Is There a Breakdown of Fermi-Liquid Behavior in the Two-Dimensional Fermi Gas?

Recently Anderson[1] has reexamined the question of the validity of Fermi-liquid theory in a low-density Fermi system in 2D, which was studied earlier by the authors.[2] We have shown that, in spite of the existence of unexpected nonperturbative effects such as a new collective bound state,[2] there is no breakdown of Fermi-liquid behavior in the dilute limit. Anderson arrived at the opposite conclusion. In this Comment we wish to shed some light on the confusion that has resulted from these conflicting claims. In particular, we prove the analog of Levinson's theorem, and show that the Fermi-surface phase shift computed in Ref. 1 violates this theorem.

Consider the retarded vertex part[2] $\Gamma(q,\omega)\equiv|\Gamma|e^{i\delta}$ in the (s-wave) particle-particle channel within a ladder approximation.[3] (Here q is the center-of-mass momentum, and ω the energy of the two particles.) The phase shift $\delta(q,\omega)$ describes two-particle scattering in the dilute Fermi gas. We (a) prove that the phase shift[4] of Ref. 1 is equivalent to the δ defined above, and then turn to the two points on which there is disagreement: (b) whether the Fermi surface phase shift for two fermions with $q=2k_F$ and zero relative momentum, i.e., $\delta(2k_F,\omega\to 0^+)$, is zero or not; and (c) how the phase shift affects, for example, the single-particle self-energy.

(a) Equivalence of the phase shifts.—Anderson defines[4] δ in terms of differences in the energy levels of the interacting and noninteracting systems in a finite box, analogous to ordinary potential scattering theory.[5] We show that, in the infinite-volume limit, this reduces to the above definition.

In a finite box $\Gamma=V/(1-V\chi)$ has poles at the two-particle excitation energies of the interacting system and has zeros at the excitation energies of the noninteracting system (i.e., the poles of χ). Thus Γ^{-1} has the same structure as the ratio of determinants in Fredholm theory. Then using manipulations which are formally identical to the ones in Ref. 5, we obtain the desired result.

(b) Levinson's theorem.—The analogy with potential scattering theory suggests that there must be a connection between the phase shift at the bottom of the two-particle band (at ω_q^*, corresponding to zero relative momentum for a given q), and the number of bound states n peeled off the continuum. We show that $\delta(q,\omega_q^*)=n\pi$.

To prove this result, consider the integral of $d\ln\Gamma(q,z)/dz$ over a closed contour in the complex z plane obtained from a circle of infinite radius, deformed to circumvent the branch cut on the real axis from ω_q^* to infinity. This integral clearly counts the number of states n, for a given q, which have been pulled down below the band. Further, using $\Gamma(q,\omega\pm i\eta)=|\Gamma|\times\exp(\pm i\delta)$ just above and below the branch cut we obtain the result stated above.[6]

For $q=2k_F$, the Fermi surface $\omega=0$ is also the bottom of the two-particle band $\omega_{2k_F}^*=0$. Anderson's result $\delta=\pi/[2\ln(k_Fa)]$ is then impossible, since it is not equal to $n\pi$, $n=0,1,\ldots.$ In other words one cannot pull down a fractional number of states below the band.

Our result[7] $\delta(2k_F,\omega)=\sqrt{\omega}/[2\ln(k_Fa)]$ goes to zero as $\omega\to 0$, and is consistent with the theorem since no collective bound state exists for $q=2k_F$. Note that for every $q<2k_F$, there is a bound-state pole and the phase shift does go to π at the bottom of the band which no longer coincides with $\omega=0$. However, this does not give rise to a nonzero Fermi surface phase shift either.[2]

(c) Phase shifts and the single-particle self-energy.—The two results obtained above suggest a close analogy between the phase shift of the potential scattering problem and the two-body phase shift of the dilute Fermi gas. The final point of this Comment is to caution that the connection between the quasiparticle residue Z and the two-body phase shift [Eq. (13) of Ref. 2] is completely different from that between the overlap matrix element and the phase shift in the single-impurity problem. In particular, in the interacting-fermion problem, the value of the phase shift at the Fermi surface does not[2] completely determine Z.

In conclusion, there is no evidence for a breakdown of Fermi-liquid theory in 2D within a low-density expansion of the vertex part considered in Refs. 1 and 2.

J.R.E. was supported by NSF Grant No. DMR88-22688.

Jan R. Engelbrecht
Department of Physics
University of Illinois at Urbana-Champaign
1110 West Green Street
Urbana, Illinois 61801

Mohit Randeria[a]
Department of Physics
State University of New York at Stony Brook
Stony Brook, New York 11794-3800

Received 11 December 1990
PACS numbers: 75.10.Lp, 74.65.+n, 74.70.Vy

[a]Now at Argonne National Laboratory, Argonne, IL 60439.

[1]P. W. Anderson, Phys. Rev. Lett. **65**, 2306 (1990).

[2]J. R. Engelbrecht and M. Randeria, Phys. Rev. Lett. **65**, 1032 (1990).

[3]Although Ref. 1 refrains from using the term "ladder diagrams," the solution of the two-particle excitation energies through its Eq. (2) is formally identical to solving for poles of Γ.

[4]See top of column 2, p. 2306, Ref. 1.

[5]K. Gottfried, *Quantum Mechanics* (Benjamin, New York, 1966), Sec. 49.

[6]This result would not be obtained if, for example, poles had moved off the real axis, signaling an instability.

[7]See text below Eq. (12) of Ref. 2.

