Radiative Corrections to Top-Quark Decay

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We calculate all radiative corrections to one-loop order for the main decay of the top quark, $t \rightarrow b+W$, in the standard model, retaining exact dependence on all masses. For $m_t = 150$ GeV and $M_H = 100$ GeV we find a -2.9% (-6.9%) correction with a very weak dependence on the Higgs-boson mass, in renormalization schemes that use α, G_F, and M_Z (G_F, M_W, and M_Z) as input parameters. Out of the above results, -8.5% is due to QCD. The m_t and M_H dependence is given up to 300 and 1000 GeV, respectively. The inadequacy of a leading m_t calculation is pointed out.

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The top quark, according to recent analyses, is around the corner. From Collider Detector at Fermilab experiments,[1] $m_t > 89$ GeV, and a maximum-likelihood analysis of recent data[2] has given the most likely value of $m_t \sim 150$ GeV. Other studies[3] prefer $m_t \sim 130$ GeV, while a most recent analysis[4] of some—but not all—higher-order (beyond one-loop) effects shows reasonable probability distributions up to $m_t \sim 300$ GeV. Within the framework of the standard model (SM) discussed here, the main decay of the top quark is $t \rightarrow bW$, with a tree-level width given by

$$\Gamma_0 = \frac{\alpha |V_{tb}|^2 w(m_t, m_b, M_W)}{16 m_t^3 s_W^2} G_0 , \tag{1}$$

where

$$w(x,y,z) = [(x^2 - y^2 - z^2)^2 - 4y^2 z^2]^{1/2} ,$$
$$G_0 = m_b^2 + m_t^2 - 2M_W^2 + \frac{(m_b^2 - m_t^2)^2}{M_W^2} , \tag{2}$$

and the square of the Weinberg angle is defined as $s_W^2 = 1 - M_W^2/M_Z^2$. Once the top quark is discovered more accurate experiments will search for radiative corrections to the tree-level width. We present here the full radiative corrections to one-loop order to the main decay mode $t \rightarrow bW$.

The motivation for this work should be clear. Since the top quark has a unique mass scale, being (very likely) the heaviest in the SM, precise tests of its properties against the predictions of the SM represent a unique opportunity to search for the effects of mass scales beyond the SM. Radiative corrections are, of course, powerful tests of gauge theories. Furthermore, the mixing angle V_{tb} must be deduced from experiments by studying the decays of the top quark. At this point we remark that while the quantum chromodynamics (QCD) part of our calculation confirms previous results,[5,6] the electroweak (EW) component of the radiative corrections has not been presented in full. Only leading results valid in the limit $m_t \gg M_W, M_H$ have been previously presented.[6] The leading m_t calculation fails to describe the correct dependence on the relevant masses, and does not reproduce the exact results even for high m_t and low M_H, as is shown below.

Before the current lower limit on m_t was established, a complete first-order calculation of $W^+ \rightarrow t\bar{b}$ had been presented in Ref. 7. We have independently done that calculation and will adopt the notation of Ref. 7, hereafter referred to as DS (Denner and Sack), and emphasize all the necessary modifications needed for the transformation from $W \rightarrow tb$ to $t \rightarrow Wb$.

Define the relative correction

$$\delta = (\Gamma - \Gamma_0)/\Gamma_0 , \tag{3}$$

where Γ is the width including first-order, i.e., one-loop, corrections. Then, the total correction can be separated as

$$\delta_{\text{total}} = \delta_{\text{EW}} + \delta_{\text{QCD}} . \tag{4}$$

The EW correction is further given as

$$\delta_{\text{EW}} = \delta_f + \delta_W + \delta_v + \delta_b \tag{5}$$

with contributions from fermion and W-boson wave-function renormalizations, vertex corrections, and photon bremsstrahlung without a cutoff on photon energies. The QCD first-order correction is separated into virtual (including fermion wave-function renormalization and vertex corrections) and bremsstrahlung corrections, with gluons replacing photons,

$$\delta_{\text{QCD}} = \delta_{\text{virt,QCD}} + \delta_{b,\text{QCD}} . \tag{6}$$

The masses and charges m_2,m_1 and Q_2,Q_1 in DS are here m_t,m_b and $\frac{2}{3}, -\frac{1}{3}$, respectively. Then,

$$G_0=-G_0^{DS}, \quad G_i=-G_i^{DS} \quad (i=1,2,3) \tag{7}$$

(for $G_1^{DS},\ldots,G_3^{DS}$ and other undefined symbols below, see DS).

δ_f.— Except for the above changes in masses, charges, and thus in the weak isospin and couplings $g_i^{\pm}$ $(i=1,2)$, our δ_f equals δ_f^{DS}.

δ_W.— Here, as in DS, the wave-function renormalization correction is taken from Ref. 8.

δ_v.— Again, transform masses, charges, and couplings, and in addition, change the signs of $Q_i,g_i^{\pm}$ $(i=1,2)$. Such a sign change is irrelevant in δ_f, where only squares are present.

δ_b.— In the bremsstrahlung correction, few modifications of DS are necessary. First, the three-body decay width $t(p_2)\to W(k)b(p_1)\gamma(q)$ is given by

$$\Gamma_b=\frac{1}{2}\frac{\alpha}{(2\pi)^4 m_t}\int\frac{d^3q}{2q_0}\frac{d^3p_1}{2p_{10}}\frac{d^3k}{2k_0}\delta^4(q+p_1+k-p_2)\sum_{\rm pol}|M_b|^2 . \tag{8}$$

In $\sum_{\rm pol}|M_b|^2$ [see Eq. (34) of DS] substitute our masses, charges, and G_0 [see Eq. (7) above], use

$$N_0=2k\cdot q, \quad N_1=2p_1\cdot q, \quad N_2=-2p_2\cdot q , \tag{9}$$

and finally add an overall sign change. Then δ_b is altered the same way as $\sum_{\rm pol}|M_b|^2$, with the additional change of $4M_W^2/\kappa\to 4m_t^2/w(m_t,m_b,M_W)$. The bremsstrahlung integrals are defined as in DS, but with our N_0,N_1,N_2 as in Eq. (9). Therefore, our integrals are obtained from those of DS with $m_2\to M_W$, $M_W\to m_t$, keeping all the 1 indices as 1, and changing 0 to 2 indices, and vice versa. All bremsstrahlung integrals can then be taken, after the above-mentioned transformations, from Eqs. (A.48)-(A.57) of DS, except one integral (I_{00}^{12}), which we calculate separately.

$\delta_{\rm virt,QCD}$.— The virtual QCD correction is given by Eq. (37) of DS, with $m_1=m_b$, $m_2=m_t$.

$\delta_{b,\rm QCD}$.— It is given by our δ_b as described before, after setting $Q_1=Q_2=1$, replacing α by α_s, and multiplying the result by $C_F=\frac{4}{3}$.

Before presenting our results, let us elaborate on the renormalization schemes used. In the α scheme, α, G_F, and M_Z are used as input parameters and M_W is determined by solving the implicit equation

$$M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_F}\frac{1}{1-\Delta r} , \tag{10}$$

where Δr, which depends on α, M_W and all the fermion masses, M_H, and V_{ij}, is given in Ref. 9. For processes dominated by mass scales larger than M_W, it becomes more appropriate to use the G_F scheme, where G_F, M_W, and M_Z are used as input parameters. Then,

$$\Gamma_0(G_F)=\frac{\Gamma_0(\alpha)}{1-\Delta r} . \tag{11}$$

In addition, define the first-order width as

$$\Gamma(G_F)=\frac{\Gamma(\alpha)-\Delta r\Gamma_0(\alpha)}{1-\Delta r} . \tag{12}$$

While the difference between $\Gamma_0(G_F)$ and $\Gamma_0(\alpha)$ is of first order, $\Gamma(G_F)$ as defined by Eq. (12) differs only in second order from $\Gamma(\alpha)$ and from the width calculated with G_F, M_W, and M_Z as input parameters. From Eqs. (3), (11), and (12) we find

$$\delta(G_F)=\delta(\alpha)-\Delta r . \tag{13}$$

Our numerical results are obtained with[10] M_Z $=91.177$ GeV. For the mixing angles we take the central values of the Particle Data Group table[11] except for $V_{tb}=1$ and use other masses and couplings as in DS.[12] In Fig. 1 we present the electroweak relative correction as a function of m_t, in both the α and G_F renormalization schemes. The dashed line shows $\delta_{\rm EW}(G_F)$ as calculated in Ref. 6 (which assumes $m_t\gg M_W,M_H$). This leading-order result is off, especially in the larger m_t region where it is expected to be a good approximation, and it does not display the proper dependence on m_t. In Fig. 2, $\delta_{\rm total}=\delta_{\rm EW}+\delta_{\rm QCD}$ is depicted as a function of m_t (both Figs. 1 and 2 are for $M_H=100$ GeV). For $m_t=100$, 150, 200, 250, and 300 GeV, $\delta_{\rm QCD}=-0.070$, -0.085, -0.084, -0.083, and -0.082, respectively. We observe in both $\delta_{\rm EW}(\alpha)$ and $\delta_{\rm total}(\alpha)$ a very strong m_t dependence, while $\delta_{\rm EW}(G_F)$ and $\delta_{\rm total}(G_F)$ change very mildly with m_t, especially for $m_t\gtrsim 120$ GeV. As

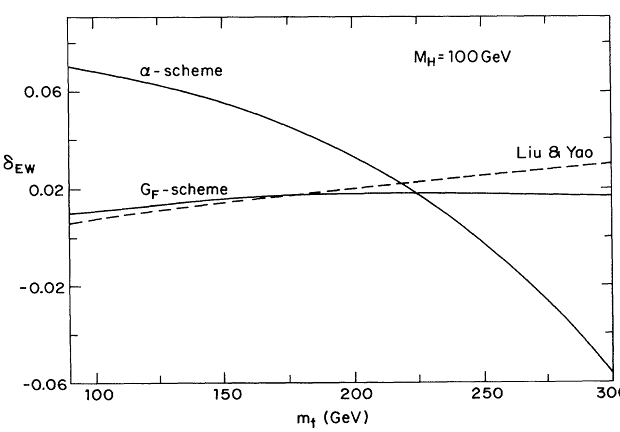

FIG. 1. Electroweak (EW) relative first-order correction as a function of m_t, in the α and G_F renormalization schemes (solid lines). Dashed line: Leading correction, as in Ref. 6. $M_H=100$ GeV.

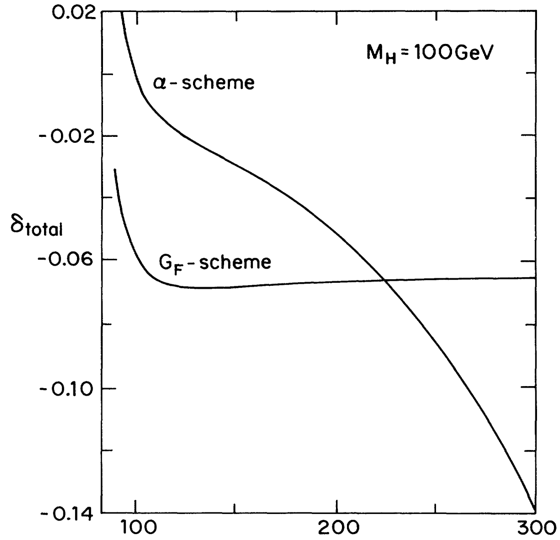

FIG. 2. Total (i.e., electroweak plus QCD) relative first-order correction as a function of m_t, in the α and G_F renormalization schemes, for $M_H = 100$ GeV.

remarked before, the G_F renormalization scheme is more appropriate for our case. In Fig. 3, the M_H dependence of δ_{EW} is displayed for $m_t = 150$ GeV. Again, the leading m_t result (dashed line) as calculated in Ref. 6 fails to reproduce the correct behavior, this time deviating significantly from the full result $[\delta_{\mathrm{EW}}(G_F)]$. From inspection of our intermediate results, it is clear why the leading m_t calculation (of Ref. 6) fails so badly. The problem is that in that reference only diagrams with Yukawa couplings are retained. However, we find that diagrams that are ignored by Ref. 6 give a non-negligible contribution even for $m_t = 300$ GeV and for (unphysical) low M_H. Furthermore, even when considering a specific diagram proportional to m_t, one cannot use reliably the leading form of the scalar functions involved.[13] Finally, in Table I, results for lowest-order and first-order widths are presented, in both schemes. One should not be surprised to observe that $\Gamma_{\mathrm{total}} < \Gamma_{\mathrm{EW}}$, since most of the first-order corrections result from interference between

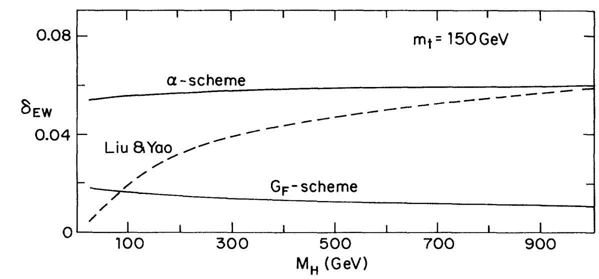

FIG. 3. Electroweak (EW) relative first-order correction as a function of M_H, in the α and G_F renormalization schemes (solid line). Dashed line: Leading correction, as in Ref. 6. $m_t = 150$ GeV.

tree and loop diagrams. We also note that while $\Gamma_0(\alpha)$ differs significantly from $\Gamma_0(G_F)$, the first-order results are almost scheme independent.

We have subjected our rather complicated calculations to many tests. We reproduce the results of DS (after proper modifications). All parts of δ_{EW} agree with the results of Ref. 8, our results are λ independent (where λ is a small photon or gluon mass) and $\epsilon = n - 4$ (where n represents the number of dimensions) independent, δ_{QCD} agrees with previous results,[5,6] and our scalar functions have been thoroughly checked and compared with other programs.

To summarize, we find that first-order corrections to the main decay of the top quark in the standard model are of the order of a few percent. They show weak dependence on m_t in the G_F scheme, and a strong m_t dependence in the α scheme. In both schemes there is almost no M_H dependence. Once the top quark is observed, radiative corrections to $t \to bW$ should be experimentally determined. It will also be very interesting to investigate the effects of scenarios beyond the standard model on the first-order radiative corrections.

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TABLE I. M_W and lowest-order and first-order widths as a function of m_t, in the α and G_F renormalization schemes. EW and total denote the electroweak and total (i.e., including QCD) contributions, respectively. $M_H = 100$ GeV. All masses and widths are in GeV.

m_t	M_W	$\Gamma_0(\alpha)$	$\Gamma_0(G_F)$	$\Gamma_{\mathrm{EW}}(\alpha)$	$\Gamma_{\mathrm{EW}}(G_F)$	$\Gamma_{\mathrm{total}}(\alpha)$	$\Gamma_{\mathrm{total}}(G_F)$
100	79.99	0.0888	0.0942	0.0949	0.0953	0.0887	0.0887
150	80.30	0.8487	0.8836	0.8958	0.8977	0.8238	0.8228
200	80.69	2.3979	2.4351	2.4761	2.4773	2.2740	2.2722
250	81.21	5.0617	4.9632	5.0494	5.0496	4.6275	4.6360
300	81.91	9.3551	8.7139	8.8231	8.8596	8.0525	8.1418

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[12]For the strong coupling constant we use $\alpha_s = 1.47/\ln(m_t/0.18)^2$ with m_t in GeV. We also note that there is no need to renormalize the mixing angles; see A. Denner and T. Sack, Nucl. Phys. **B347**, 203 (1990).

[13]As an example take the finite part F of the scalar function B_0 as defined in Ref. 8 with arguments $M_Z^2;m_t,m_t$. For $m_t \gg m_Z$, $F=1$, which is the leading-order result. However, the exact result is $F=0.21,0.50,0.78$, for $m_t = 100,300,1000$ GeV, respectively. Thus, for example, even for $m_t = 300$ GeV and $M_H = 10$ GeV the exact and leading results for $\delta_{\mathrm{EW}}(G_F)$ differ by more than a factor of 3.