Constructing the Three-Dimensional Gross-Neveu Model with a Large Number of Flavor Components

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(Received 4 February 1991)

We construct the ultraviolet limit of the massive Gross-Neveu model in three dimensions and with a large number of flavor components N.

PACS numbers: 11.10.Gh, 03.70.+k, 05.30.Fk

Traditional treatments of quantum field theory have laid stress on the concept of perturbative renormalizability. However, recent progress[1] on renormalization theory helped to change our view of holding this criterion as fundamental for a local model to exist. On the one hand, a theory may present a good renormalized perturbation series and fail to exist under physically acceptable conditions. For instance, this is believed to happen with the φ_4^4 model.[2] On the other hand, a theory may be perturbatively nonrenormalizable and make sense, as for the planar φ_4^4 model with modified propagator $p^{-2+\varepsilon/2}$ ($\varepsilon > 0$) and negative coupling constant,[3] and the two-dimensional Gross-Neveu model with propagator $\not{p}/p^{2-\varepsilon}$.[1] Unfortunately, these two theories possibly violate Osterwalder and Schrader (OS) axioms.

Using these new ideas, as the first "genuine" perturbatively nonrenormalizable model, in this Letter we outline the main steps for controlling the UV limit of the massive Gross-Neveu[4] model in Euclidean dimension $d=3$ and with large N. A detailed version of this work is in preparation.[5] Until the present time, we cannot say anything about the OS axioms. As for Refs. 3 and 4, reflection positivity should be the most delicate point to prove, but we have no reason to believe it should fail.

The model is formally defined by the Lagrangian

$$L=\bar{\psi}(x)(i\zeta\not{\partial}+m)\psi(x)+\lambda[:\bar{\psi}(x)\psi(x):]^2/2N\,, \tag{1}$$

where $\lambda, m, \zeta \in \mathbb{R}_+$ are respectively the coupling constant, the fermion mass, and field strength, and the γ matrices are either 2×2 or 4×4.

For $d>2$, perturbative nonrenormalizability is seen in the negative mass dimension of λ. It turns out, however, that if we sum all chains with the bubble graph

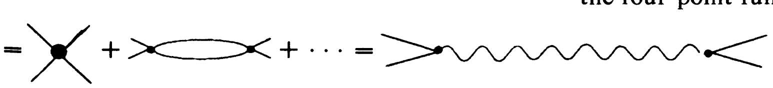
$p \rightarrow$  $= -\lambda^2\pi(p)/N$

in the bare series, before any renormalization, we obtain a just-renormalizable series in the parameter $1/N$.[6] This strong property reflects on the structure of the renormalization group (RG) for the model, allowing us to verify the existence of a nontrivial and stable UV fixed point.

To analyze the UV limit, instead of renormalizability in λ, the proper question is whether or not one can find functions $m_\rho = m(\rho,\lambda_{\rm ren},m_{\rm ren},\zeta_{\rm ren},N)$, etc., of an UV cutoff ρ, N, and positive and finite (renormalized) parameters, such that the $2p$-point Schwinger functions $S_{2p}(\lambda_\rho,m_\rho,\zeta_\rho,\rho,N)$ exist when $\rho\rightarrow\infty$, describing a nontrivial theory. Moreover, in all non-super-renormalizable examples we know, we satisfy these conditions only when we can relate the theory to a stable fixed point, and the construction of the Schwinger functions using methods like cluster expansions may be envisaged for the weakly coupled case. As we show, this is exactly the case for this model. The fixed point is given by $1/N$ times $\beta^*(N)>0$ satisfying $\beta^*(N)\rightarrow\beta^*$ as $N\rightarrow\infty$. It fixes a small (perturbation) parameter if N is large. With these ideas, for Λ being a compact box in $\mathbb{R}^3$, we prove the following theorem.

Theorem.—One can exhibit a family of parameters m_ρ, ζ_ρ, and λ_ρ such that, for N sufficiently large, the normalized Schwinger functions $S_{2p}(\lambda_\rho,m_\rho,\zeta_\rho,\rho,\Lambda,N)$ exist when the volume $|\Lambda|$ and $\rho\rightarrow\infty$.

The renormalization structure of the model.—Let us drop here the volume dependence and consider first the leading order in $1/N$. To this approximation, the only connected functions are the free-fermion propagator and the chains of bubbles. UV divergences appear only in the four-point functions. Let $C_\rho(p)$ be defined by

$C_\rho(p) =$  $= \dfrac{\lambda_\rho/N}{1+\lambda_\rho\pi_\rho(p)}\,,$ (2)

where, for $S_\rho(p)=(\zeta\not{p}+m)^{-1}\eta_\rho(p)$ being the fermion propagator with UV cutoff $\eta_\rho(p)=\theta(1-|p|/M^\rho)$ and $M\in\mathbb{N}_+-\{1\}$, we have

$$\pi_\rho(p)=\int\frac{d^3k}{(2\pi)^3}\,{\rm tr}_{\rm spin}S_\rho(k+p)S_\rho(k)=O(1)\int\frac{d^3k(-\zeta_\rho^2k^2-\zeta_\rho^2k\cdot p+m_\rho^2)\eta_\rho(k)\eta_\rho(k+p)}{[\zeta_\rho^2(k+p)^2+m_\rho^2](\zeta_\rho^2k^2+m_\rho^2)}\,. \tag{3}$$

Let us consider the ansatz $\lambda_\rho^{-1}=\lambda_{\rm ren}^{-1}-\pi_\rho(0)$, $m_\rho=m_{\rm ren}$, and $\zeta_\rho=\zeta_{\rm ren}$. The two last conditions recall the finiteness of S_ρ at this level. To analyze the four-point functions, we first remark that $\pi_\rho(p=0)=-b(\tilde{\mu}_\rho^2)M^\rho\zeta_\rho^{-2}$, with $\tilde{\mu}_\rho=\mu_\rho M^{-\rho}=m_\rho\zeta_\rho^{-1}M^{-\rho}$, and $b(\mu)>0$, if ρ is taken large. With this, since $\pi_\rho^{\rm ren}(p)=\pi_\rho(p)-\pi_\rho(0)\rightarrow\pi_\infty^{\rm ren}(p)$ when

$\rho\to\infty$,

$$C_\rho(p)\to C_\infty(p)=\{[\lambda_0^{-1}+\pi_\infty^{\rm ren}(p)]N\}^{-1}.$$

That $\pi_\infty^{\rm ren}(p)$ is positive may be checked from $|\pi_\rho(p)|\le -\pi_\rho(0)$, assuring that $C_\infty(p)$ is well defined. Also,

$$\pi_\infty^{\rm ren}(p)=(\beta^*)^{-1}|p|+{\rm const}+O(1/|p|),\quad |p|\gg 1,$$

shows the existence of a nontrivial UV fixed point $\beta^*/N=\lim_{p\to\infty}pC_\infty(p)$. We can also show that the bubble summation is convergent since $|\lambda_\rho\pi_\rho(p)|\le|\lambda_\rho\pi_\rho(0)|<1$. In other words, both the model as a whole and its perturbation series in λ_ρ exist *at this order* in $1/N$.

The above ansatz arises as a solution of the RG equations. To see this, we take a formalism where the momentum space is chopped. We decompose the cutoff η_ρ into partitions $\eta^i(p)=\eta_i(p)-\eta_{i-1}(p)$, with $\eta_{-1}(p)\equiv 0$, and slice the fermion propagator $S_\rho(p)=\sum_{i=1}^\rho S^i(p)$. Note that $S^i(p)=S_\rho(p)\eta^i(p)$ has support on $M^{i-1}<p\le M^i$. Next, we decompose $\pi_\rho(p)$ as a sum of $\pi^i(p)=\pi_i(p)-\pi_{i-1}(p)$ ($=0$ if $|p|>M^i$).

In this formalism, the idea of summing bubbles is to start from λ_ρ and, after summing $\pi^\rho(p)$, define $\lambda_{\rho-1}=\lambda_\rho+\delta\lambda_\rho$, etc., until we get $\lambda_0=\lambda_{\rm ren}$. That is,

$$C_\rho(p)=\frac{(\lambda_\rho/N)\{1+\lambda_\rho[\pi^\rho(0)+\cdots+\pi^{i+1}(0)]\}^{-1}}{1+\lambda_\rho[\pi^\rho_{\rm ren}(p)+\cdots+\pi^{i+1}_{\rm ren}(p)+\pi_i(p)]\{1+\lambda_\rho[\pi^\rho(0)+\cdots+\pi^{i+1}(0)]\}^{-1}}.\tag{4}$$

Renormalization and resummation of bubbles fix the evolution of the renormalization flow for λ_ρ, the only nontrivial one at this approximation. From (4), we see that the effective coupling at slice i satisfies

$$\lambda_i^{-1}=\lambda_\rho^{-1}+[\pi^\rho(0)+\cdots+\pi^{i+1}(0)].\tag{5}$$

As before, for large i, one can check that

$$\pi^{i+1}(0)=-b(\tilde\mu_{i+1})M^{i+1}\zeta_{\rm ren}^{-2}(1-M^{-1})<0,$$

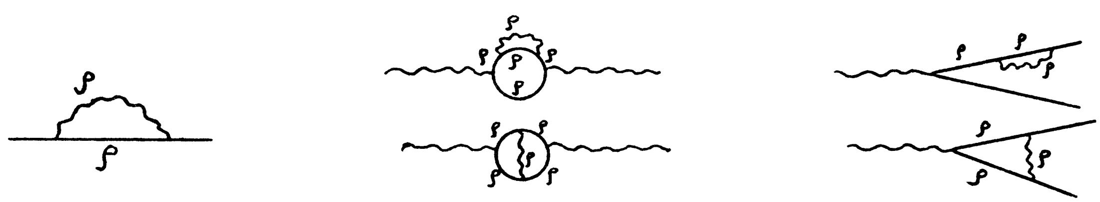
with $\tilde\mu_{i+1}=\mu_{\rm ren}M^{-(i+1)}$, $\mu_{\rm ren}=m_{\rm ren}/\zeta_{\rm ren}$. It follows that $\pi^i_{\rm ren}(p)=\pi^i(p)-\pi^i(0)\ge 0$. Also, as for λ_ρ, if we suppose that $\lambda_{i+1}\approx[\zeta^2_{\rm ren}/b(\tilde\mu_{i+1})]M^{-(i+1)}$ asymptotically in i, then (5) leads to $\lambda_i\approx\zeta^2_{\rm ren}M^{-1}/b(\tilde\mu_i)$, which shows the ansatz is stable up to negligible corrections.

Let β_i stand for the ratio $\lambda_iM^i/\zeta^2_{\rm ren}$. The fixed-point condition is also seen in the fact that β_i has a positive limit when $i\to\infty$. In terms of this variable, (5) reads

$$\beta_{i-1}=\frac{\beta_i/M}{1-\beta_ib(\tilde\mu_i)(1-1/M)}=f(\beta_i)\tag{6}$$

and has a fixed point $\beta=\beta^*$. The stability of this nontrivial solution is a consequence of $f'(\beta^*)=M>1$, and $\lambda_\rho(\lambda_{\rm ren})$ emerging from (5) gives the above ansatz.

Let us recover all orders in $1/N$. The nonrenormalizability in λ_ρ is surmounted when considering the expansion in $C_\rho(p)$. Taking the wavy line in (2) as a "true" propagator, the only divergent contributions come from the two-fermion, the two-wavy-line, and the three-point (with two fermions and one wavy line) functions, which give corrections in $\bar\psi(x)\psi(x)$ and $[:\bar\psi(x)\psi(x):]^2$. Fortunately, the apparently divergent three-wavy-line functions are convergent avoiding a disallowed renormalization in $[:\bar\psi(x)\psi(x):]^3$. With this, all parameters depend on N [$m_\rho\to m_\rho(N)$, etc.] and we require them to recover the above solution when $N\to\infty$. For positive g_i and v_i, set $\lambda_i=g_i^2v_i^{-1}$ and ask $g_i(N\to\infty)=1=g_{\rm ren}(N=\infty)$. Let $\beta_i(N)$ denote $g_i(N)^2M^i/v_i(N)\zeta_i(N)^2$. At the first order in $1/N$, for the slice ρ, corrections come from



$\delta m,\delta\zeta$ $\delta\kappa$ δg

For example, direct calculation yields $\delta m_\rho\approx N^{-1}\beta_\rho a^{(3)}m_\rho$, where $a^{(3)}$ is approximated by a constant for ρ large. We have similar results for the other corrections and δv_ρ is given by $g_\rho^2\pi^\rho(0)+\delta\kappa_\rho$. Let $a^{(1)}$, $a^{(2)}$, and $a^{(4)}$ be the coefficients appearing in δg_ρ, $\delta\zeta_\rho$, and $\delta\kappa_\rho$, respectively. Corrections to β describe a renormalization flow resulting in the following fixed-point equation:

$$\beta^2a^{(4)}/N-\beta[(1-1/M)b-2(a^{(2)}-a^{(1)})/N]+(1-1/M)=0.\tag{7}$$

This has one solution $\beta^*(N)$ converging to β^* for $N\to\infty$. The derivative of the corresponding mapping is larger than 1 at $\beta^*(N)$. This guarantees the stability of the solution for the full theory.

About the proof.— We write the normalized $2p$-point Schwinger functions with cutoffs as

$$S_{2p,\rho,\Lambda}(E)=Z_{\rho,\Lambda}^{-1}\int d\mu_{C_\Lambda}(\sigma_\rho)\,{\rm Tr}\{\Lambda^p([1+\mathcal{K}_{\rho,\Lambda}]^{-1})X_\rho^{(p)}(E)\}\,{\rm det}_3(1+\mathcal{K}_{\rho,\Lambda}),\tag{8}$$

where $Z_{\rho,\Lambda}$ is the partition function, σ_ρ is a boson field, and $d\mu_{C_\Lambda}(\sigma_\rho)$ is a Gaussian measure with covariance C_ρ given by a cutoff version of (2) with λ_ρ^{-1} replaced by v_ρ and supported on Λ. $\mathcal{K}_{\rho,\Lambda}$ is a trace class operator on $\mathcal{H}=\mathcal{L}^2(d^3p,\mathbb{R})\otimes\mathbb{C}^{2\text{ or }4}\otimes\mathbb{C}^N$ given by

$$\mathcal{K}_{\rho,\Lambda}(x,y)=(g_\rho/\sqrt{N})\Lambda(x)S_\rho(x,y)\sigma_\rho(y)\Lambda(y)\,. \quad (9)$$

$S_\rho(x,y)$ is the Fourier transform of $S_\rho(p)$, $\Lambda(x)$ is the characteristic function of Λ, and $\det_n(1+\mathcal{O})$ means $\det(1+\mathcal{O})\exp\{-\mathrm{Tr}[\cdots+(-1)^{n+1}\mathcal{O}^n/n]\}$. Finally, $X_\rho^{(p)}(E)$ is a projector on $\wedge^p\mathcal{H}$ which includes the set E of external legs.

That (8) describes the Schwinger functions of this model is given by the following steps. First, we introduce an ultralocal field σ by splitting the four-fermion vertex. The interaction $\lambda[:\bar\psi(x)\psi(x):]^2/2N$ is replaced by $\sqrt{\lambda/N}\sigma(x):\bar\psi(x)\psi(x):$. Then, we scale σ to absorb the part v of λ in its covariance and integrate the fermions out. This yields a Matthews-Salam formula with interaction $\det_2(1+\mathcal{K})$. $\mathrm{Tr}\mathcal{K}^2$ is the bubble contribution, and is absorbed in the boson covariance.

The proof of the theorem is based on multislice cluster expansions.[7] As for S_ρ, C_ρ is sliced, σ^i being measured by $d\mu_{C^i}(\sigma^i)$. Using lattice coverings for Λ, with spacing aM^{-i} $(a>1)$, we perform horizontal expansions for the unnormalized and the partition functions. This is done by interpolating S^i and C^i and testing couplings between cells of one same covering. With vertical expansions, we test couplings between a cell of the slice i and a cell of the slice $j<i$. Inductively, Mayer expansions factorize the vacuum graphs for each slice.

We list the main difficulties and their solutions. The major problem is the control of the large fields. It appears in two ways: (i) domination of the badly localized fields; (ii) measurability of the interaction in (8) by $d\mu_{C_\Lambda}(\sigma_\rho)$.

Inspired by Ref. 8, (i) is solved with an effective-potential technique. Badly localized fields are fields σ^j produced by expansions of slices $i>j$. To afford their domination, for $i\in I=\{j/a^{3/4}N^{1/4}m_j\zeta_j^{-1}M^{-j}<O(1)\}$, Δ a cube of slice i, and $\Delta(x)$ its characteristic function, we consider the operator $K^i_{P,\Delta}$ on $\mathcal{H}$,

$$K^i_{P,\Delta}(x,y)=(g_i/\sqrt{N})\Delta(x)S^i_P(x-y)\bar\sigma^{(i)}_{i-q-1}\Delta(y)\,, \quad (10)$$

with m_i, ζ_i, and g_i being parameters corrected up to the slice i and q a positive integer. S^i_P is the fermion propagator with periodic conditions in Δ and $\bar\sigma^{(i)}_{i-q-1}$ denotes $\sigma_{i-q-1}=\sum_{j=1}^{i-q-1}\sigma^j$ averaged in Δ. Since $S^i_P\bar\sigma^{(i)}_{i-q-1}$ is a multiplication operator in momentum space, $\det_3(1+K^i_{P,\Delta})$ may be evaluated. If $0<\alpha<50$, $0.5<\gamma<1$, and for $\tau=(g_i/\zeta_i\sqrt{N})\bar\sigma^{(i)}_{i-q-1}$, we get

$$|\det_3(1+K^i_{P,\Delta})|\le O(1)\times\begin{cases}\exp[O(1)a^3(m_i/\zeta_i)^4M^{-4i}N]=O(1), & |\tau|\le am_i/\zeta_i\,,\\ \exp[-O(1)a^3M^{-4i}|\tau|^4N], & am_i/\zeta_i<|\tau|<(1-\gamma)M^i,\\ \exp[-O(1)a^3M^{-2i}|\tau|^2N], & |\tau|\ge(1-\gamma)M^i\,.\end{cases} \quad (11)$$

We use this to dominate [apply $\sigma^n\exp(-\sigma^{2k})\le\mathrm{const}^n\times(n/2k)!$] badly localized fields. The reason why $K^i_{P,\Delta}$ is a good approximation for $\mathcal{K}_{\rho,\Lambda}$ in the same support is seen as follows. First, $S^i-S^i_P$ is small if the constant a is large. Second, $\delta\sigma_{i-q-1}=\sigma_{i-q-1}-\bar\sigma_{i-q-1}$ is small because σ has been averaged in a cube smaller than the natural one. This effect is weakened for large a. Thus, we introduced the shift q and we set $M^q\gg a$. The not very badly localized fields $\sigma^{i-1},\dots,\sigma^{i-q}$, produced at slice i, are integrated at a price of accumulation factors $\mathrm{const}^{3q/2}$ (see below).

Roughly speaking, the bound (11) is enough because we essentially have to control $\mathrm{Tr}(\mathcal{K}_{\rho,\Lambda})^3$ which appears from perturbation of $\det_3$. A bound per dominated σ field arising from (11) is $O(1)(\zeta_i/g_i)a^{-3/4}M^iN^{1/4}$. $\mathrm{Tr}(\mathcal{K}_{\rho,\Lambda})^3$ gives a triangle graph with bare fermions running along the sides and three external bosons. Its leading part is diagonal in the slice indices and cube localizations because of renormalization and distance falloff of the propagators. For the slice i, it behaves as $O(1)\times a^3(g_i\sigma/\zeta_i)^3m_iM^{-4i}N^{-1/2}$. Integration of a field σ^j, produced in slice i, gives $(M^{3j/2}/v_j^{1/2})\times M^{3(i-j)/2}$. (The second term is the accumulation factor giving the square root of the number of cubes of slice i contained in a cube of slice j, and the first term is the usual power counting coming from the propagator C^j.) The field $\delta\sigma_{i-q-1}$ behaves as a well localized field σ^i. Altogether, in the leading case with three badly localized fields, we can keep a factor $O(1)a^{3/4}N^{-1/10}$ if we integrate one field and dominate the other two. And we set $1\ll a^3\ll M^q\ll N^{1/100}$, which makes this factor small. The same inequalities ensure small factors for the other cases.

Now we consider the measurability of the interaction. The difficulty in showing that we can integrate the interaction in (8) comes from bounding $|\ln[\det_3(1+\mathcal{K}_{\rho,\Lambda})]|$. The usual bound $\mathrm{Tr}(\mathrm{Re}\mathcal{K}_{\rho,\Lambda})^2$ gives $(\sigma_\rho,\Lambda\pi_\rho^{\mathrm{ren}}\Lambda\sigma_\rho)/2$, which is also in $d\mu_{C_{\rho,\Lambda}}$. However, both come with different cluster interpolations and can hardly be compared.

To solve this, with a large-field expansion, we define "unpainted" cubes, where the covariance C gives a small factor, and "painted" cubes for which it behaves as in (4). We use this distinction to construct expansion cells under the following rule: We do not expand horizontal or vertical couplings between too closely painted cubes. In this way, we get a correspondence between expansion cells and regions in phase space where we can have leading bubbles. By construction, these are not greatly affected by interpolations, and integrability of the "noninterpolated" traces follows by inspection.

This recipe works provided we can associate small factors to the elementary cubes. So, this expansion is applied to the covariances C^i. For each slice, this precedes any other expansion. It is local, in contrast to the ones linking different cells. However, the way it gives convergent factors is also by vertex productions.

If we complete the above procedure by adopting entirely inductive cluster expansions (horizontal expansions following the Brydges-Battle-Federbush scheme, and vertical expansions using the tree polymers resulting from the former), we can exploit the positivity of the renormalized bubbles as operators to prove

$$\mathrm{Tr}(\mathcal{H}^*_{\rho,\Lambda}+\mathcal{H}_{\rho,\Lambda})^2(h,v) \le \mathrm{Tr}(\mathcal{H}^*_{\rho,\Lambda}+\mathcal{H}_{\rho,\Lambda})^2+(\text{boundary or small terms})\,, \qquad (12)$$

where h and v indicate the horizontal and vertical parameters, absent in the right-hand side. Similar results hold for the boson covariances. Integrability of the interaction follows from $\mathrm{Tr}C\pi_{\mathrm{ren}} \le \text{const}$ per cube and $\|C\pi_{\mathrm{ren}}\| < 1$, since $C^{-1} = v + \pi_{\mathrm{ren}}$. This is applied to

$$|\det{}^{-1}(1-C\pi_{\mathrm{ren}})| \le \exp[(1-\|C\pi_{\mathrm{ren}}\|)^{-1}\mathrm{Tr}C\pi_{\mathrm{ren}}]\,.$$

Combining this with the small factors previously obtained, we can prove convergence of the whole expansion, uniformly in ρ and Λ.

To conclude, let us underline the main property of our construction. In renormalizable and asymptotically free models, asymptotic freedom leads to an expansion in the neighborhood of a free field. For the UV problems, this provides a standard guideline through the perturbative expansion of the Callan-Symanzik β function. Here, the renormalizable behavior is obtained by an expansion around a solvable but not free theory. What measures the shift from this solvable theory is not a renormalized coupling constant, as in asymptotically free models, but $1/N$. Thus, convergence holds only for large N, and the inductive determination of the ansatz is not related to the first perturbative orders of β.

P.A.F.V. thanks the Coordenação de Aperfeiçoamento de Pessoal do Nivel Superior-Ministério da Educação, Brazil, for initial support, the Ecole Polytechnique for its hospitality, and acknowledges support from the National Science Foundation under Grant No. DMS/PHY 88-16214. He also thanks Professor J. Imbrie and Professor A. Jaffe for an invitation to Harvard University. All the authors thank Professor C. Borgs, Professor J. Imbrie, Professor S. Janowsky, Professor P. Renouard, and Professor V. Rivasseau for discussions.

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