be extended to macroscopic systems and that dissipation always reduces the tunneling rate. The possibility of quantum tunneling of vortices in thin superconducting films has been proposed by Glazman and Fogel.[10] Recently, Fisher, Tokuyasu, and Young[11] have studied quantum vortex creep in disordered thin-film superconductors and have found a variable-range-hopping resistivity with a non-Arrhenius low-temperature behavior characterized by an exponent $\sim \frac{2}{3}$. Here we are studying vortex tunneling in *bulk* superconductors for the case of a weak-pinning potential, which is believed to be the main source of pinning in the oxide superconductors.[12]

Within collective-pinning theory and for low magnetic fields a single vortex is pinned by the collective action of defects within a characteristic pinning length L_c producing a potential barrier U_c against motion to the adjacent metastable state. In an elementary tunnel process a vortex segment of length $\sim L_c$ tunnels under the barrier U_c to its neighboring state. At higher fields the tunneling object is a vortex bundle. The tunneling rate γ is determined by the (effective Euclidean) action S_E^{eff} of the process, $\gamma \propto \exp(-S_E^{\text{eff}}/\hbar)$, and for the weak-field, strong-dissipation limit we find that tunneling is favored by a small coherence length ξ and by a large normal-state resistivity ρ_n, $S_E^{\text{eff}}/\hbar \propto (\hbar/e^2)\xi/\rho_n$. A summary of the results is given in Table I.

In the following we will construct the Lagrangian for a single vortex and present an estimate for the vortex mass. We describe how to determine the tunneling rate in terms of the saddle-point solution of the Euclidean action S_E. The results are then generalized to include dissipation, and finally the case of large fields, where vortex bundles are tunneling, is discussed. For the sake of simplicity, we consider here the isotropic case and defer the discussion of the effects due to anisotropy to a forthcoming paper.

The Lagrangian generating the classical dynamic equations of the vortex is

$$\mathcal{L}[\mathbf{u}] = \int dz \frac{M}{2}(\partial_t \mathbf{u})^2 - \mathcal{F}[\mathbf{u}] , \qquad (1)$$

with the free energy functional

$$\mathcal{F}[\mathbf{u}] = \int dz \left[\frac{\varepsilon_l}{2}(\partial_z \mathbf{u})^2 + U_{\text{pin}}(z,\mathbf{u}) \right] . \qquad (2)$$

Here the two-dimensional field $\mathbf{u}(z,t)$ is the displacement of the vortex from its equilibrium position and plays the role of the macroscopic variable describing the system. Furthermore, M is the vortex mass per unit length, ε_l is the line tension of the vortex, and U_{pin} denotes the pinning potential.

An expression for the vortex mass M can be obtained by calculating the kinetic energy of a moving vortex or by studying the response of the vortex to an external force. Using different kinds of time-dependent Ginzburg-Landau theories corresponding calculations have been done by Suhl[13] and by Kupriyanov and Likharev[14] with similar results. Whereas their approach should be valid for gapless superconductors or for temperatures near the superconducting transition, the applicability of

TABLE I: Geometric dimensions $(R_\perp, R_\parallel, L_c)$ of the collective-pinning volume and effective Euclidean action $(S_E^{\rm eff})$ for the cases of single-vortex collective pinning at low magnetic fields $(L_c < a)$, and for collective pinning of vortex bundles at intermediate $(a < R_\perp < \lambda$, regime of strong dispersion) and large magnetic fields $(\lambda < R_\perp)$. c and c' denote numerical constants and L_c is the single-vortex collective-pinning length.

Regime	$R_\perp$	$R_\parallel \simeq L_c^b$	$S_E^{\rm eff}/\hbar$
$L_c < a$	...	...	$\frac{\hbar}{e^2}\frac{\xi}{\rho_n}\left(\frac{j_0}{j_c}\right)^{1/2}$
$a < R_\perp < \lambda$	$a\exp\left[c\left(\frac{L_c}{a}\right)^3\right]$	$R_\perp \frac{R_\perp}{a}$	$\frac{\hbar}{e^2}\frac{a}{\rho_n}\exp\left\{c'\left[\frac{\xi}{a}\left(\frac{j_0}{j_c}\right)^{1/2}\right]^3\right\}$
$\lambda < R_\perp$	$\lambda\left(\frac{L_c}{a}\right)^3$	$R_\perp \frac{\lambda}{a}$	$\frac{\hbar}{e^2}\frac{\lambda}{\rho_n}\left(\frac{\lambda}{a}\right)^4\left(\frac{\xi}{a}\right)^9\left(\frac{j_0}{j_c}\right)^{9/2}$

the results to a superconductor with a finite gap at low temperatures is unclear. We therefore present a simple estimate for the vortex mass based on general arguments which should hold for our situation. Our derivation reproduces the results of Refs. 13 and 14 and thus extends their regime of validity down to low temperatures. The basic idea is that the electronic contribution to the vortex mass is due to the local change in dispersion within the vortex core. The number of electrons exposed to this change is given by $N(0)\pi\xi^2\Delta$, with $N(0)$ the density of states at the Fermi level and Δ denoting the energy gap. These electrons experience a relative change of their effective mass which is of the order of $m\Delta/\varepsilon_F$, where m and ε_F denote the effective mass and the Fermi energy, respectively. Expressing the density of states $N(0)$ by the electron density n and the Fermi energy ε_F, we obtain the electronic contribution to the vortex mass $M_{\rm el} = (3\pi/2)mn\xi^2(\Delta/\varepsilon_F)^2$, reproducing Suhl's result. Further simplification leads to $M_{\rm el} = (2/\pi^3)mk_F$ (k_F = Fermi wave vector) with agrees with the result of Kupriyanov and Likharev. Besides this electronic contribution, a second term $M_{\rm em}$ of electromagnetic origin[13] contributes to the vortex mass. Typically $M_{\rm el} \gg M_{\rm em}$ and in the following we use the estimate $M \simeq M_{\rm el}$. Below we will find that in the limit of strong dissipation the tunneling rate becomes independent of the vortex mass and thus our qualitative estimate will not influence the accuracy of our final results.

Within collective-pinning theory each segment of length $\sim L_c$ is pinned independently, with L_c determined by minimization of the free energy (2). For a pinning potential with a minimal characteristic length $r_p \simeq \xi$ and producing a mean-squared random force W, the result is[6]

$$L_c \simeq \pi[\Phi_0^4\xi^2/(2\pi)^2 a^2 W\lambda^4]^{1/3}, \qquad (3)$$

with $\Phi_0 = hc/2e$ the flux quantum, $a = (\Phi_0/B)^{1/2}$ the mean distance between neighboring vortices, and λ the London penetration depth. Note that a^2W and hence L_c is independent of the magnetic field B. The result (3) can be obtained from a simple dimensional estimate: The solution minimizing the free-energy functional (2) is characterized by the equality between the elastic and the pinning energy, $\varepsilon_l(\xi/L_c)^2 L_c \simeq \xi(Wa^2L_c)^{1/2} \equiv U_c$, where we have used that adjacent metastable states are $\sim\xi$ apart such that u varies by ξ on a length scale L_c. The pinning potential U_c is obtained by adding up the random forces due to the individual pins along the collective pinning length L_c. Using the estimate $\varepsilon_l \simeq (\Phi_0/4\pi\lambda)^2$ reproduces the result (3) up to a numerical factor.

The mean-square random force W is not directly accessible by experiment. In order to express the pinning length L_c through experimentally accessible quantities we can use the condition of a vanishing effective pinning barrier $U_{\rm eff} \simeq U_c - j_c\Phi_0 L_c\xi/c$ at the critical current density j_c and express the collective-pinning length L_c by j_c and the depairing current density j_0, $L_c \simeq \xi(j_0/j_c)^{1/2}$.

Whereas the classical thermal creep rate is determined by the saddle-point solution of the free-energy functional (2), the quantum tunneling rate is given by the saddle-point solution of the corresponding Euclidean action,[8,15]

$$S_E = \int dt \left\{ \int dz \frac{M}{2}(\partial_t \mathbf{u})^2 + \mathcal{F}[\mathbf{u}] \right\}. \qquad (4)$$

In order to find the effective action of the saddle-point solution we can again use the method of dimensional estimates.[7] It turns out that quantum creep in d dimensions can be mapped onto the problem of thermally activated, i.e., classical, creep in $d+1$ dimensions. The time axis of the problem can be viewed as simply adding an additional dimension to our minimization task. The geometric part of the solution (L_c) has already been found above. The estimate for the characteristic tunneling time t_c is obtained by equating the kinetic and elastic energy densities in (4), $\varepsilon_l(\xi/L_c)^2 \simeq M(\xi/t_c)^2$. The result is $t_c \simeq L_c(M/\varepsilon_l)^{1/2}$ and the corresponding action becomes

$$S_E \simeq t_c U_c \simeq \xi^2(M\varepsilon_l)^{1/2}. \qquad (5)$$

In the limit of weak fields and vanishing dissipation the result is independent of the collective-pinning length L_c and hence of the strength of the pinning potential.

We have checked the correctness of the above approach by calculating exactly the Euclidean action for the instanton solution of an elastic string of length L_c trapped in a cubic model potential with a (maximal) barrier height $U_{\rm pin}=U_c/L_c$. The exact result agrees with the above estimate up to a numerical factor of order 1. Contrary to the situation encountered in the problem of MQT of the superconducting phase in a SQUID, the appropriate model potential is not known in the present case. Hence, our estimates should actually provide a result as precise as we can hope to obtain, considering our ignorance about the actual pinning potential.

In a next step we generalize our result to include dissipation into the model. As shown by Caldeira and Leggett[8] the interaction with the environment can be accounted for by adding a term

$$\frac{\eta}{4\pi}\int dt\int dt'\int dz\left[\frac{\mathbf{u}(z,t)-\mathbf{u}(z,t')}{t-t'}\right]^2 \quad (6)$$

to the Euclidean action (4) which results in the so-called effective action $S_E^{\rm eff}$ of the vortex. Here η denotes the viscous drag coefficient, $\eta\simeq\Phi_0 H_{c_2}/c^2\rho_n$, with H_{c_2} the upper critical field. The expression (6) is nonlocal in time and in order to treat this term we transform the effective action to Fourier space,

$$S_E^{\rm eff}=\int\frac{d\omega}{2\pi}\int\frac{dq}{2\pi}\left\{\frac{1}{2}\left[\left[M+\frac{\eta}{|\omega|}\right]\omega^2+\varepsilon_l q^2\right]|\mathbf{u}(q,\omega)|^2+U_{\rm pin}(q,\mathbf{u})\right\}. \quad (7)$$

According to (7) the inclusion of dissipation into the model leads to an increase in the (effective) mass, $M_{\rm eff}=M(1+\eta/M|\omega|)>M$. This is similar to the increase in capacitance of the junction due to dissipation as found by Eckern, Schön, and Ambegaokar in the case of MQT in a SQUID, where the role of the mass is played by the junction capacitance. Note that the inclusion of dissipation leads to a dispersive mass resembling the situation where the interaction between the vortices leads to a dispersion in the elastic moduli.[16]

The saddle-point solution of (7) produces a larger tunneling time t_c which is found by solving the quadratic equation $M_{\rm eff}(\omega_c)\omega_c^2=\varepsilon_l q_c^2$, $q_c=2\pi/L_c$, for the time $t_c=2\pi/\omega_c$. The mass enhancement factor $1+\eta/M|\omega_c|$ becomes equal to $1+2/[(1+\nu)^{1/2}-1]$, with $\nu=4M\varepsilon_l q_c^2/\eta^2$. Usually, the motion of the vortex is strongly overdamped,[17] i.e., $\eta/M|\omega_c|\gg 1$. A rough estimate for ν using parameters appropriate to oxide superconductors leads to $\nu\simeq 0.1$, $\eta/M|\omega_c|\simeq 40$. In order to obtain this result we express ν by the ratio $j_c/j_0\simeq 10^{-2}$ of the critical and depairing current densities, the resistivity $\rho_n\simeq 100\ \mu\Omega\,{\rm cm}$, the coherence length $\xi\simeq 15$ Å, and the Fermi wave vector $k_F\simeq 0.5$ Å^{-1},

$$\nu=(32/3\pi^4)(e^2\rho_n/\hbar\xi)^2(k_F\xi)^4 j_c/j_0\,.$$

In the limit of strong dissipation the tunneling time becomes $t_c\simeq L_c^2\eta/\varepsilon_l$ and the final result for the effective Euclidean action is

$$\frac{S_E^{\rm eff}}{\hbar}\simeq\frac{\hbar}{e^2}\frac{\xi}{\rho_n}\left[\frac{j_0}{j_c}\right]^{1/2}. \quad (8)$$

Our result predicts a large tunneling rate for materials characterized by a small coherence length ξ and a large normal-state resistivity ρ_n. The quantum unit of resistance is $\hbar/e^2=4.1\ {\rm k}\Omega$ and thus quantum creep can be observed in superconductors with a high ratio ρ_n/ξ of the order of 1 kΩ. In particular, the oxide superconductors are good candidates for the observation of quantum creep at low temperatures: Using the above values for ξ, ρ_n, and j_c/j_0, we obtain typical relaxation rates $(1/M_0)dM/d\ln t\simeq\hbar/S_E^{\rm eff}\simeq 1\%$. This compares favorably with the experimental findings of Mota *et al.*[1] and of Fruchter *et al.*[2]

Finally, we generalize our results to the case of large magnetic fields where the tunneling object is a collectively pinned vortex bundle. The crossover from single-vortex pinning at low fields to pinning of vortex bundles takes place when the interaction between the vortices becomes important, which is the case when $L_c\simeq a$. In order to obtain the action for the vortex bundle we have to substitute M and η by their corresponding densities M/a^2 and η/a^2 and the free-energy functional $\mathcal{F}[\mathbf{u}]$ becomes

$$\mathcal{F}=\int d^3r\left[\frac{c_{11}-c_{66}}{2}(\nabla\mathbf{u})^2+\frac{c_{66}}{2}(\nabla_\perp\mathbf{u})^2+\frac{c_{44}}{2}(\partial_z\mathbf{u})^2+U_{\rm pin}(\mathbf{r},\mathbf{u})\right]. \quad (9)$$

In a static configuration the vortices adjust to the pinning potential by shear and tilt deformations alone. Therefore the minimal solution of (9) defining the collective-pinning volume V_c^b is characterized by the equality of the shear $[c_{66}(\xi/R_c)^2]$, tilt $[c_{44}(\xi/L_c^b)^2]$, and pinning energy densities $[\xi(W/V_c^b)^{1/2}]$, where the bundle volume is $V_c^b=R_c^2L_c^b$. We obtain for the bundle radius $R_c\simeq L_c^b(c_{66}/c_{44})^{1/2}$, for the bundle length $L_c^b\simeq\xi^2c_{66}c_{44}/W$, and for the pinning energy $\xi(WV_c^b)^{1/2}=U_c^b$. However, in the regime of (quantum) creep, i.e., for current densities $\lesssim j_c$, the moving bundles hop into neighboring states, leading to a considerable local compression along the direction of the hop within the (static) neighborhood. To minimize this compression a number of bundles will hop together, their number being determined by the equality of the compression and shear energy densities in (9), $c_{11}(\xi/R_\parallel)^2=c_{66}(\xi/R_\perp)^2$, $R_\perp=R_c$. Here $R_\parallel$ is the size of the superbundle in the direction of the hop, $R_\parallel$

$=R_\perp(c_{11}/c_{66})^{1/2}$. The tunneling object thus consists of $(c_{11}/c_{66})^{1/2}$ bundles of volume V_c^b, where each bundle has to tunnel under the barrier U_c^b. Note that the critical current density is determined by the pinning energy of one bundle U_c^b.

For inhomogeneous displacement fields characterized by short wavelengths the nonlocality of the elastic constants c_{11} and c_{44} has to be taken into account. In principle, transforming to Fourier space, the problem of finding the correct bundle dimensions reduces to an algebraic one by using[16] $c_{11} \simeq c_{44} \simeq (B^2/4\pi)(1+k_\perp^2\lambda^2)^{-1}$, with $k_\perp \sim 1/R_\perp$ the largest wave vector in the problem. However, in the regime of strong dissipation, where $a<R_\perp<\lambda$, the length $R_\perp$ drops out of the problem: Simple dimensional estimates cannot provide the correct length scale $R_\perp$ of the bundle. This implies that the power-law relation between the mean displacement $u = \langle|\mathbf{u}(r)-\mathbf{u}(r+R_t)|^2\rangle^{1/2}$ and the transverse distance R_t is replaced by a logarithmic one. The latter appears in the Larkin-Ovchinnikov formula[6] which is valid for $u<\xi$:

$$u \simeq \xi\left(\frac{a}{L_c}\right)^{3/2}\left(\frac{R_t}{\lambda}+\ln\frac{R_t}{a}\right)^{1/2}. \qquad (10)$$

The logarithmic dependence in (10) is due to the dispersion which is large within the intermediate-field regime $a<R_\perp<\lambda$. The condition $u(R_t=R_\perp)\simeq\xi$ determines the transverse bundle size $R_\perp$ and the remaining length scales $R_\parallel$ and L_c^b are found by using $c_{11}\simeq c_{44}$ and $c_{44}/c_{66}\simeq R_\perp^2/a^2$ for $a<R_\perp<\lambda$, whereas $c_{44}/c_{66}\simeq\lambda^2/a^2$ for $\lambda<R_\perp$. The results are summarized in Table I.

We have now solved the geometric part of the problem and can proceed with the dynamic aspect. As mentioned above, the tunneling object is a collection of $R_\parallel/R_\perp$ bundles which tunnel simultaneously. The tunneling time t_c^b is thus given by that of a single bundle of size V_c^b tunneling under the barrier U_c^b. Equating the kinetic and shear energy densities in the effective action we obtain in the limit of strong dissipation the result $t_c^b \simeq R_\perp^2\eta/a^2c_{66}$. The action of the tunneling superbundle is given by the action of a single bundle multiplied by the number of simultaneously hopping bundles, $S_E^{\text{eff}}=t_c^bU_c^bR_\parallel/R_\perp$. The final expressions for the three cases of weak, intermediate, and large fields are summarized in Table I. With increasing magnetic field the tunneling rate is suppressed: The vortex lattice becomes more rigid due to the interaction between the flux lines. This leads to a larger pinning volume and as a consequence the effective action increases. Here we have not considered the geometric problem of the field penetration into the sample arising in a magnetic relaxation experiment. In a comparison with experiment care must be taken to account correctly for the two opposing effects of increasing the penetration depth of the Bean state and decreasing the tunneling rate when the magnetic field is increased.

In summary, we have presented the theory of quantum collective creep for current densities $\lesssim j_c$ and for arbitrary values of the magnetic field. For strong dissipation and for moderate magnetic fields the main parameter determining the tunneling rate is the ratio ρ_n/ξ. The strength of the pinning potential is less important as it enters the action only through the factor $(j_0/j_c)^{1/2}$. Increasing the field, the tunneling rate is suppressed as the interaction between the vortices becomes essential.

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[17]For dirty superconductors the vortex mass can be interpreted as originating from a dispersion in the viscosity, $\eta(\omega) \simeq \eta(0)[1-i\hbar\omega/\Delta]$ (Ref. 10). Because of the weakness of the pinning potential the typical frequency ω involved in tunneling is less than $\Delta/\hbar$ and thus the viscous term is dominating.