Localization in Interacting, Disordered, Bose Systems

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(Received 11 February 1991)

We use quantum Monte Carlo techniques to study a one-dimensional, disordered, interacting, Bose Hamiltonian. The effect of disorder on the Mott-insulator portion of the phase diagram is determined. We observe the destruction of superfluidity by disorder at incommensurate densities, for the first time demonstrating the emergence of a "Bose-glass" phase. In addition to these strong-coupling phases, we observe an unanticipated reentrance into an Anderson-type localized regime for weak couplings.

PACS numbers: 64.70.Pf, 05.30.Jp, 05.70.Jk

The metal-insulator transition is one of the central problems of condensed-matter physics. Interactions alone can drive localization of electrons[1] via the Mott transition. On the other hand, disorder alone can induce localization of the eigenstates, often referred to as the Anderson transition.[2] The exact nature of the interplay between these two scenarios has been the subject of intense study recently.[3] While these questions have most widely been considered for fermions, the understanding of the analogous interacting, disordered, Bose systems is also of considerable interest both experimentally and theoretically. A primary motivation is to describe the superconductor-insulator phase transition, which takes place in disordered thin films and wires.[4,5] In this case the Cooper pairs can be thought of as the bosons of the model. Additionally, such boson models have direct realizations in ^{4}He adsorbed in Vycor[6] or carbon black, and have relevance to the motion of flux lines in superconducting materials.[7] While we will study a one-dimensional system, we expect the phase diagram and physics to be qualitatively similar in higher dimensions.

In a recent paper[8] we explored the superfluid-to-insulator transition in the one-dimensional, interacting, Bose Hamiltonian

$$H = -t\sum_l (a_{l+1}^\dagger a_l + a_l^\dagger a_{l+1}) + V\sum_l n_l^2 . \qquad (1)$$

Here, a_l and $a_l^\dagger$ are Bose destruction and creation operators, t is a boson transfer strength, V is a soft-core repulsion, and $n_l = a_l^\dagger a_l$ is the boson number operator on site l. This boson Hamiltonian cannot be solved by the Bethe ansatz,[9] and hence our knowledge about its properties is limited even in 1D. In Ref. 8, we mapped out the phase diagram, and computed the critical interaction strength $(t/V)_c$ required to localize the bosons at commensurate fillings, the gap to excitations as a function of t/V for $t/V < (t/V)_c$, and the critical exponents for the density-controlled transition from superfluid to Mott insulator. These latter agreed well with the predictions from scaling arguments by Fisher *et al.*[10]

In this work we simulate the effects of disorder in the Hamiltonian by adding a random on-site energy term:

$$\Delta H = \sum_l \epsilon_l n_l . \qquad (2)$$

The random energies ϵ_l are uniformly distributed in the interval $(-\Delta,\Delta)$. As suggested in Ref. 10, the *cooperation* of interactions and disorder can lead to the emergence of a gapless, insulating, "Bose-glass" phase in the strong-interaction regime. Here we report the first direct observation of such a phase and of a new Anderson-type insulating region ("Anderson glass") at weak couplings. We suggest that in the latter case the interaction has a *delocalizing* effect which then *competes* with the disorder; thus the physics of the two localized regions is rather different. The existence of two such separate insulating phases has been conjectured by Giamarchi and Schulz.[11]

We perform our Monte Carlo simulations using the world-line algorithm in the canonical ensemble. To make contact with the grand canonical ensemble, we use the definition of the chemical potential $\mu(N_b) = E_0(N_b+1) - E_0(N_b)$, where $E_0(N_b)$ is the ground-state energy for N_b bosons. One quantity of interest is the filling N_b as a function of chemical potential μ. The vanishing of the compressibility, $\kappa = \partial N_b/\partial\mu$, signals the presence of a gap and the Mott-insulating phase. The critical behavior of κ prior to this plateau is characterized by the scaling law $\kappa \approx (\rho-\rho_c)^\alpha$. For the ordered case in one dimension, $\alpha < 0$ so that κ diverges before it vanishes.

The superfluid density ρ_s is the order parameter for the phase transition, and is proportional to $\langle W^2\rangle$, the mean-square winding number.[12] $\langle W^2\rangle$ is the $\omega=0$ value of the Fourier transform of $J(\tau)$, the imaginary-time momentum-momentum correlation function.[8] $J(\tau)$ measures the correlation between the number of left-moving minus the number of right-moving bosons at imaginary time 0 and some later time τ. Because we work in the zero-winding-number ensemble, the value $\tilde{J}(\omega=0)$ vanishes identically, but the $\omega\to 0$ limit nevertheless has a well-defined nonzero value in the superfluid phase.[8]

In the ordered case, the phase diagram[8,10,13] in the μ/V vs t/V plane consists of a set of lobes for small t/V. These describe Mott-insulating phases of fixed commensurate density $\rho = 1,2,3,\ldots$ The density-driven phase transition has mean-field exponents while the interaction-strength–driven transition, in 1D at fixed integer

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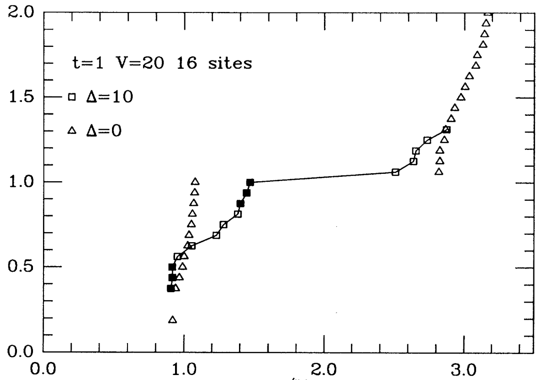

FIG. 1. The density ρ as a function of the chemical potential μ for $t=1$, $V=20$, and a 16-site lattice. The triangles are for the ordered case $\Delta=0$, and the squares, the disordered case $\Delta=10$.

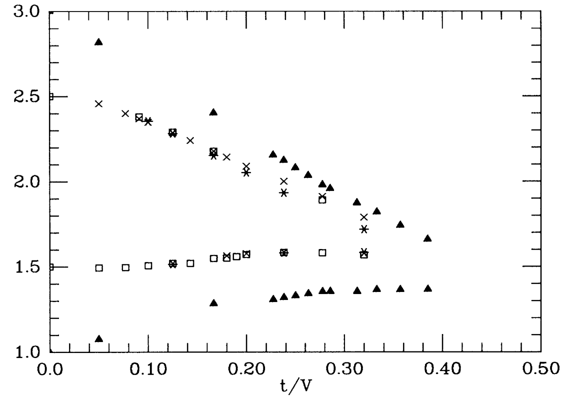

FIG. 2. The boundary of the Mott-insulator phase. The solid triangles give the boundary for the ordered case. The other symbols are for $\Delta/V=1/2$ and $N=64$ (crosses), 128 (squares), and 256 (stars).

filling, is in the universality class of the 2D XY model.[10] We begin by looking at the manner in which these lobes shrink with disorder. In Fig. 1 we show the density ρ as a function of the chemical potential μ for $t=1$ and $V=20$ on an $N=16$ site lattice. The triangles correspond to the ordered case, $\Delta=0$, and the squares to one realization of random site energies with $\Delta=10$. The difference between the open and the solid squares will be discussed below. We see that the gap was shrunk by the disorder, as is expected. For $\Delta=0$ the compressibility $\kappa=\partial\rho/\partial\mu$ diverges with an exponent $\alpha=-0.5$ upon approaching the Mott lobe.[8,10] For $\Delta>0$, this divergence is smeared out.

By constructing the same plot as in Fig. 1 for different values of V, we mapped out the phase boundary of the shrunken Mott-insulating regime in the presence of disorder. This is shown in Fig. 2. The triangles again are for the ordered case, while the crosses, squares, and stars represent a single realization of disorder with $\Delta/V=1/2$, and system sizes 64, 128, and 256 sites, respectively. The consistency of the boundaries of the Mott phase for the different size lattices of Fig. 2 suggests that, at least for the energies, lattices of this size are reasonably self-averaging. It is possible that other quantities may be more sensitive to the particular realization of the randomness. Nevertheless, we have found that the values of observables, such as the energies and ρ_s, are the same to within 10% for different choices of the site energies on lattices of $N\geq 100$ sites. While such fluctuations do not affect qualitative features of the phase diagram, they make addressing more quantitative issues like the evaluation of exponents difficult.

Next we measure the superfluid density by plotting $\tilde{\mathcal{J}}(\omega)$ vs ω and extrapolating to zero frequency, as shown in Fig. 3(a) ($\Delta=0$) and Fig. 3(b) ($\Delta=10$) for $t=1$, $V=20$, $N_b=96$, and $N=128$. The extrapolation is performed with simple polynomial fits. Clearly, for this *incommensurate* filling, in the absence of disorder, $\tilde{\mathcal{J}}(\omega\to 0)$ is nonzero signaling superfluidity, while for $\Delta=10$, $\tilde{\mathcal{J}}(\omega\to 0)=0$ indicating the destruction of superfluidity. Furthermore, we see from Fig. 1 that the system is still compressible at finite incommensurate fillings in the presence of disorder. Therefore, this new phase is distinguished from the Mott phase by a nonvan-

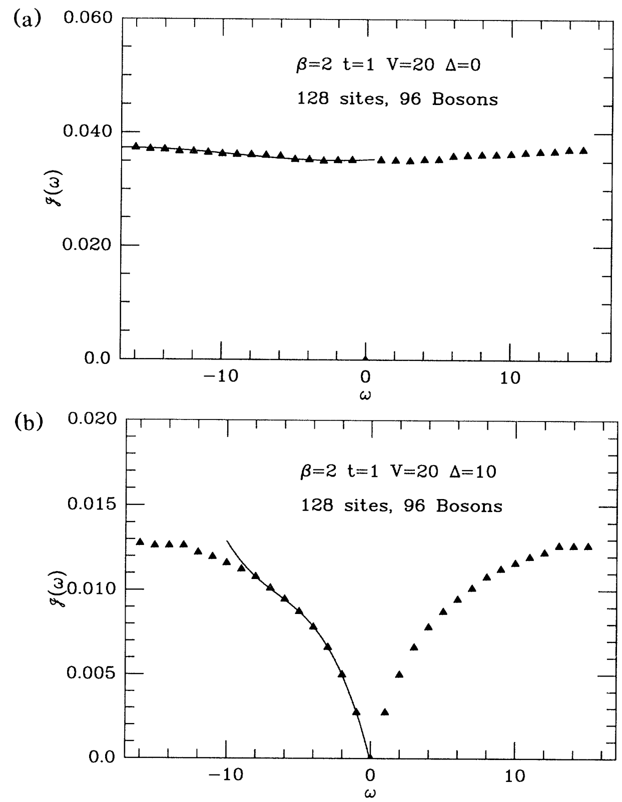

FIG. 3. The Fourier transform of the imaginary-time momentum-momentum correlation function at $t=1$, $V=20$, and $N=128$ sites, for (a) $\Delta=0$ and (b) $\Delta=10$. The solid lines are cubic fits to four points which do *not* include $\omega=0$.

ishing compressibility, and from the superfluid phase by a vanishing superfluid density. These are the properties of the Bose-glass phase discussed in Ref. 10 and the present simulations constitute its first numerical observation. We have unambiguously established the existence of a phase transition from Bose glass to Mott insulator only for large values of V. It will require further studies to answer the important question of whether the Bose-glass phase completely surrounds the shrunken Mott lobe, as suggested in Ref. 10.

We note that for small lattices the localization length might exceed the lattice size, resulting in a false signal for superfluidity in the form of a nonzero ρ_s. To distinguish this case from the true superfluid phase we also measured a superfluid susceptibility χ_s by measuring the response of ρ_s to an applied external momentum, i.e., a nonzero winding number. Whereas χ_s is small in the superfluid phase, it is expected to diverge in the Bose glass.[10] Indeed this is what we have found. When the system was truly superfluid, such as in the ordered case (triangles in Fig. 1), ρ_s changed little in the presence of nonzero winding number, resulting in a small χ_s. The behavior was very different for the disordered case shown in Fig. 1 (squares), where the measured χ_s was between 15 and 30 times larger than the superfluid phase. In this way, by measuring both ρ_s and χ_s, we conclude that the triangles in Fig. 1 describe a superfluid state, while the squares correspond to the Bose glass. For the open squares both ρ_s and χ_s indicate an insulator, while for the solid squares ρ_s has a small nonzero value while χ_s is still very large. As mentioned above, our interpretation for this is that the localization length is larger than the size of the system. In fact, when we perform simulations at the same densities but on larger lattices we find that the extrapolated ρ_s approaches zero.

So far we have focused on the strong-coupling regime. In the other extreme limit, for noninteracting systems, the disorder is able to localize the one-particle eigenstates below the mobility edge E_c—independently of the statistics.[2] Using a renormalization-group treatment, Giamarchi and Schulz raised the possibility that such an Anderson glass might extend to finite values of the interaction V in the present bosonic system.[11] To develop a simple physical picture we recall that for $V=0$ *all* bosons occupy the single lowest-energy eigenstate. For finite but small V, one can treat the interaction in a Hartree-Fock approximation.[14] This maps the problem onto a noninteracting one, with shifted one-particle energies. For small enough filling and interaction strength all states remain below the mobility edge E_c and hence are localized. Upon increasing either of the two parameters, the renormalized energy levels cross E_c, and the states become extended in high enough dimensions. Thus in this regime the interaction *delocalizes* the particles and therefore *competes* with the disorder. This scenario is rather different from the physics of the Bose glass, and provides a more specific picture underlying the conjecture of Ref. 11. Since the lower critical dimension is 1,[10] we expect this picture to apply even in the present case. Of course the concept of the mobility edge is not applicable directly in one dimension, and a more elaborate description of this phase is called for.

A possible insight can be gained from our numerical simulations. In Fig. 4 we show a plot of ρ_s for the whole range of couplings in the presence of disorder, for a system of 40 bosons on a lattice of 64 sites. Again, we see that ρ_s vanishes for large V/t as the system is in the Bose-glass phase. As V/t decreases, ρ_s increases, levels off, and then drops dramatically, vanishing at small but *finite* V/t. In contrast, in the absence of disorder, ρ_s increases from zero as V/t decreases, and reaches its *maximum* for $V/t \rightarrow 0$. Clearly, disorder is localizing the bosons, even at finite values of the coupling strength. We expect that the Anderson glass will be further stabilized by a near-neighbor Coulomb attraction which would, for example, be appropriate to model the ^{4}He Lennard-Jones potential.

Are these two disordered insulators really distinct phases? There are several arguments to support the view that they may be. The first is based on the difference in the mechanism driving the insulating behavior. At strong coupling one envisions a *cooperation* between disorder and interactions, as the boson hard core introduces steric constraints which aid the disorder in localizing the particles. In contrast, at weak coupling as we have just demonstrated, the interaction *competes* with the disorder, as it tries to *delocalize* the bosons. Second, these phases differ substantially in the nature of the boson density distribution. In the Bose glass the density is reasonably uniform. On the other hand, since the Anderson glass is distinguished by the interaction scaling to zero,[11] boson density correlations are expected to decay exponentially. Indeed, we do observe substantial clumping of the boson density in our simulations. Also, unlike the

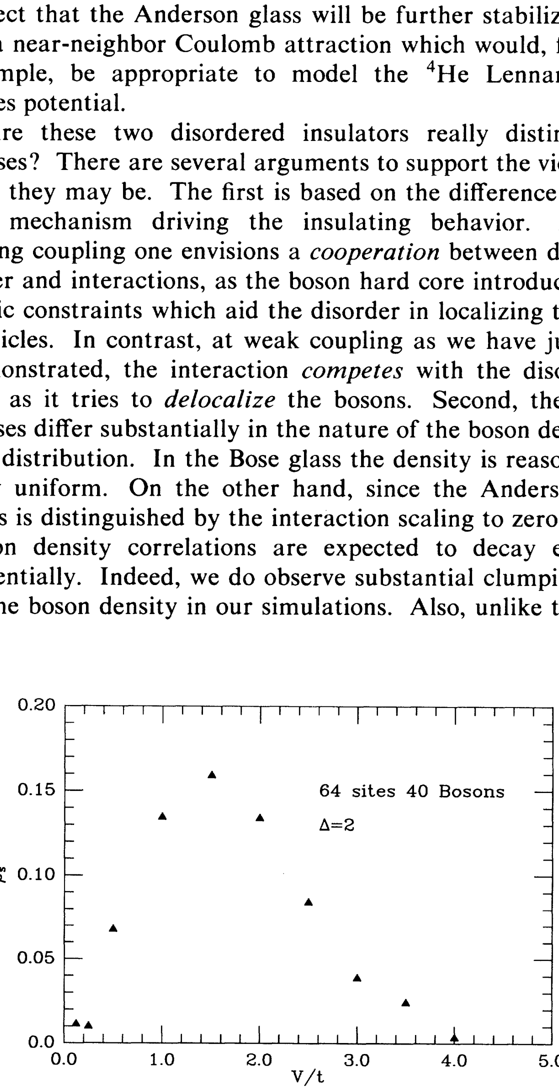

FIG. 4. The superfluid density at fixed density $\rho=0.625$ and varying coupling. ρ_s is zero in the Bose-glass phase at strong coupling, but goes to zero again at weak coupling in the Anderson phase.

Bose-glass phase, the observed susceptibility does not diverge. Still, it is conceivable that again a characteristic length of the problem exceeds our sample size. Thus we cannot exclude the possibility that on a strongly coarse-grained length scale the density might appear more uniform. But should this be the case, this Anderson glass is an exponentially wide crossover region of the model, and therefore should bear experimental relevance.

To summarize, in this paper we have reported on the first quantum simulation of the disordered boson Hubbard model. We have characterized the reduction of the Mott insulating phase by disorder and have shown the existence of the compressible, insulating, Bose glass at strong coupling. We have also presented the first convincing evidence for the existence of a second insulating region, the Anderson glass, which we argue has substantially different features from the Bose glass.

We would like to thank M. P. A. Fisher, C. Y. Fong, T. Giamarchi, K. Runge, and P. B. Weichmann for useful conversations, and the hospitality of the Aspen Center for Physics. G.G.B.'s work was performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract No. W-7405-ENG-48. G.T.Z. was partially supported by a Grant from NATO, No. CRG 900692.

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